BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 61, Number 2, April 2024, Pages 331–342
https://doi.org/10.1090/bull/1825
Article electronically published on February 15, 2024

AUTOMATION COMPELS MATHEMATICIANS TO REFLECT ON OUR VALUES

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Abstract. The author takes colleagues to task for failing to "live deliberately," and specifically for failing to pay attention to debates over technology and artificial intelligence in the wider society, when contemplating a mechanical future for mathematics.

We're just a biological speculation
Sittin' here, vibratin'
And we don't know what we're vibratin' about
— George Clinton, *Funkadelic*, 1972 [7]

1. Entering the woods

The Venkatesh essay provides a unique opportunity to reflect on our values. Toward the end of his talk at the 2022 Fields Medal Symposium which gave rise to this volume of *Bulletin of the AMS*, Jeremy Avigad found a helpful way to look at our current assignment in Thoreau's *Walden* [24]. Thoreau

> *went to the woods because [he] wished to live deliberately, to front only the essential facts of life...*

What would it mean for mathematics to "live deliberately" in the face of the existential challenge posed by artificial intelligence (AI)? At a minimum, living deliberately[1] entails knowing where we, as a community, want to go; and this in turn entails knowing what mathematics is for, "what we're vibratin' about."

A necessary first step is to free ourselves from the grip of misleading metaphors. The "intelligence" part of AI, for example, can only be a metaphor, a particularly tenacious one, since there is no agreed definition that embodies everything that may go by the name of "intelligence." The consequence is that discussion tends to be dominated by the definitions promoted most aggressively—not, coincidentally, those preferred by the industry that stands to profit from turning "intelligence" into something monetizable. But "community" is also a metaphor: it's more accurate to describe mathematics as a landscape of overlapping villages. Geometers, logicians, analysts, and applied mathematicians differ in the potential reasons they may welcome AI, or be curious, or be indifferent, or feel threatened. Certainly we don't all expect to be "vibratin' " about the same thing.

Received by the editors July 30, 2023.
2020 *Mathematics Subject Classification.* Primary 00A99, 68T99.

[1]I intend to try the reader's patience by squeezing as much as I can out of Thoreau's expression; it's really the most apt description I know of what the profession needs at this turning point (if it is one), as much for the character of the person who wrote it as for what it says.

A second step, one that is apparently particularly challenging for mathematicians, is to recognize that other disciplines have much to teach us. The Columbia library catalogue lists 4,628 books with "automation" in the title. Even accounting for duplication this is a substantial literature that reflects a vast variety of approaches to the topic, some of which are likely to be directly relevant to mathematics, even though the books are written from the standpoint of history, economics, political science, sociology, and even the humanities.

When mathematicians talk about automated proof verification, automated theorem proving, or the other challenges or opportunities to be expected from information technology, there is no acknowledgment of this vast literature [20].[2] The discussion instead tends to be limited to the sort of comments that economist Robert J. Shiller cites in what he calls the (viral) narrative "Automation and artificial intelligence replace almost all jobs"[3]—in other words, the pop culture version of technological determinism, which is also pervasive in Silicon Valley [21]. I have no objection to quoting pop culture; George Clinton, for example, quoted above, is as astute an observer of the contemporary condition as any academic. But living "deliberately" should entail seeking out and engaging with any coherent thoughts that may shed light on the most urgent questions about our future.

Venkatesh's essay marks a break: it is the first explicit call for mathematicians to examine the implications of AI for what we value most in the practice of our profession and to take action to defend our acknowledged values. Among other benefits, it provides an opportunity to *agree on what these values are*—Venkatesh helpfully points out that these values are constructed, and indicates a few "mechanisms" that contribute to this construction—and to frame definitions of *intelligence* suited to our own priorities. Like most of the other authors represented in this issue, I intended my text to be a response to Venkatesh's call. I will not, however, respond directly to the explicit questions that inform his essay. Instead, my text is a lengthy detour through some ways of thinking that are not often practiced in connection with mathematics but that might help us understand our shared values.

2. Crisis of vocation, crisis of institutions

The kind of reflection described here is extremely common in the literary disciplines and social sciences, and is related to a perceived legitimacy crisis of the institutions in which we work.

