

## THE FISCHER-CLIFFORD MATRICES OF A MAXIMAL SUBGROUP OF $Fi'_{24}$

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ABSTRACT. The Fischer group  $Fi'_{24}$  is the largest sporadic simple Fischer group of order

$$1255205709190661721292800 = 2^{21} \cdot 3^{16} \cdot 5^2 \cdot 7^3 \cdot 11 \cdot 13 \cdot 17 \cdot 23 \cdot 29 .$$

The group  $Fi'_{24}$  is the derived subgroup of the Fischer 3-transposition group  $Fi_{24}$  discovered by Bernd Fischer. There are five classes of elements of order 3 in  $Fi'_{24}$  as represented in ATLAS by  $3A$ ,  $3B$ ,  $3C$ ,  $3D$  and  $3E$ . A subgroup of  $Fi'_{24}$  of order 3 is called of type  $3X$ , where  $X \in \{A, B, C, D, E\}$ , if it is generated by an element in the class  $3X$ . There are six classes of maximal 3-local subgroups of  $Fi'_{24}$  as determined by Wilson. In this paper we determine the Fischer-Clifford matrices and conjugacy classes of one of these maximal 3-local subgroups  $\bar{G} := N_{Fi'_{24}}(\langle N \rangle) \cong 3^7 \cdot O_7(3)$ , where  $N \cong 3^7$  is the natural orthogonal module for  $\bar{G}/N \cong O_7(3)$  with 364 subgroups of type  $3B$  corresponding to the totally isotropic points. The group  $\bar{G}$  is a nonsplit extension of  $N$  by  $G \cong O_7(3)$ .

### 1. INTRODUCTION

Let  $\bar{G} = N \cdot G$  be an extension of  $N$  by  $G$ , where  $N$  is a normal subgroup of  $\bar{G}$  and  $\bar{G}/N \cong G$ . Let  $\bar{g} \in \bar{G}$  be a lifting of  $g \in G$  under the natural homomorphism  $\bar{G} \rightarrow G$  and let  $[g]$  be a conjugacy class of elements of  $G$  with representative  $g$ . Let  $X(g) = \{x_1, x_2, \dots, x_{c(g)}\}$  be a set of representatives of the conjugacy classes of  $\bar{G}$  from the coset  $N\bar{g}$  whose images under the natural homomorphism  $\bar{G} \rightarrow G$  are in  $[g]$  and we take  $x_1 = \bar{g}$ . Let  $\{\theta_1, \theta_2, \dots, \theta_t\}$  be a set of representatives of the orbits of  $\bar{G}$  on  $\text{Irr}(N)$  such that for  $1 \leq i \leq t$ , we have  $\bar{H}_i = I_{\bar{G}}(\theta_i)$  with the corresponding inertia factors  $H_i$  and let  $\psi_i$  be a projective character of  $\bar{H}_i$  with the factor set  $\bar{\alpha}_i$  such that  $(\psi_i)_N = \theta_i$ . By Gallagher [18] we have

$$\text{Irr}(\bar{G}) = \bigcup_{i=1}^t \{(\psi_i \bar{\beta})^{\bar{G}} \mid \beta \in \text{IrrProj}(H_i), \text{ with the factor set } \alpha_i^{-1}\},$$

where  $\alpha_i$  is obtained from  $\bar{\alpha}_i$  and  $\bar{\beta}$  from  $\beta$ . Without loss of generality, suppose that  $\theta_1 = 1_N$  is the identity character of  $N$ . Then  $\bar{H}_1 = \bar{G}$  and  $H_1 = G$ . Now choose  $y_1, y_2, \dots, y_r$  to be the representatives of the  $\alpha_i^{-1}$ -conjugacy classes of elements of  $H_i$  that fuse to  $[g]$  in  $G$ . Since  $y_k \in H_i$  for  $1 \leq k \leq r$ , then we define  $y_{l_k} \in \bar{H}_i$  such that  $y_{l_k}$  ranges over all representatives of the conjugacy classes of elements of  $\bar{H}_i$

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which map to  $y_k$  under the homomorphism  $\bar{H}_i \rightarrow H_i$  whose kernel is  $N$ . Now by using the formula for induced characters, we have

$$\begin{aligned} (\psi_i \bar{\beta})^{\bar{G}}(x_j) &= \sum_{1 \leq k \leq r} \sum_{\ell}' \frac{|C_{\bar{G}}(x_j)|}{|C_{\bar{H}_i}(y_{\ell_k})|} \psi_i \bar{\beta}(y_{\ell_k}) \\ &= \sum_{1 \leq k \leq r} \sum_{\ell}' \frac{|C_{\bar{G}}(x_j)|}{|C_{\bar{H}_i}(y_{\ell_k})|} \psi_i(y_{\ell_k}) \bar{\beta}(y_{\ell_k}) \\ &= \sum_{1 \leq k \leq r} \left( \sum_{\ell}' \frac{|C_{\bar{G}}(x_j)|}{|C_{\bar{H}_i}(y_{\ell_k})|} \psi_i(y_{\ell_k}) \right) \beta(y_k) \end{aligned}$$

where  $\sum_{\ell}'$  is the summation over all  $\ell$  for which  $y_{\ell_k} \sim x_j$  in  $\bar{G}$ . Now we define a matrix  $M_i(g)$  by  $M_i(g) = (a_{uv})$ , where  $1 \leq u \leq r$  and  $1 \leq v \leq c(g)$ , and

$$a_{uv} = \sum_{\ell}' \frac{|C_{\bar{G}}(x_j)|}{|C_{\bar{H}_i}(y_{\ell_k})|} \psi_i(y_{\ell_k}).$$

Then we get

$$(\psi_i \beta)^{\bar{G}}(x_j) = \sum_{1 \leq k \leq r} a_{uv} \hat{\beta}(y_k).$$

