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Probability on graphs: random processes on graphs and lattices, by Geoffrey Grimmett, Institute of Mathematical Statistics Textbooks, Vol. 1, Cambridge University Press, Cambridge, 2010, xii+247 pp., ISBN 978-0-521-14735-4, paperback, US$38.99

Growth of the field of mathematics called *Interacting Particle Systems* (IPS) was, to a great extent, catalyzed by Liggett's 1985 monograph [12]. Thirty years later this IPS field (together with the closely related *percolation* field) remains an active area within mathematical probability and the theorem-proof end of statistical physics, with its MSC primary classification 60K35 containing about 170 papers a year. In brief, IPS studies models over a graph: each vertex (or *site*) is in some state; vertices interact with neighboring vertices at random intervals and update their states according to some rule. For instance in the *contact process*, a natural toy model for epidemics, a vertex is either *infected* or *healthy*: healthy sites become infected at rate λ times the number of infected neighbors, and infected sites become healthy at constant rate one (*rate* means probability per unit time). Liggett's book gave a masterful account of work on several models over the previous 15 years, for which a major original motivation had been rigorous study of the statistical physics notion of *phase transitions*, formalized as qualitative changes in behavior of a process defined on the infinite d-dimensional lattice as parameters pass through a critical value. For instance in the contact process, the epidemic will die out if λ is smaller than the critical value, but has a chance to survive forever if λ is larger.

Not only has IPS remained an active field within statistical physics and mathematical probability, but similar models have been used in a surprising range of other quantitative disciplines. To cite just two recent graduate texts, [14] treats models relevant to topics within computational complexity (satisfiability problems) and information theory (error-correcting codes); and [7] shows how continuum analogs of percolation models are fundamental to mathematical study of wireless network communication. A more familiar context is provided by *social networks*. Whatever you think a social network does, it involves some notion of exchange of information, and toy mathematical models are inevitably somewhat similar to the models from IPS (for instance there are many variants of the *voter model* for how your opinions are affected by your friends' opinions). See [3] for a wide-ranging overview of such literature, and [1] for novel toy models in the spirit of IPS. In all these broader disciplines the appropriate underlying "geometry" (described by the graph) is context-dependent, not the 3-dimensional lattice used classically in physics models for phenomena such as ferromagnetism.

Writing a subsequent book which resembled Liggett's in being both introductory and comprehensive seems quite impractical, not only because of the size of the field but also because it would need to duplicate much of Liggett's material. So the subsequent book literature consists of a small set of lecture notes and specialized monographs, such as [4, 6, 8, 9, 13]. This makes it unfortunately difficult for a newcomer to know how to get started in the field—how to start learning the techniques, models and results behind today's research frontier. The book under review serves

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admirably for this "getting started" purpose. It provides a rigorous introduction to a broad range of topics centered on the percolation-IPS field discussed above. The prefatory sentence "based in part on courses [for] Part III of the Mathematical Tripos at Cambridge University" will provide some readers with a clue as to its style. Part III corresponds roughly to the first year of a U.S. mathematics Ph.D. program, but is taught in a somewhat different style. This book, like a typical Part III course, requires only undergraduate background knowledge but assumes a higher level of general mathematical sophistication. It also requires active engagement by the reader. As I often tell students, "Mathematics is not a spectator sport—you learn by actually *doing* the exercises!"

For the reader who is willing to engage the material and is not fazed by the fact that some proofs are only outlined or are omitted, this style enables the author to cover a lot of ground in 247 pages. I can best illustrate the coverage by mentioning a highlight from each chapter: Pólya's theorem (recurrence of simple random walk) via electric networks; brief introduction to the Schramm–Löwner evolution as the limit of loop-erased random walk; existence of a critical point for two-dimensional oriented percolation; the influence theorem for product measures; uniqueness of the infinite cluster in supercritical percolation; bounds for the critical value of the contact process on trees and lattices; the equivalence of Markov random fields and Gibbs random fields; existence of the infinite-volume random cluster model; the quantum Ising model and its connection to random-cluster models; introduction to other IPS models; size of the giant component in the Erdős–Rényi random graph; the square Lorentz gas. There are many brief references to hard research results, so in places it reads more like a survey article than a textbook.

This is indeed a lot of ground, so inevitably the coverage will seem patchy to experts, but for almost all these topics there are specialized monographs available for further reading. Everyone in the field will have their own taste regarding other topics they would have liked to have been included. To my own taste, while the book treats both the finite graph and the infinite graph cases, it tends to present results in finite settings as ingredients for studying the infinite case, instead of studying the finite cases in their own right, by analogy with the modern literature on mixing and hitting times for finite Markov chains [2, 11]. The Facebook Friends graph has a billion vertices, but it's just not helpful to treat it as a step on a road to some infinite graph.

The book's selection of material spotlights a much broader issue that concerns many probabilists like myself. Over the last 30 years the number and breadth of fields involving (mathematically sophisticated) probability theory has expanded dramatically. But the standard first-year graduate course in probability has changed little, still focussing on topics like laws of large numbers, central limit theorems, countable Markov chains, martingales, and Brownian motion [5]. No one could object to these topics, in that any researcher in probability should become familiar with them; on the other hand they are not such direct prequels to current research as they were 30 years ago. To assess how the range of this book relates to current research, I looked at the themes of the 17 invited sessions at the 2013 Stochastic Processes and Their Applications conference. Four of those themes are directly addressed in this book (IPS, random graphs, self-avoiding walk, Schramm–Löwner evolutions) and another two are touched upon (mixing rates for Markov chains, random combinatorial structures). This surely indicates that the book covers as broad a range as any book at this length and level could cover. In my eyes the ideal

curriculum for probability at the first-year-graduate level would require two year-long courses: one year to cover the standard classical material of [5], half a year on a range of "modern discrete probability" topics of the kind in the book under review, and a final half year to cover stochastic calculus, diffusions and finance in the style of [10].

Cambridge University Press maintains its traditional high quality of typography, though some bizarre bug has caused the index in my copy to point to nonexistent pages like 1912 (the fix: delete the final digit).

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