> *Both insofar as the mind has clear and distinct ideas, and insofar as it has confused ideas, it strives, for an indefinite duration, to persevere in its being and it is conscious of this striving [conatus] it has.*[4]
>
> — Spinoza, *Ethics*, Part III, Proposition 9 [17]

You might think that a profession's very first steps, upon pledging to live deliberately, would be to clarify the nature of the "being" in which it "strives"—its

[2]The bibliography in Marcus du Sautoy's *The Creativity Code*, which "wonder[s], with the onslaught of new developments in AI, if the job of mathematician will still be available to humans in decades to come," includes exactly one item with "automation" in the title.

[3]Robert J. Shiller, *Narrative Economics*, Chapter 14. One contributor to this volume once wondered in public whether I might be a Luddite—a character of the narrative treated in Shiller's Chapter 13.

[4]Translation from *Stanford Encyclopedia of Philosophy*.

Spinozian *conatus*—to persevere; and, having done that, to reflect on the material conditions without which its striving is futile. For the profession of mathematics, the *conatus* is largely concerned with the values highlighted in Venkatesh's essay. It is exceedingly rare, however, for mathematicians to undertake such a reflection. The most recent such reflection that comes to mind is the exchange in response to the 1993 Jaffe and Quinn article on "Theoretical Mathematics" [13]. I'm sure one reason these texts—especially Thurston's article "On Proof and Progress in Mathematics"—are so widely cited is that such wide-ranging discussions among mathematicians are so infrequent.

As for the material conditions, they are primarily provided by the research university,[5] an institution that only relatively recently (compared to the history of mathematics) adopted its present form, and whose fragile balance of conflicting priorities has been evident for my entire career (see [4]). I know of no serious attempt on the part of mathematicians to come to grips with the deep instability of the model of higher education that provides our livelihood. To grasp the scale of the endlessly postponed crisis, we need to leave the departmental silo and cross the campus to meet our colleagues in the humanities and social sciences. It's only a slight exaggeration to say that for years they have been speaking of nothing else than this vocational crisis.

The crisis in higher education as a whole is reflected in contradictory metaphors. Is higher education itself an *industry*, that "shapes who our future leaders and builders will be"?[6] Or is it a *public good* [23]? Is it primarily the means by which its self-reproducing elites (like the one described at greater length below) reproduce themselves? Or is it the primary locus of the debate in which the contours of this and other elites are questioned?

Material conditions for disciplinary self-reproduction are already in crisis at institutions like mine. Here is how Columbia administrators responded recently to faculty questions about the decision to reduce graduate admissions, particularly in humanities and social sciences (my emphasis):

> *. . . admissions numbers set decades ago were in many cases not reflective of the current realities of our disciplines.*
>
> *The median figure for that hope [to pursue an academic career] was 89%; and yet the median figure for the actual academic placements for those same departments is 58%. What do we tell our students about their unattainable futures?* ***Our pedagogical dependence on our PhD students*** *cannot take precedence over their desires and hopes for their own professional lives. . . all peer graduate schools have already rethought or are currently rethinking the number of PhDs produced, as well as their responsibilities to those students.*

Mathematicians don't need to be reminded of our departments' "pedagogical dependence" on graduate students. Discussions of their "unattainable futures,"

[5]At some point the population of mathematicians beyond academia will have to be included in this deliberate living. The National Security Agency (NSA), for example, home to "the largest collection of mathematicians and linguists in the country and possibly the world," according to James Bamford's Body of Secrets, published in 2001, is entirely absent from this reflection.

[6]From an article by Anne Lee Skates [22], on the Andreessen–Horowitz website, in which we learn that "Education is a 1.2 trillion dollar industry" and "It's easier than ever to start a Learning Economy company."

when they do occur, mainly take the form of reminders that mathematics PhDs have many options, often lucrative, outside the academy, more, in any case, than those in many other fields. I have never seen their "desires and hopes" placed at the center of such a discussion—nothing like this:

> *In society as we know it, it is a rare privilege to be offered the conviction that a meaningful vocation awaits you. A meaningful vocation doesn't await everyone. And most people know it.* (Robbins [19, p. 40]).

Venkatesh acknowledges that if automation does provoke a crisis in mathematics—which I think is likely, given the balance of forces—it will be, in the first place, a crisis of meaning. This understanding is rare in mathematics but it is commonplace for scholars in the humanities. (The *Chronicle of Higher Education* has just published a collection of essays [15], its second in three years, on the "collapse of the discipline" of literary studies.) The above quotation is taken from one of two new books, by colleagues in Columbia's English and Architecture departments, that I read last fall. Both books, unsurprisingly, are exercises in disciplinary deliberate living. Reinhold Martin's *Knowledge Worlds* [16] treats the material basis for the university in the broadest possible terms, as

> *. . . a world of gates, screens, departments, papers, reports, and other media,*

and emphasizes how each of these factors, and the architectural structure of the campus, is an expression of the values of the social forces behind the university's creation [16, p. 7].[7] Bruce Robbins's *Criticism and Politics* deals with his discipline's material existence more directly.

