



# BULLETIN

(NEW SERIES)

*of the*

AMERICAN MATHEMATICAL SOCIETY

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Susan Friedlander, *Chief Editor*

*Book Reviews*

Volodymyr Nekrashevych

VOLUME 60

NUMBER 4

OCTOBER 2023

*Available electronically at*  
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## Bulletin (New Series) of the American Mathematical Society

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*Bulletin (New Series) of the American Mathematical Society* (ISSN 0273-0979 (print); ISSN 1088-9485 (online)) is published quarterly by the American Mathematical Society at 201 Charles Street, Providence, RI 02904-2213 USA. Periodicals postage is paid at Providence, Rhode Island, and additional mailing offices. POSTMASTER: Send address changes to Bulletin (New Series) of the American Mathematical Society, American Mathematical Society, 201 Charles Street, Providence, RI 02904-2213 USA.

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Printed in the United States of America.

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Courtesy of Akram Aldroubi

Sir Vaughan F. R. Jones (1952–2020)

**Honors and Awards include**

Fields Medal (1990)

Fellow of the Royal Society (1990)

Rutherford Medal of the Royal Society of New Zealand (1991)

Fellow of the Australian Academy of Sciences (1992)

Fellow of the American Academy of Arts and Sciences (1993)

Fellow of the US National Academy of Sciences (1999)

Distinguished Companion of the New Zealand Order of Merit (2002)

Knight Companion of the New Zealand Order of Merit (2009)

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PROVIDENCE, RHODE ISLAND USA

ISSN 0273-0979

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0273-0979(202310)60-4\*;1-2