By doing so for all  $1 \leq i \leq t$  such that  $H_i$  contains an element in  $[g]$  we obtain the matrix  $M(g)$  given by

$$M(g) = \begin{bmatrix} M_1(g) \\ M_2(g) \\ \vdots \\ M_t(g) \end{bmatrix},$$

where  $M_i(g)$  is the submatrix corresponding to the inertia group  $\bar{H}_i$  and its inertia factor  $H_i$ . If  $H_i \cap [g] = \emptyset$ , then  $M_i(g)$  will not exist and  $M(g)$  does not contain  $M_i(g)$ . The size of the matrix  $M(g)$  is  $l \times c(g)$  where  $l$  is the number of  $\alpha_i^{-1}$ -regular conjugacy classes of elements of the inertia factors  $H_i$  for  $1 \leq i \leq t$  which fuse into  $[g]$  in  $G$ , and  $c(g)$  is the number of conjugacy classes of elements of  $\bar{G}$  which correspond to the coset  $\bar{g}N$ . Then  $M(g)$  is the *Fischer-Clifford matrix* of  $\bar{G}$  corresponding to the coset  $\bar{g}N$ . We will see later that  $M(g)$  is a  $c(g) \times c(g)$  nonsingular matrix. Let

$$R(g) = \{(i, y_k) \mid 1 \leq i \leq t, H_i \cap [g] \neq \emptyset, 1 \leq k \leq r\},$$

and we note that  $y_k$  runs over representatives of the  $\alpha_i^{-1}$ -conjugacy classes of elements of  $H_i$  which fuse into  $[g]$  in  $G$ . Following the notation used by Fischer [9], Mpono [25] and Whitely [30] we denote  $M(g)$  by writing  $M(g) = Cl(N\bar{g}) = (a_j^{(i,y_k)})$ , where

$$a_j^{(i,y_k)} = \sum_{\ell}' \frac{|C_{\bar{G}}(x_j)|}{|C_{\bar{H}_i}(y_{\ell_k})|} \psi_i(y_{\ell_k}),$$

with columns indexed by  $X(g)$  and rows indexed by  $R(g)$ . Then the partial character table of  $\bar{G}$  on the classes  $\{x_1, x_2, \dots, x_{c(g)}\}$  is given by

$$\begin{bmatrix} C_1(g)M_1(g) \\ C_2(g)M_2(g) \\ \vdots \\ C_t(g)M_t(g) \end{bmatrix}$$

where the Fischer-Clifford matrix  $M(g)$  is divided into blocks with each block corresponding to an inertia group  $\bar{H}_i$  and  $C_i(g)$  is the partial projective character table of  $H_i$  with the factor set  $\alpha_i^{-1}$  consisting of the columns corresponding to the  $\alpha_i^{-1}$ -regular classes that fuse into  $[g]$  in  $G$ . We obtain the characters of  $\bar{G}$  by multiplying the relevant columns of the projective characters of  $H_i$  with the factor set  $\alpha_i^{-1}$  by the rows of  $M(g)$ . We can also observe that the number of irreducible characters of  $\bar{G}$  is the sum of numbers of projective characters of the inertia factors  $H_i$  with the factor set  $\alpha_i^{-1}$ , for all  $i, 1 \leq i \leq t$ .

In the present paper we determine Fischer-Clifford matrices and conjugacy classes of  $3^7 \cdot O_7(3)$ . Then the character table of  $3^7 \cdot O_7(3)$  can be constructed by using these matrices and the partial character tables of the inertia factor groups.

Let  $K$  be a group and  $A \leq \text{Aut}(K)$ . Then by Brauer’s theorem (see [14])  $A$  acts on the conjugacy classes of elements of  $K$  and on the irreducible characters of  $K$  resulting in the same number of orbits.

**Lemma 1.1.** *Suppose we have the following matrix describing the above actions:*

$$\begin{matrix}
 & 1 = l_1 & l_2 & \cdots & l_j & \cdots & l_t \\
 \begin{matrix} s_1 \\ s_2 \\ \vdots \\ s_i \\ \vdots \\ s_t \end{matrix} & \begin{pmatrix} 1 & 1 & \cdots & 1 & \cdots & 1 \\ a_{21} & a_{22} & \cdots & a_{2j} & \cdots & a_{2t} \\ \vdots & \vdots & & \vdots & & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{ij} & \cdots & a_{it} \\ \vdots & \vdots & & \vdots & & \vdots \\ a_{t1} & a_{t2} & \cdots & a_{tj} & \cdots & a_{tt} \end{pmatrix}
 \end{matrix}$$

where  $a_{1j} = 1$  for  $j \in \{1, 2, \dots, t\}$ ,  $l_j$ ’s are lengths of orbits of  $A$  on the conjugacy classes of  $K$ ,  $s_i$ ’s are lengths of orbits of  $A$  on  $\text{Irr}(K)$  and  $a_{ij}$  is the sum of  $s_i$  irreducible characters of  $K$  on the element  $x_j$ , where  $x_j$  is an element of the orbit of length  $l_j$ . Then the following relation holds for  $i, i' \in \{1, 2, \dots, t\}$ :

$$\sum_{j=1}^t a_{ij} \overline{a_{i'j}} l_j = |K| s_i \delta_{ii'}.$$

*Proof.* This result has been proved as Lemma 2.2.2 in [27] and as Lemma 4.2.2 in [30]. □

For arithmetical properties the weights are important and hence we attach to  $M(g)$  the corresponding *weights*. Let  $x_j \in X(g)$ . For a fixed coset  $X = \bar{g}N \in \bar{G}/N$ , we define  $m_j = [N_{\bar{G}}(X) : C_{\bar{G}}(x_j)]$ .

The Fischer-Clifford matrix  $M(g)$  is partitioned row-wise into blocks, where each block corresponds to an inertia group. The columns of  $M(g)$  are indexed by  $X(g)$  and for each  $x_j \in X(g)$ , at the top of the columns of  $M(g)$ , we write  $|C_{\bar{G}}(x_j)|$  and at the bottom we write  $m_j$ . The rows of  $M(g)$  are indexed by  $R(g)$  and on the left of each row we write  $|C_{H_i}(y_k)|$ , where  $y_k$  fuses into  $[g]$  in  $G$ . Then, in general, we can write  $M(g)$  with corresponding weights for rows and columns as follows, where blocks corresponding to the inertia groups are separated by horizontal lines:

$$\begin{array}{c}
 |C_{\bar{G}}(x_1)| \quad |C_{\bar{G}}(x_2)| \quad \cdots \quad |C_{\bar{G}}(x_{c(g)})| \\
 |C_G(g)| \quad \left( \begin{array}{cccc}
 a_1^{(1,g)} & a_2^{(1,g)} & \cdots & a_{c(g)}^{(1,g)} \\
 \hline
 |C_{H_2}(y_1)| & a_1^{(2,y_1)} & a_2^{(2,y_1)} & \cdots & a_{c(g)}^{(2,y_1)} \\
 |C_{H_2}(y_2)| & a_1^{(2,y_2)} & a_2^{(2,y_2)} & \cdots & a_{c(g)}^{(2,y_2)} \\
 \vdots & \vdots & \vdots & \vdots & \vdots \\
 \hline
 |C_{H_i}(y_1)| & a_1^{(i,y_1)} & a_2^{(i,y_1)} & \cdots & a_{c(g)}^{(i,y_1)} \\
 |C_{H_i}(y_2)| & a_1^{(i,y_2)} & a_2^{(i,y_2)} & \cdots & a_{c(g)}^{(i,y_2)} \\
 \vdots & \vdots & \vdots & \vdots & \vdots \\
 \hline
 |C_{H_t}(y_1)| & a_1^{(t,y_1)} & a_2^{(t,y_1)} & \cdots & a_{c(g)}^{(t,y_1)} \\
 |C_{H_t}(y_2)| & a_1^{(t,y_2)} & a_2^{(t,y_2)} & \cdots & a_{c(g)}^{(t,y_2)} \\
 \vdots & \vdots & \vdots & \vdots & \vdots \\
 \hline
 \end{array} \right) \\
 m_1 \quad m_2 \quad \cdots \quad m_{c(g)}
 \end{array}$$

The following result gives the orthogonality relation for  $M(g)$ . Its proof was obtained from Whitley [30], Proposition 4.2.3.