> *Always eager to slash public budgets and make education a matter of private investment in consumer preference, public decision-makers might decide they can dispense with such an esoteric and unprofitable branch of knowledge production.*
>
> *The fact that market relations have tightened their grip on both [journalism and literary criticism] since the eighteenth century. . . provok[e] both journalists and academics to recall or invent a more independent past. . . [But] [t]here was no earlier autonomy. The critics/journalists depended on the market. They were never truly autonomous.* (Martin [16, p. 82] and [16, p. 90])

I quote these texts because the disciplines housed on the other side of campus routinely practice self-examination on a scale that mathematicians only encounter when faced with extraordinary challenges. If we learn from our colleagues in the humanities and social sciences we may be better prepared to face the challenge of automation. It's true that literary scholars have for quite some time been made aware of the material conditions of their discipline by its very subject matter. Here, for example, is the illiterate publisher Dauriat, in Balzac's *Lost Illusions*, published around 1840 [3]:

[7]Reinhold Martin, *Knowledge Worlds*, On pp. 62–63 of this book, for example, I learned something that every American mathematician deserves to know—that chalk and blackboard made their first appearance at an American campus at the US Military Academy at West Point, as an introduction of French methods of training future officers in the basics of geometry.

> *. . . from this day forward. . . If anybody comes here with manuscripts . . . ask him whether it is poetry or prose; and if he says poetry, show him the door at once. Verses mean reverses in the booktrade.*

Disciplinary values remain central when our literary colleagues address the implications of the new technology, even when they are not talking about crises. The document announcing an upcoming conference on **AI-technology as interactional human culture** [1] lists the following "Research Questions," among others, as topics for the conference:

- How do social values and cultural traditions, among them beliefs about machines, commercial interests, technological affordances and notions of language, frame the development of AI technology?
- How do traditions of writing, established language norms, the dominance of English and people's beliefs about language shape the programming of speech-enabled AI or translation technologies?
- And, finally, what do we learn from all this with regards to the question what constitutes democratic, culturally sensitive and human-centred AI?

All of these questions, suitably adapted, are relevant to mathematics as well. Is the debate on AI within mathematics concerned with "Language, Data Practice, and Social Struggle"—the conference's subtitle? If not, why not?

Venkatesh's analysis shares an attention to value with the above list of "Research Questions," and with the torrent of articles that have appeared in practically every venue,[8] especially since the release of ChatGPT, about the implications of AI for one discipline or another. The only visible reaction of mathematicians, apart from the Fields Medal Symposium and this special issue, is in the conference on Machine Assisted Proofs that was taking place at IPAM as I wrote these lines, and at the upcoming workshop, organized by the National Academies of Sciences, Engineering, and Medicine, on "AI to Assist Mathematical Reasoning" [2]. The word "value" appears nowhere in the *overview* of the IPAM program; nor is there any mention of values, traditions, or cultural sensitivity in the blurb for the National Academies program. Both programs seem designed to show mathematicians "vibratin'" in unison with the tech industry. Moreover, they sidestep the very real possibility that obtaining the massive datasets needed to train AI systems to formalize or generate proofs may involve copyright infringement, a concern already raised by class action lawsuits against AI corporations by artists [9]. A failure to live deliberately could hardly be more blatant.

3. Industry

The automation of mathematics is largely driven by the tech industry, whose values are not generally aligned with those of mathematicians, but no one is talking about this. Is the drive to automate mathematics primarily a reaction to a perceived need on the part of the vocation, a contemporary expression of our *conatus*, or does it represent a sequence of intermediate steps in the implementation of a business plan? The role of industry in shaping scientific research and the institutions in which it takes place is well documented. In contrast, private philanthropy has long been a factor in support of pure as well as applied

[8] Among others, in the *MIT Technology Review*, *The New Yorker*, *Times Higher Education*, and *The Chronicle of Higher Education*.

mathematics. Michael Barany highlights the role of the Rockefeller Foundation and the Carnegie Corporation in creating some central institutions and adds:

> *The predominance of military and philanthropic funders... meant that a relatively small collection of socially and institutionally well-connected men had a disproportionate influence on the prerogatives that governed access to resources. This shaped not just the fields that were studied and the institutions that grew and thrived, but also the socioeconomic, racial, and gender composition of [the] mathematics community...* (Barany [5])

Direct corporate sponsorship of mathematical research seems to have been less common in the past [10].[9] A 2009 symposium in Paris failed in its primary goal: to generate interest on the part of corporations. The following reactions by executives were typical (see [12]):

> *I don't believe in pure research, nor do I believe... that direct corporate investment in research is the right way to go.*
>
> *... Mathematics for the sake of mathematics is none of the business of corporations.*

So the tech industry's new attention to pure mathematics—as evidenced by the increasingly visible participation of Silicon Valley engineers in conferences on automated mathematics,[10] and the creation of the multi-million dollar Hoskinson Center for Formal Mathematics at Carnegie-Mellon University[11]—appears to be unprecedented. But it's a safe bet that the industry is no more interested in "pure research" or "mathematics for the sake of mathematics" than were the French executives in 2009.

Nor is the industry likely to be interested in protecting the free exchange of ideas that (for the most part) characterize our discipline. "Colleges and universities," writes Reinhold Martin, "still offer precariously sheltered islands of sanctuary and dissent... despite their contradictions," [16] he adds. Silicon Valley is a kind of archipelago but it is also a place where dissent is ruthlessly suppressed. Readers probably know that Google fired engineer Timnit Gebru, former technical co-lead of their ethical AI team, when she refused to remove her name from an article about the dangers, of bias in particular, inherent in large language models (LLM).

> *An internal investigation has since uncovered a larger pattern of Google silencing research that casts its technologies in a negative light. Over 2,700 Google employees signed a petition demanding more transparency and denouncing "the unethical and undemocratic incursion of powerful and biased technologies into our daily lives."*
>
> — (O'Gieblyn [18])

Given the balance of forces, any convergence between mathematics and industry is likely to be realized on the basis of the latter's values. This logically means subordination of critical thinking to the need to maintain profits and a steady flow of investment. This may already be happening. A mathematician who joined a

[9] Feffer traces some of this history. I thank Michael Barany for this reference.

[10] See, for example, the February 2023 IPAM Workshop on Machine Assisted Proofs, as well as the MATH-AI Workshops at the annual NeurIPS conferences, whose lists of sponsors speak for themselves.

[11] Crypto billionaire Charles Hoskinson was clear about his objectives in his speech at the opening ceremony of his eponymous center.

Silicon Valley project showed me the confidentiality and nondisclosure agreement (NDA) he was required to sign. The conditions in the NDA, which apparently has no expiration date, prevented him from signaling his agreement with an article I published on my Substack newsletter.

Living deliberately also entails modesty on the part of the authors of this collection of essays. How will we argue that we are entitled to speak for "mathematics" as such? What will be the weight of our influence in comparison with the massive carrots and sticks the industry has at its disposal?

4. History

Historians of mathematics have been vigorously questioning the narratives around proof that are dominant in the writings of mathematicians; but these narratives, misleading as they often are, are at the basis of much of what is claimed regarding the implications of automation. It gives me no pleasure to acknowledge that too many of my colleagues among mathematicians believe history of mathematics can only be written by mathematicians. This is not merely because of the many years of experience required for competence in the use of the specialized vocabulary; it is primarily because, they believe, only mathematicians truly understand our objectives. This incidentally raises an interesting question: will AI ever understand our objectives? In other words, is mathematics subject to what is known in the AI literature as the *alignment problem*? To take an extreme example (but one similar to those typical of the literature), might an AI world government "solve" the Riemann hypothesis by killing everyone who cares about the answer?

But do we even agree on our objectives? One perspective has dominated media coverage of both the question of proof verification and that of automation of theorem proving, including the coverage in publications of mathematical societies. This perspective, which corresponds roughly to what I call the *Central Dogma*,[12] provides the primary underpinning for the sense of urgency that drives much of the discourse around the nature and role of proof in mathematics; it says roughly that a proof is valid if (and only if) it can be expressed in a formal symbolic language. I call it a *dogma* because it endows projects of formal proof verification with an epistemological centrality that I don't think is justified.

