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ABOUT THE COVER: TWO THEOREMS ON GEOMETRIC CONSTRUCTIONS

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Jean-Victor Poncelet (1788–1867) was a professional engineer and sometimes mathematician who is probably best known for reviving an interest in projective geometry which had long languished since the golden age of the subject in the 17th century, the time of Pascal and Desargues. As we see in the extensive article by Dragović and Radnović in this issue, Poncelet's provocative "porism" theorem has prompted much subsequent work and remains a startling example of a theorem where if a single solution exists, then an infinite number exist [6]. With the wonders of modern technology we can view this phenomenon in animated form on Wolfram Mathworld (`http://mathworld.wolfram.com/PonceletsPorism.html`).

There is however another theorem in geometry that bears Poncelet's name, the Poncelet–Steiner theorem, that answers, in part, a question anyone might raise when studying Euclidean constructions in a high school course: Can a construction by straightedge and compass be carried out without one of the two instruments—in the first case, without the straightedge? It is understood that the straightedge is just that. It has no markings on it and the desired objects are points, so one can determine a line without actually drawing it. The lines are determined only by sets of points. The first widely available answer to this question was given by Lorenzo Mascheroni (1750–1800), who, in his *Geometria del Compasso* of 1797, proved that any construction with the above conditions can be carried out by compass alone [4]. And that is where the problem stood until 1928 when a student browsing in a rack of books in a Copenhagen bookshop found a small book by Georg (Jørgen) Mohr, an obscure Danish mathematician. It was called *Euclides Danicus* and was published in 1672 [5]. It contained a proof of what was then called Mascheroni's theorem. The contents of this volume had remained totally unknown. Mohr had given the book a Latin title but wrote the text in Danish and, as it turns out, also in Dutch for a simultaneous edition that was published in Amsterdam. (Mohr lived for much of his life in Holland.) Neither edition had any impact on the mathematical

EUCLIDES DANICUS,

Beſtaende in twee Deelen:

Het eerſte Deel: Handeldt van de Meetkonſtige Werckſtucken, begrepen in de ſes eerſte Boecken *Euclidis*:

Het tweede Deel: Geeft aenleyding om verſcheyde Werckſtucken te maecken, als van Snijding, Raecking, Deeling, Perſpective en Sonnewijſers:

Alleenig met een *Paſſer* te wercken (ſonder Rye ofte Liniael te gebruycken) door Snijding van Ronden.

Voorgeſtelt

Door

GEORG MOHR.

t'AMSTERDAM,

Gedruckt by *Iacob van Velſen*, voor de *Autheur*, welcke geen Exemplaren voor de zijne erkent, als die hy ſelver met zijn handt onderteeckent heeft; 1672.

Figure 1. Clear copy of the cover. The title page of the Amsterdam issue of Mohr's book. The flourish under Mohr's signature follows a long Spanish tradition of certifying an author's signature on documents, possibly a holdover from the Spanish Habsburgs' influence in the Low Countries that lasted into the early 18th century.

world at all. There is a lesson here for all of us. In the 17th century the scientific *lingua franca* was clearly Latin and anyone interested in Mohr's result would have been able to read the text in Latin and recognize what he had done. But in Danish or Dutch? It didn't happen. Subsequently, of course, the theorem has been renamed the Mohr–Mascheroni theorem. After Mohr's work was discovered, a facsimile was published in 1928 and a year later there was a translation into German, which at that time was probably the *lingua franca* of mathematics, that or French. Today all of these languages have been replaced by English. We observe this when we track the languages permitted in presentations at International Congresses of Mathematicians: Italian was once fairly common but disappeared long ago, only to be replaced for a short time by Russian in the mid-20th century. German and French survived until much later but have now practically disappeared completely from congress proceedings. We realize that French is doomed as an international language when even French diplomats negotiate today in English, when for centuries, French was the standard for international diplomacy. The next question is, What will eventually replace English? Perhaps we should return to Latin.