**Proposition 1.2** ([30], [25] (*Column orthogonality*)). *Let  $\bar{G} = N \cdot G$ . Then*

$$\sum_{(i,y_k) \in R(g)} |C_{H_i}(y_k)| a_j^{(i,y_k)} \overline{a_{j'}^{(i,y_k)}} = \delta_{jj'} |C_{\bar{G}}(x_j)| \quad .$$

*Proof.* The partial character table of  $\bar{G}$  at classes  $x_1, \dots, x_{c(g)}$  is given by

$$\begin{bmatrix} C_1(g)M_1(g) \\ C_2(g)M_2(g) \\ \vdots \\ C_t(g)M_t(g) \end{bmatrix} .$$

By column orthogonality of the character table of  $\bar{G}$ , we have

$$\begin{aligned}
 |C_{\bar{G}}(x_j)|\delta_{jj'} &= \sum_{i=1}^t \sum_{\beta_i \in \text{IrrProj}(H_i)} \left( \sum_{y_k: (i, y_k) \in R(g)} a_j^{(i, y_k)} \beta_i(y_k) \right) \overline{\left( \sum_{y'_k: (i, y'_k) \in R(g)} a_{j'}^{(i, y'_k)} \beta_i(y'_k) \right)} \\
 &= \sum_{i=1}^t \sum_{\beta_i \in \text{IrrProj}(H_i)} \left( \sum_{y_k} a_j^{(i, y_k)} \overline{a_{j'}^{(i, y'_k)}} \beta_i(y_k) \overline{\beta_i(y'_k)} \right) \\
 &\quad + \sum_{y_k} \sum_{y'_k \neq y_k} a_j^{(i, y_k)} \overline{a_{j'}^{(i, y'_k)}} \beta_i(y_k) \overline{\beta_i(y'_k)} \\
 &= \sum_{i=1}^t \left( \sum_{y_k} a_j^{(i, y_k)} \overline{a_{j'}^{(i, y_k)}} \sum_{\beta_i \in \text{IrrProj}(H_i)} \beta_i(y_k) \overline{\beta_i(y_k)} \right) \\
 &\quad + \sum_{y_k} \sum_{y'_k \neq y_k} a_j^{(i, y_k)} \overline{a_{j'}^{(i, y'_k)}} \sum_{\beta_i \in \text{IrrProj}(H_i)} \beta_i(y_k) \overline{\beta_i(y'_k)} \\
 &= \sum_{i=1}^t \left( \sum_{y_k} a_j^{(i, y_k)} \overline{a_{j'}^{(i, y_k)}} |C_{H_i}(y_k)| + 0 \right) \\
 &= \sum_{(i, y_k) \in R(g)} a_j^{(i, y_k)} \overline{a_{j'}^{(i, y_k)}} |C_{H_i}(y_k)|.
 \end{aligned}$$

□

**Theorem 1.3.**  $a_j^{(1, g)} = 1$  for all  $j \in \{1, 2, \dots, c(g)\}$ .

*Proof.* For  $y_{\ell_k} \sim x_j$  in  $\bar{G}$ , we have  $|C_{\bar{G}}(x_j)| = |C_{H_1}(y_{\ell_k})|$ . Thus we obtain that

$$a_j^{(1, g)} = \sum_{\ell} \frac{|C_{\bar{G}}(x_j)|}{|C_{H_1}(y_{\ell_k})|} \psi_1(y_{\ell_k}) = \sum_{\ell} 1 = 1.$$

Hence the result.

□

**Proposition 1.4** ([19], [30]). *The matrix  $M(1_G)$  is the matrix with rows equal to the orbit sums of the action of  $\bar{G}$  on  $\text{Irr}(N)$  with duplicate columns discarded. For this matrix we have  $a_j^{(i, 1_G)} = [G : H_i]$ , and an orthogonality relation for rows:*

$$\sum_{j=1}^t \frac{1}{|C_{\bar{G}}(x_j)|} a_j^{(i, 1_G)} a_j^{(i', 1_G)} = \frac{1}{|C_{H_i}(1_G)|} \delta_{ii'} = \frac{1}{|H_i|} \delta_{ii'}.$$

*Proof.* See [25].

□

As a consequence of Lemma 1.1, Proposition 1.3 and from Fischer [10], we have the following properties:

- (a)  $|X(g)| = |R(g)|$ ,
- (b)  $\sum_{j=1}^{c(g)} m_j a_j^{(i, y_k)} \overline{a_j^{(i', y'_k)}} = \delta_{(i, y_k), (i', y'_k)} \frac{|C_G(g)|}{|C_{H_i}(y_k)|} |N|$ ,
- (c)  $\sum_{(i, y_k) \in R(g)} a_j^{(i, y_k)} \overline{a_{j'}^{(i, y_k)}} |C_{H_i}(y_k)| = \delta_{jj'} |C_{\bar{G}}(x_j)|$ ,
- (d)  $M(g)$  is square and nonsingular.

2. SPLIT COSETS

From now on suppose that  $N$  is an elementary abelian normal  $p$ -subgroup of  $\bar{G}$  and  $\bar{g}N = X$  is a fixed coset of  $\bar{G}/N \cong G$ . Let  $M = C_{\bar{g}} = N_{\bar{G}}(X)$ . We define

$$N_{\bar{g}} := \langle [\bar{g}, n], n \in N \rangle.$$

With this notation we have the following lemma.

- Lemma 2.1.** (i)  $N_x = N_{\bar{g}}$  for all  $x \in X$  and  $[\bar{g}, u_1].[\bar{g}, u_2] = [\bar{g}, u_1u_2]$  for all  $u_1, u_2 \in N$ .  
 (ii)  $N_{\bar{g}} \triangleleft M$  and  $N_{\bar{g}} \leq N$ .  
 (iii) If  $\varphi \in \text{Irr}(N)$ , then  $N_{\bar{g}} \leq \ker(\varphi)$  or  $I_{\bar{G}}(\varphi) \cap \bar{g}N = \emptyset$ .