This perspective, like everything else connected to mathematics, or anything touched by human creativity, has its own history. Here professional mathematicians —who are generally exposed to this history in a highly diluted form, if at all, adequate for motivation and moralizing but not for intellectual rigor—would benefit from "the use of the resources of history to query the self-evidence of our contemporary habits of thought"[13] when seeking to come to grips with the implications of the new technologies for the future of the discipline (see [8, p. 22]). Again, it's unreasonable to expect everyone with an opinion about the mechanization of mathematics to be deeply familiar with this history.[14] But intellectual modesty should compel anyone who offers an opinion for public consideration to acknowledge that

[12]Reuben Hersh called it the *Mainstream* (as opposed to *Mavericks*).

[13]Daston continues: "One of the uses of history. . . is to unsettle present certainties and thereby enlarge our sense of the thinkable." Living deliberately inevitably requires that present certainties be unsettled.

[14]It's only because I wrote to historian Stephanie Dick that I became aware of work by Daston and other historians on nineteenth century mathematicians who were on opposite sides of a debate

ignorance—of professional standards as well as of isolated facts—is as unappealing in history as in mathematics. Moreover, it hardly requires specialized training to be aware that there was mathematics *before* formalism and thus that mathematics is likely to continue *after* this perspective has outlived its usefulness. Formalism as an ideology represents only a blip in the long history of mathematics; its expiration date, in other words, is likely to arrive well before that of human mathematics.

In her introduction to a thick volume entitled *The History of Mathematical Proof in Ancient Traditions*, Karine Chemla is harsh on the failure of philosophers and mathematicians to reckon with the fetishization of certain standards of proof during the colonial period, "in the process of shaping 'European civilization' as superior to the others."

> *What is important for us to note... is that through such nonmathematical uses of mathematical proof the actor's... perception of proof has been colored by implications that were foreign to mathematics itself. This observation may help to account for the astonishing emotion that often permeates debates on mathematical proof... ordinary ones as well as more academic ones... while other mathematical issues meet with indifference.*
>
> — (Chemla [6, p. 4])

In particular, the "now standard narrative of the early history of proof"—that asserts, among other things, "that mathematical proof, as it is practised today, is inherited exclusively from... Greek ancestors"—was itself shaped by the ideological preferences of the period.

> *The narrative belongs to its time and the time may have come that we need to replace it.*
>
> — (Chemla [6, p. 12])

For those willing to take the time to read it, the book Chemla edited is a remedy to the blinkered view of proof that is pervasive in mainstream philosophy of mathematics. As an example, I point to a hypothesis Chemla formulates in her introduction, based on a reading of several analyses of ancient texts:

> *that some proofs were to be read as a kind of* ***paradigm****, the text of a proof was not read only as establishing a proposition, but also as a possible source for working techniques.*[15]
>
> — (Chemla [6, p. 31])

Conceiving mathematics on the model of a video game, or a rugby match, where the only goal is to win, misses this kind of reading, where the apprehension of the text by a necessarily human reader is the reason for its existence.

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on the value of algebraic methods in geometry. Only one side of the debate was at all compatible with the future emphasis on formalism.

[15]One doesn't have to look to *Ancient Traditions* to observe the variability of standards of proof. The Society for Industrial and Applied Mathematics brochure *Careers in the Mathematical Sciences* lists more than fifty mathematical careers but only mentions the word "proof" once. Even among branches of academic mathematics one encounters very different styles of proof; drawings, often elaborate, are much more central in published articles in geometric topology than in commutative algebra, for example. (In response to the inevitable objection that the drawing is merely an aid to intuition, or that the eversion of the 2-sphere has now been formalized, I can only say that, in the actual practice of geometric topology, drawing on the blackboard or on the page is central to the process of proving.)

5. Democracy

The concealed but deeply embedded hierarchical nature of mathematics makes it more vulnerable to takeover by the explicitly hierarchical forces of industry.

> *. . . If the change is accompanied by some hardship, so is every step in the progress of the human race. . .*
>
> — (*Scientific American*, [14])

Much of the debate over AI focuses on the threats it may pose to democracy, the concern that decision-making will be taken out of the hands of the concerned public and entrusted to automata whose priorities have been defined by the corporations that designed them. It is no secret that mathematics is not run on purely democratic principles. Decisions that determine the priorities of the discipline—in recruiting students, hiring colleagues, publishing articles, or awarding prizes, among other functions—are made by a self-reproducing elite; the material conditions that guarantee the field's survival are obtained by an opaque process of negotiation with public and private funding sources of which no one, I think, has ever attempted to provide a global synthetic account.

