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James Joseph Sylvester: Life and work in letters, by Karen Hunger Parshall, Clarendon Press, Oxford, 1998, xviii + 321 pp., $98.00, ISBN 0-1985-0391-1

The British mathematician James Joseph Sylvester (1814–1897) is primarily associated with nineteenth-century invariant theory. Contemporary students of mathematics may encounter his work and inventive terminology in elimination theory (the "dialytic method"), in the theory of real quadratic forms (the "law of inertia of quadratic forms"), in partition theory ("Sylvester's Theorem"), in matrix studies (the "law of nullity"), in multidimensional algebras ("nonions"), and in the study of differential invariants ("reciprocants"), among others. Readers of mathematical biography may picture him as a volatile, often impetuous, bachelor given to bursts of oratory and poetry, whose career was fragmented by personal idiosyncracies as well as by exclusion as a Jew from honors and positions restricted to Anglicans. Historians of science who have followed the writings of Karen Hunger Parshall during the last decade may think of him as a formative force in the establishment of an American mathematical research community whose thoughts on research and teaching resonate to this day.

Sylvester received mathematical training at the University of London, the Royal Institution in Liverpool, and St. John's College, Cambridge. Although placing second in the mathematical tripos examination in 1837, he could not obtain degrees, prizes, fellowships or teaching positions at Cambridge because he could not support the required Articles of the Church of England. In 1838 he joined the faculty at University College, London. In 1841, after taking both B.A. and M.A. degrees at Trinity College in Dublin, he accepted a post at the University of Virginia. His stay there was short and, failing to gain another academic position in the United States, he returned to England in 1843. There he earned his living as an actuary and private tutor for a number of years. In 1846 he commenced law studies and was called to the bar in 1850. This period marked the beginning of his lifelong friendship with Arthur Cayley. From 1855 to 1870 he held the mathematics chair at the Royal Military Academy at Woolwich. In 1876 he answered a call to the United States to become the first professor of mathematics at the newly founded Johns Hopkins University in Baltimore. Remaining there until 1883, he completed his career as Savilian professor of geometry at Oxford University.

His earliest publications in the 1830s dealt with problems of applied mathematics and gained him admission as a Fellow of the Royal Society. They were followed

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in the 1840s by notable work on elimination theory, elementary divisors, theory of determinants, and Sturm's functions. His work in 1851 on the canonic representation of binary forms initiated a four-year period of intense research devoted to invariant theory. Beginning about 1855 he turned his attention to problems in partition theory, to which he would return periodically over the next three decades. In the 1860s he also published his proof on Newton's Rule in the theory of equations. Results on enumerative methods in invariant theory date from the 1870s, and his contributions to the theory of differential invariants were made in the 1880s. Additional scattered results involve the theory of groups, linkages, and arithmetic progressions, among others. Sylvester edited the *Quarterly Journal of Mathematics* from 1855 to 1878 and initiated the *American Journal of Mathematics* in 1878. He was elected corresponding member of most of Europe's major scientific societies, in addition to holding office in the Royal Society, the London Mathematical Society, the British Association for the Advancement of Science, and others.

Whereas this sparse chronology has been generally established since Sylvester's death, characterizations of the man and an assessment of his work have varied over the years. This is due partly to Sylvester's strong personality, partly to changing dominants in mathematics. The authors of the most influential biographical papers written about Sylvester had interacted with him personally or mathematically or were close to those who had: Arthur Cayley, who had produced a brief biographic sketch and numerous references to Sylvester, was closest to, and most tolerant of, him personally and mathematically; P. A. MacMahon, who was charged with writing the death notice for the Royal Society, would best appreciate his contributions to partition theory but had been involved in a priority dispute not long before Sylvester's death; Fabian Franklin and George B. Halsted, two of his American students, were mindful of their legacy and attested to his mark on many causes they championed. Max Noether, Gordan's colleague in Erlangen, while presenting perhaps the most interesting mathematical survey of Sylvester's work as it relates to his contemporaries and while clearly describing Sylvester's consistent attempts at algebrization and constructive generalizations, decried a lack of cohesiveness and discipline. Although Noether praised Sylvester's intuitive talent, he damned him by observing that one could not demand of anyone that they wade through the body of his work [1]. That work was published in four volumes, but after the fourth volume of Sylvester's *Collected Mathematical Papers* had appeared in 1912, with a biography compiled by H. F. Baker, anecdotes were beginning to overshadow biographic facts. Many potential students of the mathematics were put off from closer study of the papers when faced with the task of working through extensive computations of expressions for invariants known not to be error free. Taking into account the anticombinatorial, anticonstructivist, anticomputational sentiments of the next decades, it is not surprising that by 1930 Sylvester's contributions to mathematics had been substantially distorted. In 1936 three publications appeared that attempted to capture the readers' appreciation for Sylvester. David Eugene Smith, in the *Dictionary of American Biography*, reconstructed the facts of Sylvester's life as previously published. Raymond Clare Archibald attempted a factually accurate, well-documented biographic compilation, with the addition of previously unpublished materials and clarification of some of the facts surrounding Sylvester's stay in Virginia [2]. Eric Temple Bell, in *Men of Mathematics*, presented what Parshall calls "the most colorful and widely known folkloric account of Sylvester's life and

work" (p. vii). Subsequent publications did little to produce a clearer image of the man or his mathematics.