*Proof.* (i) Let  $x = \bar{g}n \in \bar{g}N$  and  $u \in N$ , then

$$\begin{aligned} [x, u] &= [\bar{g}n, u] = n^{-1}(u^{-1})^{\bar{g}}nu \\ &= (u^{-1})^{\bar{g}}u \quad \text{since } N \text{ is abelian} \\ &= \bar{g}^{-1}u^{-1}\bar{g}u = [\bar{g}, u] \end{aligned}$$

which implies that

$$N_x = N_{\bar{g}} \quad \text{for all } x \in \bar{g}N.$$

Also, since  $N$  is abelian, we obtain for all  $u_1, u_2 \in N$ ,

$$\begin{aligned} [\bar{g}, u_1].[\bar{g}, u_2] &= (u_1^{-1})^{\bar{g}}u_1(u_2^{-1})^{\bar{g}}u_2 \\ &= (u_1^{-1}u_2^{-1})^{\bar{g}}u_1u_2 \\ &= [\bar{g}, u_1u_2]. \end{aligned}$$

Hence

$$[\bar{g}, u_1].[\bar{g}, u_2] = [\bar{g}, u_1u_2] \quad \text{for } u_1, u_2 \in N.$$

- (ii) Since  $[\bar{g}, u] = (u^{-1})^{\bar{g}}u \in N$ , we obtain  $N_{\bar{g}} \leq N \leq M$ . Conversely, let  $m \in M$ . Then

$$\begin{aligned} [\bar{g}, u]^m &= m^{-1}[\bar{g}, u]m \\ &= (\bar{g}^{-1})^m(u^{-1})^m\bar{g}^m u^m \\ &= (\bar{g}^m)^{-1}(u^m)^{-1}\bar{g}^m u^m \\ &= [\bar{g}^m, u^m] \in N_{\bar{g}}. \end{aligned}$$

Hence  $N_{\bar{g}} \triangleleft M$ .

- (iii) Let  $\varphi \in \text{Irr}(N)$  be fixed. Then

$$\begin{aligned} N_{\bar{g}} \leq \text{Ker}(\varphi) &\Leftrightarrow \varphi([\bar{g}, u]) = \varphi(1) = 1 \quad \text{for all } u \in N \\ &\Leftrightarrow \varphi(\bar{g}^{-1}u^{-1}\bar{g}u) = \varphi((u^{-1})^{\bar{g}}u) = 1 \\ &\Leftrightarrow \varphi((u^{-1})^{\bar{g}}) = (\varphi(u))^{-1} = \varphi(u^{-1}) \\ &\Leftrightarrow \varphi^{\bar{g}}(u^{-1}) = \varphi(u^{-1}) \\ &\Leftrightarrow \varphi^{\bar{g}} = \varphi \\ &\Leftrightarrow \bar{g}N \subseteq I_{\bar{G}}(\varphi) \\ &\Leftrightarrow \bar{g}N \cup I_{\bar{G}}(\varphi) \neq \emptyset. \end{aligned}$$

□

*Remark 2.2.* We can easily show that  $\langle X \rangle/N_{\bar{g}}$  is abelian and  $X/N_{\bar{g}}$  is a coset of  $\langle X \rangle/N_{\bar{g}}$ .

**Lemma 2.3** ([10]). *The rows of the Fischer-Clifford matrix  $Cl(X)$  can be identified with restrictions of  $M$ -invariant characters of  $\langle X \rangle / N_{\bar{g}}$  to  $X / N_{\bar{g}}$ .*

*Proof.* This is Lemma 5.2 in [10]. □

*Remark 2.4.* In the above lemma, the rows of  $Cl(X)$  will be an independent set of orbit sums, under the action of  $M$  on  $\langle X \rangle / N_{\bar{g}}$ . This observation was first given in Fischer [10].

**Definition 2.5.** A coset  $X$  is said to be a *split coset* if it contains an element  $x$  such that  $M = N.C_{\bar{G}}(x)$ .

Note that we do not require  $\langle x \rangle \cap N = \langle 1 \rangle$  in the above definition.

**Lemma 2.6** ([28]). *If the extension splits, then every coset is a split coset.*

*Proof.* Let  $X = \bar{g}N$  and  $h \in C_{\bar{G}}(\bar{g})$ . Then  $h(\bar{g}n)h^{-1} = (h\bar{g}h^{-1})(hnh^{-1}) = \bar{g}hnh^{-1} = \bar{g}n^h \in \bar{g}N$ . Now, since  $N \leq M$  and  $C_{\bar{G}}(\bar{g}) \leq M$ , then  $M \geq N.C_{\bar{G}}(\bar{g})$ . Let  $C$  be the complement of  $N$  in  $\bar{G}$  such that  $\bar{g} \in C$ . Let  $m \in M$ . Then  $m = n.k$ , for some  $k \in C$ . Since  $M = N_{\bar{G}}(\bar{g}N)$ ,  $(\bar{g}N)^m = \bar{g}N$ . Hence

$$\begin{aligned} \bar{g}N &= (\bar{g}N)^m = m(\bar{g}N)m^{-1} \\ &= n(k\bar{g}Nk^{-1})n^{-1} = n(k\bar{g}Nk^{-1})n^{-1} = n(\bar{g}N)^k n^{-1}, \end{aligned}$$

so that  $n^{-1}(\bar{g}N)n = (\bar{g}N)^k$  and  $n^{-1}\bar{g}N = (\bar{g}N)^k$ . Hence  $\bar{g}N = (\bar{g}N)^k$ . It follows that  $\bar{g}N = (\bar{g}N)^k = \bar{g}^k N$ , which implies that  $\bar{g}^k \in \bar{g}N$ . Hence  $\bar{g}^k \in C \cap \bar{g}N = \{\bar{g}\}$  and so  $k \in C_{\bar{G}}(\bar{g})$ , which implies that  $m = n.k \in N.C_{\bar{G}}(\bar{g})$  and so  $M \leq N.C_{\bar{G}}(\bar{g})$ . Thus  $M = N.C_{\bar{G}}(\bar{g})$ . □

The following result is of fundamental importance and very helpful to fill the entries of Fischer-Clifford matrices.

**Lemma 2.7** ([9]). *Let  $X$  be a split coset. Then the rows of  $Cl(X)$  can be identified with  $M$ -invariant characters of  $N/N_{\bar{g}}$  multiplied by a  $p$ -th root of unity.*

*Proof.* See [9] and [28]. □

**Lemma 2.8.** *Let  $X = \bar{g}N$  be a split coset and  $N_{\bar{G}}(X) = N.C_{\bar{G}}(x)$  for  $x \in X(g)$ . Then we have the following:*

- (i)  $a_1^{(i,y_k)} = \frac{|C_{\bar{G}}(g)|}{|C_{H_i}(y_k)|}$ ,
- (ii)  $|a_j^{(i,y_k)}| \leq |a_1^{(i,y_k)}|$  for all  $1 \leq j \leq r$ ,
- (iii) If  $|N| = p^w$ , then  $a_j^{(i,y_k)} \equiv a_1^{(i,y_k)} \pmod{p}$ .

