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Random walks and random environments, by Barry D. Hughes, Clarendon Press, Oxford, New York. Volume 1: *Random walks*, 1995, xxi+631 pp., $95.00, ISBN 0-19-853788-3; Volume 2: *Random environments*, 1996, xxiv+526 pp., $115.00, ISBN 0-19-853789-1

There is a rich class of discrete mathematical models that appear at first glance to be simple but turn out to be deep and of profound importance both to probability and statistical physics. Hughes' ambitious pair of volumes, totalling well over a thousand pages, offers a wide-ranging introduction to two of the most fundamental of these models: random walk and percolation.

The most basic example of a random walk is the well-known simple random walk on the d-dimensional integer lattice $\mathbb{Z}^d$. In this model, a particle begins at the origin, chooses one of its $2d$ nearest neighbours at random, and then steps to it. The particle then randomly choses one of the $2d$ neighbours of its current location, and steps to that neighbour. The process continues in this way. Simple random walk and its variations form the subject of Hughes' Volume 1. In one variation, the integer lattice is replaced by another lattice, such as the triangular lattice. More dramatically, the lattice can be replaced by a fractal. Or the lattice can be dispatched entirely, with the walker taking random steps in $\mathbb{R}^d$. Discrete time can be replaced by continuous time. Inhomogeneities can be introduced into the walker's environment, or boundaries can be imposed, or traps can be set which, once visited, will end the walker's journey. All these variations are discussed at length in Chapters 1–6 of Volume 1.

Another modification of the simple random walk is for the walker to retain some memory of its past. The self-avoiding walk is an extreme example of this, in which all simple random walk paths taking n steps and having no self-intersections are assigned equal probability to occur. The self-avoiding walk is a good mathematical model of a long-chain polymer in a dilute solution. It is notoriously difficult to analyse. In Chapter 7, the final and by far the longest chapter of Volume 1, Hughes gives an account of the rigorous and nonrigorous work on self-avoiding walks, including numerical methods. The concepts of critical exponents and universality play key roles in the discussion.

Volume 2 concerns random environments. The most basic example, filling more than half of Volume 2, is percolation. In bond percolation on $\mathbb{Z}^d$, independent and identically distributed random variables are associated to the nearest-neighbour bonds $\{x, y\}$, with $x, y \in \mathbb{Z}^d$ separated by Euclidean distance 1. The random variables take the values 'open' with probability p and 'closed' with probability $1-p$, where $p \in [0, 1]$ is a control parameter representing the density of open bonds in the infinite lattice. The objects of interest are the connected clusters of open bonds. The principal fact about percolation is that, for dimensions $d \geq 2$, there is a critical value $p_c \in (0, 1)$ such that for $p < p_c$ there is with probability 1 no infinite connected cluster of open bonds, whereas for $p > p_c$ there is exactly one such infinite cluster. Chapters 1–4 of Volume 2 are devoted to percolation, with

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much of the discussion concerning work on the value of p_c and on the behaviour of the model near p_c, where again critical exponents and universality come into play.

Chapter 5 of Volume 2 concerns random resistor networks. A central example involves bond percolation, with open bonds assigned the same nonzero conductance, and closed bonds assigned zero conductance. The goal is an understanding of the effective conductance between the faces of a large box in the lattice, in the limit as the box diameter becomes infinite. Once again, critical exponents play a principal role.

The final chapters, Chapters 6 and 7 of Volume 2, are devoted to random walk in a random environment. In Chapter 6, a random tendency to move in a particular direction is assigned to each point in the lattice, once and for all, and then a walk is executed in this fixed environment. Most of the discussion is restricted to 1-dimensional walks. In Chapter 7, the lattice itself is modified in a random way, by restricting the walker to a connected cluster of a percolation model. This model, known as 'the ant in the labyrinth', closes Hughes' two volumes by uniting their two principal themes.

There is already a large literature on random walks and percolation, much of which appeared during the eleven years Hughes spent preparing his two volumes. Aspects of random walks are treated in the classic books by Feller [2], [3] and Spitzer [10], the latter now sadly out of print. More recent books on random walks include [1], [7], [8], [9], [12], [13]. The books [4], [5], [6], [11] are devoted entirely to percolation. Hughes' volumes overlap with all these books, but provide a unique perspective and an important addition to this literature. Their scope is broader than that of any one of [1], [4], [5], [6], [7], [8], [9], [10], [11], [12], [13].

The mathematical treatment is careful, and, in a work where mathematics and physics are both involved, the essential distinction is always made between what is proved and what is 'known' formally or numerically. The style is down-to-earth, and the notation of physicists is generally preferred to that of mathematicians. The mathematical background required of the reader is not extensive. Long and complicated proofs are omitted, with clear references to the literature. These features will make Hughes' volumes particularly accessible to applied scientists.

Historical references abound throughout the two volumes, both to the mathematics and physics literature. The historical descriptions are often very detailed, and there is an extensive bibliography. However, readers who want to use the bibliography to look up a vaguely remembered paper will be disappointed to find that the references are listed at the end of each chapter, rather than at the end of each volume, and there are sixteen lists of references to search.

These two impressive volumes contain a wealth of material. They will serve as a useful introduction to newcomers to the field and as a valuable reference to researchers, pure and applied, whose work touches on random walks or percolation.

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Gordon Slade
McMaster University
E-mail address: slade@math.mcmaster.ca