

*Mathematics of medical imaging*, by Charles L. Epstein, Prentice Hall, Upper Saddle River, NJ, 2003, 739 pp., \$92.00, ISBN 0-13-067548-2

*Inside out: Inverse problems and applications*, Gunther Uhlmann (Editor), MSRI Publications, vol. 47, Cambridge Univ. Press, Cambridge, 2004, xii + 400 pp., £50.00, ISBN 0-521-82469-9

Medical imaging became a mathematical discipline with the advent of computerized tomography in the late seventies. Simple mathematical operations such as smoothing and other techniques of image enhancement have always been used in medical imaging, but the degree of mathematical sophistication computerized tomography introduced in medical imaging was unheard of at that time.

At the beginning the mathematics of medical imaging was all integral geometry, with a bit of well established methods of image processing. Integral geometry (in the sense of Gelfand, as opposed to integral geometry in the sense of Blaschke and Santalo) has a great tradition in Russia. So it's no surprise that much of the mathematics of medical imaging used in the seventies was anticipated by Russian mathematicians such as I.M. Gelfand, S. Gindikin, V. P. Palamodov, V.G. Romanov, A.B. Goncharov, S.S. Orlov and many others. In the Western world the schools around K.T. Smith, S. Helgason and G.T. Herman developed the relevant aspects of integral geometry including the practical aspects. It was discovered that the workhorse of clinical imaging, the filtered backprojection algorithm, which was found by the electrical engineers G.N. Lakshminarayanan and A.V. Ramachandran, was just a computer implementation of Radon's 1917 inversion formula

$$f = \frac{1}{4\pi} R^* H g, \quad g = R f$$

for the Radon transform

$$(Rf)(\theta, s) = \int_{x \cdot \theta = s} f(x) dx, \quad \theta \in S^1, \quad s \in \mathbb{R}.$$

Here,  $H$  is the Hilbert transform and  $R^*$  is the backprojection

$$(R^*g)(x) = \int_{S^1} g(\theta, x \cdot \theta) d\theta.$$

Subsequently iterative reconstruction techniques have been developed, bridging the gap between integral geometry and numerical linear algebra. The prime example of the iterative methods is ART (algebraic reconstruction technique). ART is an extension of the 1937 Kaczmarz algorithm for linear systems to the equations

$$g_j = R_j f, \quad j = 1, \dots, p$$

where  $R_j$  is the part of the Radon transform corresponding to direction  $\theta_j$ . Starting out from an initial approximation  $f_0$ , ART computes approximations  $f_j$  to  $f$  by

$$f_j = f_{j-1} + w R_j^*(g_j - R_j f_{j-1}), \quad j = 1, 2, \dots$$

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Here the subscripts of  $R^*$ ,  $R$  and  $g$  have to be taken mod  $p$ . Originally ART had the reputation of being very slow, but mathematical advances such as Hamaker and Solmon [5] led to substantial improvements. A multiplicative version of ART, OSEM (ordered subsets expectation maximization) became very popular in emission tomography. The excitement of these early days of tomography can be seen in the paper Smith et al. [2], the paper [4] of Shepp and Kruskal, and Gabor Herman's classic [3].

Epstein's book is about this early stage of the mathematics of medical imaging. It's a perfect introduction for students who want to learn the basic techniques with mathematical rigor and in a mathematical language. This rigor has its price: About one half of the more than 700 pages is dedicated to material (calculus, Fourier analysis, sampling, probability, functional analysis, convolutions, filtering) that is part of a general mathematical curriculum. In spite of the length of the book, it is restricted to the most elementary aspects. MRI is treated on one half a page. Truly 3D imaging, a field well worth the effort of trained mathematicians, is dealt with on three pages, discussing rebinning in spiral CT. The treatment of iterative algorithms is restricted to the basics. Practically important aspects such as speed of convergence of ART and EM (referred to as the maximum likelihood algorithm in the book) are missing. This is in agreement with the goals of the book as described in the Preface: To be an introduction to the mathematics of medical imaging for undergraduates. The book is admirably clear and does a good job in motivating the reader. After having worked his way through these 700 pages, the student has reached the state of the art of the late seventies. He is now fit for more advanced topics.

In the eighties the mathematics of medical imaging developed into a much more general and demanding theory. It was discovered that the typical questions are in no way restricted to radiology. The same problems were coming up in distant fields, such as seismics, radar, nondestructive testing, remote sensing, astronomy, electron microscopy - in fact almost every scientific or technological field had imaging problems of its own that led to interesting mathematical problems. The word "tomography" was extended to a variety of imaging techniques, some of them only remotely reminiscent of the straight line paradigm of X-ray tomography. A suitable framework turned out to be inverse problems of partial differential equations. For instance, the Boltzmann equation turned out to be a common theoretical frame for transmission, emission and optical tomography; see Barrett and Myers [1]. On the algorithmic side things became naturally much more complicated, but the structure of the filtered backprojection algorithm and of Kaczmarz's method is still visible. Thus studying classical tomography is still helpful if not mandatory.

The volume edited by Gunther Uhlmann gives an account of this development. Less than one third of the contributions are concerned with integral geometry. The survey article of Adel Faridani gives an excellent introduction into the more advanced topics in integral geometry, covering topics such as optimal sampling, truly 3D reconstruction, and local tomography. The article of David Finch is dedicated to the latest development in the theory of the attenuated Radon transform, after the spectacular recent advances by R. Novikov and Bukgheim. The article of Finch, Lan and Uhlmann on the microlocal analysis of the X-ray transform is a nice example of the power of advanced mathematical tools, such as microlocal analysis, in

imaging. It's fair to say that the systematic study of microlocal analysis in imaging started in medical imaging, even though some microlocal aspects were present in seismic imaging long ago; see the article of Maarten V. de Hoop in the book. The thinking in terms of microlocal analysis seems to be one of the most fruitful contributions of mathematics to imaging, be it in radiology, seismics, or radar. As already mentioned above, the transport equation covers various branches of imaging. The article of Plamen Stefanov on inverse transport theory is a good example of how mathematics helps to unify seemingly different phenomena. The articles by David Colton, Petri Ola, Lassi Päivärinta, Erkki Somersalo, Claude Bardos, Vesselin Petkov and Luchezar Stohanov are about various aspects of imaging based on wave propagation. In all these works it's good to have an intimate knowledge of integral geometry, since this is what's lurking behind the rather complicated and demanding mathematics. Finally, the article of P. Scott Carney and John C. Schotland on near-field tomography gives an outlook on an imaging technique that makes use of evanescent waves, achieving - at least in the linearized case - resolution much better than the wavelength. The article of Andras Vasy on many-body scattering illuminates another universal aspect of imaging, namely the propagation of singularities. It's safe to say that Uhlmann's book is a fingerpost in mathematical imaging for some time to come.

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F. NATTERER

INSTITUT FÜR NUMERISCHE UND INSTRUMENTELLE MATHEMATIK

*E-mail address:* `nattere@math.uni-muenster.de`