*Proof.* See [28]. □

### 3. NONSPLIT EXTENSIONS

Let  $\bar{G} = N.G$  be a nonsplit extension, where  $N$  is an elementary abelian normal  $p$ -subgroup of  $\bar{G}$ . Let  $\bar{g}N$  be a conjugacy class representative of  $\bar{G}/N$  and let  $\varphi$  be a representative of  $\bar{G}$ -orbit irreducible characters of  $N$  with the projective extension  $\bar{\varphi}$  to  $\bar{G}$ . We consider the groups  $\langle \bar{g} \rangle N \leq \bar{G}$  and  $\langle \bar{g}N \rangle \leq \bar{G}/N$ .

**Lemma 3.1.**

$$\langle \bar{g} \rangle N / N = \langle \bar{g}N \rangle .$$

*Proof.* Let  $x \in \langle \bar{g} \rangle N / N$ . Then  $x = \bar{g}^m n N = \bar{g}^m N$  for some  $m \in \mathbb{Z}$ . So that  $x = (\bar{g}N)^m \in \langle \bar{g}N \rangle$ . Hence  $\langle \bar{g} \rangle N / N \leq \langle \bar{g}N \rangle$ . Conversely, let  $x \in \langle \bar{g}N \rangle$ . Then  $x = (\bar{g}N)^m = \bar{g}^m N$  for some  $m \in \mathbb{Z}$ . Hence  $x = (\bar{g}^m N) \in \langle \bar{g} \rangle N / N$ . Thus  $\langle \bar{g}N \rangle \leq \langle \bar{g} \rangle N / N$ . Therefore,  $\langle \bar{g} \rangle N / N = \langle \bar{g}N \rangle$ .  $\square$

**Lemma 3.2.** *With the above notation, we have the following:*

- (a)  $\langle \bar{g} \rangle N \leq M$ .
- (b)  $(\langle \bar{g} \rangle N)' = N_{\bar{g}}$  where  $(\langle \bar{g} \rangle N)'$  denotes the commutator subgroup of  $\langle \bar{g} \rangle N$ .
- (c)  $\langle \bar{g} \rangle N \leq I_M(\varphi)$  where  $\varphi \in \text{Irr}(N)$ .
- (d) Given  $\varphi \in \text{Irr}(N)$  there exists an extension  $\eta\beta$  to  $\langle \bar{g} \rangle N$  where  $\eta = (\bar{\varphi})_{\langle \bar{g} \rangle N}$  and  $\beta$  is a projective character of  $\langle \bar{g}N \rangle$ .

*Proof.* (a) Let  $x \in \langle \bar{g} \rangle N$ . Then  $x = \bar{g}^m N$  for some  $m \in \mathbb{Z}$ . Now

$$\begin{aligned} x(\bar{g}N) &= \bar{g}^m n(\bar{g}N) = \bar{g}^m n \bar{g} N = \bar{g}^m n N \bar{g} \quad (\text{since } N \trianglelefteq \bar{G}) \\ &= \bar{g}^m N \bar{g} = N \bar{g}^{m+1}. \end{aligned}$$

Similarly,  $(\bar{g}N)x = N \bar{g}^{m+1}$ . Hence  $x \in M = N_{\bar{G}}(\bar{g}N)$  and so  $\langle \bar{g} \rangle N \leq M$ .

- (b) First suppose that  $[\bar{g}, n] \in N_{\bar{g}}$ . Then  $[\bar{g}, n] \in (\langle \bar{g} \rangle N)'$  and thus  $N_{\bar{g}} \leq (\langle \bar{g} \rangle N)'$ . Also, for  $n \in N$ , by the definition of  $N_{\bar{g}}$ , we have

$$(\bar{g}N_{\bar{g}})(nN_{\bar{g}}) = (nN_{\bar{g}})(\bar{g}N_{\bar{g}}).$$

Therefore,  $(\langle \bar{g} \rangle N / N_{\bar{g}})$  is abelian, and hence  $(\langle \bar{g} \rangle N)' \leq N_{\bar{g}}$  and we deduce that  $(\langle \bar{g} \rangle N)' = N_{\bar{g}}$ .

- (c) Let  $\varphi \in \text{Irr}(N)$ . Then  $N_{\bar{g}} \leq \text{Ker}(\varphi)$ . Now by Lemma 2.1, we have  $\bar{g}N \cap I_M(\varphi) \neq \emptyset$ . Therefore,  $\bar{g}$  lies in  $I_M(\varphi)$  and so  $\langle \bar{g} \rangle \leq I_M(\varphi)$ . Hence  $\langle \bar{g} \rangle N \leq I_M(\varphi)$ .
- (d) Notice that by part (c),  $W = \langle \bar{g} \rangle N$  is a subgroup of  $I_M(\varphi)$ . Hence  $\varphi$  is invariant under  $W$ . So we can apply the Theorem 5.8 in [26] to  $\varphi$  and  $W$ . Let  $\chi \in \text{Irr}(\langle \bar{g} \rangle N, \varphi)$ . Then by the Clifford theorem (see [26]) we obtain  $\chi = ((\bar{\varphi})_{\langle \bar{g} \rangle N})\beta = \eta\beta$  where  $\beta$  is an  $\bar{\alpha}^{-1}$ -projective character of  $\langle \bar{g} \rangle N / N = \langle \bar{g}N \rangle$  and  $\bar{\alpha}$  is the factor set of  $\langle \bar{g} \rangle N \times \langle \bar{g} \rangle N$  obtained from  $\alpha$ . If  $N$  is abelian, then  $\chi$  is linear since  $\chi_N = \varphi$  is linear (because  $\deg(\chi) = \deg(\varphi) = 1$ ).  $\square$

**Theorem 3.3** ([11]). *Let  $\bar{g} \in \bar{G}$ , so that  $\langle \bar{g} \rangle N$  is abelian. Then  $\langle \bar{g}N \rangle \leq Z(\bar{G}/N)$  and the rows of the Fischer-Clifford matrix of  $\bar{g}N$  for regular classes of the inertia group of  $\varphi$  in  $\bar{G}$  can be regarded as restrictions to  $\bar{g}N$  of the  $\bar{G}$ -orbit sums of the (projective) extension  $\eta\beta$  to  $\langle \bar{g} \rangle N$  of  $\varphi$ .*