In the volume under review Parshall sets out to bring to the fore the man "behind the myth" (p. ix) by selecting a group of 140 letters from his correspondence that "reflect Sylvester's research activities, the range of his correspondents, the scope of his interests, and the nuances of his personality and mathematical persona" (p. ix). Of the 140 letters selected from among 1,200, more than 100 were written by Sylvester. The remainder are by George Boole, C. W. Borchardt, Arthur Cayley (6), Michel Chasles, Augustus De Morgan, D. C. Gilman (7), Paul Gordan, Joseph Henry, Charles Hermite, Camille Jordan, Christine Ladd-Franklin, C. S. Peirce, and George Salmon (6). Recipients include Lord Brougham (8), Cayley (37), Gilman (15), Henry (5), T. A. Hirst (9), as well as other mathematicians, scientists, university officials, journal editors, a social acquaintance, and a favorite niece. The letters range from 1837 to 1896 and are divided chronologically into six groups. Each of these six groups is preceded by an introduction which smoothly places the letters within the context of Sylvester's life during the period in question. These introductions serve as a minibiography in which Sylvester's mathematical interests and accomplishments are skillfully framed against the background of occupational, professional, literary, musical, social, and romantic pursuits. The device of using the letters enables Parshall to paint a picture of Sylvester from his own vantage point and to highlight common threads in his work over the years. There is relatively little emphasis on the opinion of his contemporaries or assessment of him by others, although many of the letters leave little doubt as to the reactions he provoked.

In contrast to other disciplines, relatively few editions of mathematicians' correspondence exist. Comparing those that appeared in Sylvester's lifetime with those of our day, we are struck with the difference in editorial standards and techniques. We respect the yeoman service rendered by nineteenth-century scholars who struggled to produce and see through the press clean transcription from a single archival source. Use of manuscript collections scattered over various parts of the globe, indexes, and explanatory notes were luxuries that only gradually became part of standard editing practice, along with explanations of typographic conventions, manuscript locations, and often a history of prior publication of a given letter. Parshall has been precise in supplying such information, although there is some inconsistency in her references to prior publication of letters. Thus, correspondence with Joseph Henry is carefully referenced to *The Papers of Joseph Henry*. At the same time there is no publication history for some other letters, such as those to W. J. C. Miller, which were published by Archibald in 1936, or one to R. F. Scott which appeared in *The Eagle*, the publication of St. John's College.

The letters are accompanied by a total of 949 footnotes and an extensive bibliography. The notes provide both historical and mathematical commentary. Almost every single individual mentioned in a letter is identified. In most instances this identification consists of dates of birth and death, occupation, major career dates and institutional affiliations, as well as references to further biographic information or primary publications. Occasionally, related biographic comments may startle the reader. For example, identification of the Poet Robert Browning includes the remark that he "was also known for his many plays" (p. 270); the relevance of this to Sylvester's acquaintance with the poet is not made clear. At times the notes

provide editorial comments, the relevance of which is equally mysterious; for example, an extensive identification of Francis Galton contains the remark that his book *Hereditary Genius* "established him, for better or for worse, as the founder of eugenics" (p. 266). The ample nonbiographic historical notes are likewise generous in providing peripheral information. An 1869 letter by Joseph Henry remarking on changes in north and south and the growth of the national debt leads to a reminder of the dates of the American Civil War, to a reference work concerning the period of Reconstruction, and to the precise rise of the national debt from 1860 to 1866!

The mathematical notes contain definitions of terms not familiar to the contemporary mathematician, succinct paraphrases in current terminology of selected formulations by Sylvester, and explanations of references to certain problems or theorems. Especially in the area of invariant theory Sylvester's contributions are set in the context of the work of his contemporaries. There are lucid explanations of the relationship between Cayley, Sylvester, Gordan, and Hilbert, particularly with regard to Gordan's finiteness theorem.

Some readers may be put off by an occasional tendency to underline the obvious, but such instances of force-feeding are small compared to the quantity and variety of supplementary information provided.

The 26-page bibliography contains a listing of all references appearing in the notes. Work by contemporary historians of science tends to take the place of older references. Not duplicated from the notes are cross-references from his cited papers to their location in Sylvester's *Collected Mathematical Papers*. Since these are arranged chronologically, it would have been useful to add the date range of each of the four volumes to the entry of the CMP.

If, as is assumed, this book is a prolegomenon to a full-length biography of Sylvester, the ample notes and bibliographic apparatus give us a foretaste of a volume that promises to present an intriguing panorama of the environment in which Sylvester created his mathematical world. In this present volume, Parshall has given us a convincing interpretation of Sylvester's life and work. Thanks to the choice of letters and the extent of the explanations provided, the book is intellectually accessible to a wide audience. The attractive production, the lack of abbreviations or shortened citations in the notes, the editorial comments, and Parshall's clean writing style all combine to present the reader with a convenient tour through various segments of nineteenth-century history. Unfortunately, these same attributes may limit the number of readers who will have access to the volume. The price presumably will deter from purchasing the book students and others who could most benefit from leisurely reading. One wonders whether reducing the number of peripheral notes might have kept down costs and ensured that the reader wandering through the mirrors of commentary did not lose sight of the valuable image of Sylvester that Parshall has provided.

References

[1] Max Noether, "James Joseph Sylvester", *Mathematische Annalen* **50** (1898), 133–156.

[2] Raymond Clare Archibald, "Unpublished letters of James Joseph Sylvester and other new information concerning his life and work", *Osiris* **1** (1936), 85–154.

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