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The AMS was founded in 1888

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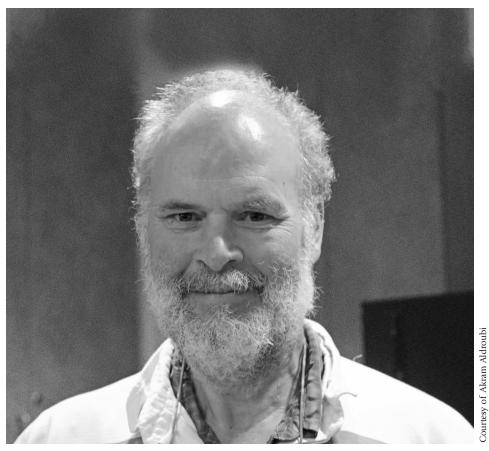
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Sir Vaughan F. R. Jones (1952–2020)

Honors and Awards include

Fields Medal (1990)

Fellow of the Royal Society (1990)

Rutherford Medal of the Royal Society of New Zealand (1991)

Fellow of the Australian Academy of Sciences (1992)

Fellow of the American Academy of Arts and Sciences (1993)

Fellow of the US National Academy of Sciences (1999)

Distinguished Companion of the New Zealand Order of Merit (2002)

Knight Companion of the New Zealand Order of Merit (2009)

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