*Proof.* See [11] and [28].  $\square$

#### 4. THE ACTION OF $O_7(3)$ ON $3^7$

We know that  $O_7(3)$  acts naturally on  $3^7$ . The action of  $O_7(3)$  on  $3^7$  gives rise to four orbits of lengths 1, 702, 728 and 756 and hence four point stabilizes  $O_7(3)$ ,  $2U_4(3)$ ,  $3^5:U_4(2)$  and  $L_4(3)$  respectively. Let  $\chi(O_7(3)|2U_4(3))$ ,  $\chi(O_7(3)|3^5:U_4(2))$  and  $\chi(O_7(3)|L_4(3))$  be the permutation characters of  $O_7(3)$  on  $2U_4(3)$ ,  $3^5:U_4(2)$  and  $L_4(3)$  respectively. Then using GAP [29] and by considering the fact that

$\chi(O_7(3)|3^7)(g) = 3^n$  for all  $g \in O_7(3)$  and for some  $n \in \{0, 1, \dots, 7\}$ , we obtain that

$$\begin{aligned}\chi(O_7(3)|2U_4(3)) &= 1a + 78a + 168a + 182a + 273a, \\ \chi(O_7(3)|3^5:U_4(2)) &= 1a + 91a + 168a + 195a + 273a, \\ \chi(O_7(3)|L_4(3)) &= 1a + 105a + 182a + 195a + 273a,\end{aligned}$$

where  $1a, 78a, 91a, 105a, 168a, 182a, 195a$  and  $273a$  are irreducible characters of  $O_7(3)$  of degrees 1, 91, 105, 168, 182, 195 and 273 respectively. Then we have

$$\begin{aligned}\chi(O_7(3)|3^7) &= 1 + I_{2U_4(3)}^{O_7(3)} + I_{3^5:U_4(2)}^{O_7(3)} + I_{L_4(3)}^{O_7(3)} \\ &= 1a + 1a + 91a + 168a + 195a + 273a + 1a + 105a + 182a + 195a \\ &\quad + 273a + 1a + 105a + 182a + 195a + 273a \\ &= 4 \times 1a + 91a + 2 \times 105a + 168a + 2 \times 182a + 3 \times 195a + 3 \times 273a\end{aligned}$$

where  $I_{2U_4(3)}^{O_7(3)}$ ,  $I_{3^5:U_4(2)}^{O_7(3)}$  and  $I_{L_4(3)}^{O_7(3)}$  are the identity characters of  $2U_4(3)$ ,  $3^5:U_4(2)$  and  $L_4(3)$  respectively induced to  $O_7(3)$ . For each class representative  $g \in O_7(3)$ ,  $\chi(O_7(3)|3^7)(g)$  will give us the number of fixed points  $k$  which we provide in Table 1.

## 5. THE INERTIA GROUPS OF $3^7 \cdot O_7(3)$

When  $O_7(3)$  acts on the conjugacy classes of  $3^7$  it produces four orbits of lengths 1, 702, 728 and 756. Hence by Brauer's theorem (see [14])  $O_7(3)$  acting on  $\text{Irr}(3^7)$  will also produce four orbits of lengths 1,  $s$ ,  $t$  and  $u$  such that  $s + t + u = 2186$ . Now by checking the indices of the maximal subgroups of  $O_7(3)$  given in the ATLAS, we can see that the only possibility is that  $s = 702$ ,  $t = 728$  and  $u = 756$ . We deduce that the four inertia groups are  $\bar{H}_i = 3^7:H_i$  of indices 1, 702, 728 and 756 in  $3^7 \cdot O_7(3)$  respectively, where  $i \in \{1, 2, 3, 4\}$  and  $H_i \leq O_7(3)$  are the inertia factors. We also observed that the inertia factors are  $H_1 = O_7(3)$ ,  $H_2 = 2U_4(3)$ ,  $H_3 = 3^5:U_4(2)$  and  $H_4 = L_4(3)$  of indices 1, 702, 728 and 756 in  $O_7(3)$  respectively. The (ordinary) character tables of  $H_1$ ,  $H_2$ ,  $H_3$  and  $H_4$  are given in the GAP [29]. We will see in Section 7 that we need to use the projective characters of  $H_2 = 2U_4(3)$ . The first step to find projective characters of a group is to find its Schur multiplier. In [15], D. Holt has developed a package called *Cohomolo* for calculating Schur multipliers of finite groups which is now available in GAP. Using the *Cohomolo* package we obtained that the Schur multiplier of  $H_2$  is the group  $C_3 \times C_6$ . The projective characters of  $H_2$  with the factor set  $\alpha^{-1}$  where  $\alpha^3 \sim 1$ , can be obtained from ordinary characters of the threefold proper covering group  $3.H_2$  of  $H_2$ . We will see later in Section 7 that the projective characters of  $H_2$  with the factor set  $\beta^{-1}$  where  $\beta^2 \sim 1$  are not required for our computations. The projective characters of  $H_2$  are not listed here, but these are just a subset of the projective characters of  $U_4(3)$  which are available in the ATLAS. In Section 7 we will use these projective characters to compute the irreducible characters of  $\bar{G} = 3^7 \cdot O_7(3)$ . Note that  $H_2$  has 31  $\alpha^{-1}$ -regular classes and 31 irreducible projective characters with the factor set  $\alpha^{-1}$ .

TABLE 1

$[g]_{O_7(3)}$	1A	2A	2B	2C	3A	3B	3C	3D	3E	3F	3G	4A
$\chi(O_7(3) 2U_4(3))$	702	2	90	6	54	90	72	0	0	18	0	0
$\chi(O_7(3) 3^5:U_4(2))$	728	0	80	8	80	62	98	26	26	8	8	0
$\chi(O_7(3) L_4(3))$	756	0	72	12	108	90	72	0	0	0	18	2
$k$	2187	3	243	27	243	243	243	27	27	9	27	3
$[g]_{O_7(3)}$	4B	4C	4D	5A	6A	6B	6C	6D	6E	6F	6G	6H
$\chi(O_7(3) 2U_4(3))$	6	2	12	12	2	2	2	18	0	12	6	6
$\chi(O_7(3) 3^5:U_4(2))$	8	0	8	8	0	0	0	8	2	2	8	14
$\chi(O_7(3) L_4(3))$	12	0	6	6	0	0	0	0	0	12	12	6
$k$	27	3	27	27	3	1	3	27	3	27	27	27
$[g]_{O_7(3)}$	6I	6J	6K	6L	6M	6N	6O	6P	7A	8A	8B	9A
$\chi(O_7(3) 2U_4(3))$	6	6	2	0	0	0	0	0	2	0	2	12
$\chi(O_7(3) 3^5:U_4(2))$	8	8	0	2	2	2	2	2	0	0	0	2
$\chi(O_7(3) L_4(3))$	12	12	0	0	0	0	0	0	0	2	0	12
$k$	27	27	1	3	3	3	3	3	3	3	3	27
$[g]_{O_7(3)}$	9B	9C	9D	10A	10B	12A	12B	12C	12D	12E	12F	12G
$\chi(O_7(3) 2U_4(3))$	6	0	0	0	2	0	6	0	0	2	0	0
$\chi(O_7(3) 3^5:U_4(2))$	14	2	2	0	0	2	8	0	0	0	2	2
$\chi(O_7(3) L_4(3))$	6	0	0	2	0	0	12	2	2	0	0	0
$k$	27	3	3	3	3	3	27	3	3	3	3	3
$[g]_{O_7(3)}$	12H	13A	13B	14A	15A	18A	18B	18C	18D	20A		
$\chi(O_7(3) 2U_4(3))$	0	0	0	2	0	2	2	2	0	0		
$\chi(O_7(3) 3^5:U_4(2))$	2	0	0	0	2	0	0	0	2	0		
$\chi(O_7(3) L_4(3))$	0	2	2	0	0	0	0	0	0	2		
$k$	3	3	3	3	3	3	3	3	3	3		

