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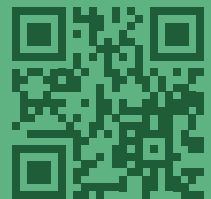
Editors

Articles

Alejandro Adem, *Chief Editor*

Book Reviews

Volodymyr Nekrashevych



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