Word of the discovery of Mohr's book traveled quickly, and by 1929 there was an enthusiastic report on its contents by the eminent Berkeley historian, Florian Cajori, who also reported on Mohr's contacts with Leibniz, whom Mohr met in 1676 [2]. In the year following, the geometer N. A. Court reviewed Mohr's book in the *Bulletin of the American Mathematical Society* and remarked that "the typography of the book is excellent", something we can see from its title page [3] (see Figure 1).

In 1822 Poncelet asked the other question: Could one dispense with the compass and do constructions with straightedge alone? That seems unlikely—somehow one needs to be able to transfer distances in a construction, though already it was known that a collapsing compass would suffice (see Figure 2). Poncelet conjectured that one could do without the compass if one has a circle somewhere in the plane and one knows the center of the circle. And it need be available only once and then can be disposed of. That is a most surprising result, and it was perhaps not surprising that Poncelet had trouble proving it. A convincing proof had to await the work of the Swiss geometer, Jakob Steiner (1796–1863), eleven years later [7]. Of course there are many variations on this theme subsequent to Steiner's proof. For example, one need not have the full circle; an arc of a circle would do. But then how much of the arc is needed? There's more. Cajori summarized these later results by pointing out that "all Euclidean constructions can be made with any one of the four ordinary instruments of geometric construction taken singly, viz., the compasses, or the ruler with parallel straight edges, or the ruler with a right angle, or the ruler with an acute angle." The word "ruler" usually implies a straightedge with markings. According to Cajori, these exercises had earlier attracted the likes of artist Albrecht Dürer and mathematician and engineer Niccolò Tartaglia. The parallel between the work of Poncelet and Steiner is interesting. Steiner had his own porism theorem: if there is a closed chain of circles that are tangent to two given nonintersecting circles, then there are infinitely many such chains.

Any bibliophile has to be inspired by the discovery of the Mohr volume in a stack of "used books", containing a geometrical theorem that would not become public for another 125 years. Needless to say, copies of Mohr's book are exceedingly scarce. In 2005 a copy of the original (accompanied by the 1928 facsimile) appeared in the catalogue of a book auction house in San Francisco. A Bay Area collector

FIGURE 2. The illustration from the title page of the second volume, Mohr's *Compendium Euclidis Curiosi* of 1673, which demonstrated that "one single opening of the compasses and a straight ruler suffice for all the propositions of Euclid's first five books." But doubt was raised about the authorship of this book in [1].

acquired it for the ridiculously low price of roughly $13,000, including the premium on hammer price. The title page of that copy shown in Figure 1 also has the signature, "G. Mohr", in the lower right corner, followed by an elaborate flourish. The before-auction estimate had been $400–$700. The experts probably did not know exactly what they had. A search turns up only one other copy in America, at the Harry Ransom Center at the University of Texas, Austin. Two copies exist in university libraries in Holland (Amsterdam and Leiden), two national libraries in Germany (Saxony and Bavaria), a copy in Aarhus, and another at Christ Church, Oxford. It is much cheaper and easier to acquire a first edition of Poncelet's most widely admired work, the *Traité* of 1822. Coincidentally, as I write this, I received a message from a Massachusetts dealer who has a copy of the 1822 Poncelet available for a mere $1,250.

A wistful echo of Mohr's experience appears in the Dragović–Radnović paper where the authors point out that "in 1870 Darboux proved the generalization of the Poncelet [Porism] Theorem for a billiard within an ellipsoid in... three-dimensional space. It seems that his work on this topic was completely forgotten until very recently."

Acknowledgments

I am grateful to Ellen Heffelfinger for her diligence in tracking down the location of copies of the 1672 editions of Mohr's book and for checking some of the details included here. And, for allowing the reproduction of Mohr's title page for the cover, I am grateful to its owner, Fry's Electronics.

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