The community of active researchers seems largely content with this state of affairs—the elitism and absence of democratic oversight.[16] Determination of winners and losers may not be as clear-cut in mathematics as in chess matches or gladiator contests, which end with one king or gladiator standing and the other horizontal on the board or dead on the sand. Nevertheless, if peer review establishes that someone has proved some theorem then the elite can be confident that their decisions have an objective basis, and they can award the job, or the grant, or the prize to an objective winner—provided the theorem is manifestly an *object of desire* on the part of the community.[17]

History teaches us that making deliberate decisions about the future of mathematics presupposes a methodological outlook consistent with that of the Chemla book already quoted:

> *We do not restrict our corpus* ***a priori*** *by reference to norms and values that would appear to us as characterizing proofs in an essential way.*

While some readers may dismiss this outlook as dangerously relativist, can't that also be said of the view, promoted by the tech industry, that the future of the discipline is necessarily in need of "disruption" [11]? Isn't it rather the case that adapting our vision of the future of the discipline to our current values is the essence of democratic practice? But democracy is hard to implement in the

[16]There is a substantial literature opposing mathematical elitism, mainly concerned with education or with questions of the familiar trinity of diversity, equity, and inclusion. The American Mathematical Society (AMS) has been encouraging discussion of these issues—see for example the AMS paraDIGMS initiative. Existing attempts to apply the insights of this literature to questions of research look awkward and half-hearted to me. But they pose serious questions that the ideologues of mechanization of mathematics have overlooked. During the Fields Medal Symposium, for example, Philip Ording asked whether a future mechanized mathematics wouldn't inevitably be even more exclusive, since only those with access to high-powered computation would be able to participate.

[17]As far as I know the account of trans-historical mathematics as a community of desire has not yet been written. It feels unlikely to me that any AI will soon be admitted to such a community.

best of circumstances; it's hard even to conceive in a scattered and fundamentally anarchistic (though deeply hierarchical) community like that of mathematics. Is this why some find it easier to entrust our future to the powerful forces of industry, forces that at least know what they're "vibratin' " about?

Most of what is written about the topic of this special issue presupposes that incorporating new technologies will allow mathematics to thrive and persevere within an unchanging *conatus*. Venkatesh's essay argues that the values of the profession will in fact be deeply affected by automation. This could be beneficial if it led to a clarification of these values and a commitment to increasing democratic practice in a way consistent with persistent core values. I have argued elsewhere that allowing industry to set the agenda—for example, by participating alongside representatives of the biggest players in Silicon Valley at conferences like those mentioned in an earlier section—is likely to short-circuit the necessary debate internal to the profession.

6. My mission, such as it is

Why I'm writing about this topic. Given how difficult it has always been, in retrospect, to predict mathematics' next move, and given the array of forces propelling the discipline in unexpected directions, why am I writing about mechanization of mathematics at all? It is because my conversations with colleagues have convinced me that in their overwhelming majority *they do not see* themselves mirrored in the narrative promoted by the ballooning literature about automation of mathematics, and they are largely indifferent, if not hostile, to the prospect of fully formalized proofs. This is not, I believe, because of the ingrown conservatism that is the hallmark of any established *conatus*, but rather because the narrative that is taking shape radically misses the point of mathematics and substitutes for it a domesticated version drained of meaning. And this, in turn, is because it is so much easier to build on an established narrative, the account of progress familiar since the beginning of the industrial revolution, than to tackle the challenge of giving a name to the drive to participate in the mathematical experience and to explain why it is a particularly gratifying way of being human.

Here's a thought to help with that challenge. Many versions of the narrative around automating mathematicians, including some represented in this special issue, conclude with the prediction that machines either will or will not replace human mathematicians. I find it enlightening to compare the prospect to a concert in which machines have replaced the *audience* rather than the *performers*.

The technological mindset banishes the contingent, repudiates the singular, obscures the ineffable, discards whatever cannot be monetized. The *understanding* at which mathematics aims is all of those things.

Chemla's introduction outlines a research programme—in history of mathematics—that aims to "attend more closely to differences between the various practices of proof, thereby breaking down what is all too often presented collectively as 'the mathematical practice'." Another aim reveals itself in the light of the present essay: to identify those practices of proof that enhance the autonomy of mathematicians and resist being monetized. Technology, including AI, may be a tool in the pursuit of these practices, but it can never replace them.

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