6. THE FUSION OF INERTIA FACTOR GROUPS INTO  $O_7(3)$

Using the permutation characters of  $O_7(3)$  on  $H_2 = 2U_4(3)$ ,  $H_3 = 3^5:U_4(2)$  and  $H_4 = L_4(3)$  of degrees 702, 728 and 756 respectively we are able to obtain partial fusions of  $H_2 = 2U_4(3)$ ,  $H_3 = 3^5:U_4(2)$  and  $H_4 = L_4(3)$  into  $O_7(3)$ . We completed the fusions by using direct matrix conjugation in  $O_7(3)$ . The complete fusions of  $2U_4(3)$ ,  $3^5:U_4(2)$  and  $L_4(3)$  into  $O_7(3)$  are given in Tables 2, 3 and 4 respectively.

TABLE 2. The fusion of  $2U_4(3)$  into  $O_7(3)$

$[h]_{2U_4(3)}$	$\rightarrow$	$[g]_{O_7(3)}$	$[h]_{2U_4(3)}$	$\rightarrow$	$[g]_{O_7(3)}$	$[h]_{2U_4(3)}$	$\rightarrow$	$[g]_{O_7(3)}$
1A		1A	2A		2A	2B		2C
2C		2B	3A		3A	3B		3C
3C		3B	3D		3F	4A		4B
4B		4C	4C		4D	5A		5A
6A		6A	6B		6C	6C		6B
6D		6K	6E		6G	6F		6D
6G		6J	6H		6F	6I		6I
6J		6H	7A		7A	7B		7A
8A		8B	8B		8B	9A		9B
9B		9B	9C		9A	9D		9A
10A		10B	12A		12B	12B		12E
14A		14A	14B		14A	18A		18C
18B		18C	18C		18B	18D		18A

TABLE 3. The fusion of  $3^5:U_4(2)$  into  $O_7(3)$ 

$[h]_{3^5:U_4(2)}$	$\rightarrow$	$[g]_{O_7(3)}$	$[h]_{3^5:U_4(2)}$	$\rightarrow$	$[g]_{O_7(3)}$	$[h]_{3^5:U_4(2)}$	$\rightarrow$	$[g]_{O_7(3)}$
1A		1A	2A		2C	2B		2B
3A		3B	3B		3A	3C		3C
3D		3A	3E		3D	3F		3E
3G		3G	3H		3A	3I		3D
3J		3E	3K		3G	3L		3B
3M		3G	3N		3G	3O		3F
3P		3C	3Q		3F	3R		3F
3S		3G	3T		3D	3U		3E
4A		4B	4B		4D	5A		5A
6A		6E	6B		6F	6C		6D
6D		6H	6E		6G	6F		6M
6G		6L	6H		6G	6I		6M
6J		6L	6K		6I	6L		6P
6M		6I	6N		6P	6O		6J
6P		6O	6Q		6N	6R		6H
9A		9A	9B		9B	9C		9B
9D		9D	9E		9C	9F		9B
9G		9D	9H		9C	12A		12A
12B		12A	12C		12H	12D		12B
12E		12G	12F		12F	12G		12B
12H		12G	12I		12F	15A		15A
15B		15A	18A		18D			

TABLE 4. The fusion of  $L_4(3)$  into  $O_7(3)$ 

$[h]_{L_4(3)}$	$\rightarrow$	$[g]_{O_7(3)}$	$[h]_{L_4(3)}$	$\rightarrow$	$[g]_{O_7(3)}$	$[h]_{L_4(3)}$	$\rightarrow$	$[g]_{O_7(3)}$
1A		1A	2A		2B	2B		2C
3A		3A	3B		3B	3C		3C
3D		3G	4A		4A	4B		4B
4C		4D	5A		5A	6A		6H
6B		6F	6C		6G	6D		6I
6E		6J	8A		8A	9A		9B
9B		9A	10A		10A	12A		12D
12B		12C	12C		12B	13A		13A
13B		13A	13C		13B	13D		13B
20A		20A	20B		20A			

7. THE FISCHER-CLIFFORD MATRICES OF  $3^7 \cdot O_7(3)$ 

Having obtained the fusions of the inertia factors  $2U_4(3)$ ,  $3^5:U_4(2)$  and  $L_4(3)$  into  $O_7(3)$  (Tables 2, 3 and 4) together with properties of the Fischer-Clifford matrices discussed in Sections 1, 2 and 3, we are able to compute the Fischer-Clifford matrices of the nonsplit extension  $3^7 \cdot O_7(3)$ .

Consider the coset corresponding to the identity of  $O_7(3)$ . Clearly this is a split coset, and since the action of  $O_7(3)$  on  $\text{Irr}(N)$  is self-dual, we can easily determine the column weights and hence the orders of the centralizers for the classes of elements of  $3^7 \cdot O_7(3)$  corresponding to this coset. We used the results given in Sections 1, 2 and 3 to complete the entries of the rows of  $M(1A)$ . We obtain four conjugacy classes of elements of  $\bar{G} = 3^7 \cdot O_7(3)$  of orders 1, 3, 3 and 3 respectively corresponding to the identity coset. We have the following Fischer-Clifford matrix for the identity of  $O_7(3)$ :

$$M(1A) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 702 & 0 & -27 & 27 \\ 728 & 26 & -1 & -28 \\ 756 & -27 & 27 & 0 \end{pmatrix}.$$

Let  $\text{Irr}(Fi'_{24}) = \{\psi_i : 1 \leq i \leq 108\}$  be the set of irreducible characters of  $Fi'_{24}$  as listed in the ATLAS. Then we have

$[x]_{Fi'_{24}}$	1A	3A	3B	3C
$\psi_2$	8671	247	-77	85
$\psi_3$	57477	534	615	210
$\psi_4$	249458	370	2705	869

Let  $\gamma_1, \gamma_2, \gamma_3$  and  $\gamma_4$  be the rows of the Fischer-Clifford matrix  $M(1A)$ . Since  $\langle(\psi_2)_N, 1_N\rangle = 91$ ,  $\langle(\psi_3)_N, 1_N\rangle = 483$  and  $\langle(\psi_4)_N, 1_N\rangle = 1392$ , we have the decompositions  $(\psi_2)_N = 91\gamma_1 + 6\gamma_2 + 6\gamma_3$ ,  $(\psi_3)_N = 483\gamma_1 + 21\gamma_2 + 30\gamma_3 + 27\gamma_4$  and  $(\psi_4)_N = 1392\gamma_1 + 105\gamma_2 + 145\gamma_3 + 91\gamma_4$ . Now by considering the coefficients of  $\gamma_2$ , we deduce that there are characters  $\chi_1, \chi_2, \chi_3 \in \text{Irr}(\bar{G})$ , with  $\deg(\chi_1) = 6 \times 702 = 4212$ ,  $\deg(\chi_2) = 21 \times 702 = 14742$  and  $\deg(\chi_3) = 105 \times 702 = 73710$ . Let  $[x_1 \ x_2 \ \dots \ x_s]$  be the transpose of the partial entries for the projective characters of  $H_2 = 2U_4(3)$  on  $1A \in O_7(3)$ . Then  $C_2(1A)M_2(1A)$  is a  $s \times 4$  matrix with the entries of the first column  $702x_1 = 4212$ ,  $702x_2 = 14742$  and  $702x_3 = 73710$ . Hence  $x_1 = 6$ ,  $x_2 = 21$  and  $x_3 = 105$ . This shows that the partial projective character table of  $H_2$  that we need to use should contain characters  $\hat{\beta}_{21}, \hat{\beta}_{22}$  and  $\hat{\beta}_{23}$  of degrees 6, 21 and 105 respectively. Now by checking the ordinary character table of  $H_2 = 2U_4(3)$  which is available in GAP and ATLAS we can see that  $\hat{\beta}_{21}, \hat{\beta}_{22}$  and  $\hat{\beta}_{23}$  do not come from the ordinary characters of  $H_2$  as there is no ordinary character of degree 6 in  $H_2$ . Since the Schur multiplier of  $H_2$  is the group  $C_3 \times C_6$ , then three distinct projective character tables occur corresponding to the factor sets  $\beta^{-1}, \alpha^{-1}$  and  $\delta^{-1}$  with  $\beta^2 \sim 1$ ,  $\alpha^3 \sim 1$  and  $\delta^6 \sim 1$ . Now by Lemma 3.2.6 in [1] we know that each irreducible projective character with a factor set  $\omega$  has its degree divisible by  $o([\omega])$ . Since  $o([\beta]) = 2$  and 2 does not divide 21 and since  $o([\delta]) = 6$  and 6 does not divide 105, we deduce that  $\hat{\beta}_{21}, \hat{\beta}_{22}$  and  $\hat{\beta}_{23}$  belong to the projective characters of  $H_2$  with the factor set  $\alpha^{-1}$  such that  $\alpha^3 \sim 1$ . Hence we need to use the projective characters of  $H_2$  with the factor set  $\alpha^{-1}$  to obtain the irreducible characters of  $\bar{G} = 3^7 \cdot O_7(3)$  corresponding to this inertia factor.

Similarly, by considering the coefficients of  $\gamma_3$  in the decompositions of  $(\psi_2)_N, (\psi_3)_N$  and  $(\psi_4)_N$ , we obtain that there are irreducible characters of  $\bar{G}$  of degrees 4368, 21840, 7280, 10920 and 65520. Let  $[y_1 \ y_2 \ \dots \ y_t]$  be the transpose of the partial entries for the projective character table of  $H_3 = 3^5 : U_4(2)$  on  $1A$ . Then  $C_3(1A)M_3(1A)$  is a  $t \times 4$  matrix and we obtain that  $y_1 = 6$ ,  $y_2 = 30$ ,  $y_3 = 10$ ,  $y_4 = 15$  and  $y_5 = 90$ . This shows that the partial projective character table of  $H_3$  needed, should contain characters  $\hat{\beta}_{31}, \hat{\beta}_{32}, \hat{\beta}_{33}, \hat{\beta}_{34}$  and  $\hat{\beta}_{35}$  of degrees 6, 30, 10, 15 and 90 respectively. Since the Schur multiplier of  $H_3 = 3^5 : U_4(2)$  is also the cyclic group of order 6, again three distinct projective character tables occur. Now by similar arguments as for  $H_2$  we deduce that  $\hat{\beta}_{31}, \hat{\beta}_{32}, \hat{\beta}_{33}, \hat{\beta}_{34}$ , and  $\hat{\beta}_{35}$  come from the ordinary characters of  $H_3$ . Hence we need to use the ordinary characters of  $H_3$ . The character table of  $H_3 = 3^5 : U_4(2)$  is available in GAP [29].

Finally, by considering the coefficients of  $\gamma_4$  in the decompositions of  $(\psi_3)_N$  and  $(\psi_4)_N$ , it can be shown that the partial projective character table of  $H_4 = L_4(3)$  needed, contains a character of degree 1. Hence this partial projective character table comes from the ordinary characters of  $H_4 = L_4(3)$ .

To summarize, for our computations of Fischer-Clifford matrices we need to use the projective characters of  $H_2 = 2U_4(3)$  corresponding to the factor set  $\alpha^{-1}$  with

$\alpha^3 \sim 1$ , the ordinary characters of  $H_1 = O_7(3)$ ,  $H_3 = 3^5:U_4(2)$  and  $H_4 = L_4(3)$ . This forces the shape of all Fischer-Clifford matrices by the results of Section 2. We produce altogether 183 conjugacy classes of elements of  $\bar{G}$ .

We use the results of Sections 1 and 2 to compute the entries of the Fischer-Clifford matrices of  $\bar{G}$  for split cosets. The fusion of  $\bar{G}$  into  $Fi'_{24}$  together with the restriction of characters of  $Fi'_{24}$  to  $\bar{G}$  forces the signs of the Fischer-Clifford matrices and the orders of the elements of  $\bar{G}$ .

For example, consider the conjugacy class  $3A$  of  $O_7(3)$ . Let  $g \in 3A$ . Since there are fusions from the  $\alpha$ -regular class  $3A$  of  $H_2$  and from classes of  $H_3$  and  $H_4$  into  $3A$  of  $O_7(3)$ , we deduce that the coset  $\bar{g}N$  is a split coset. We obtain the following form of the Fischer-Clifford matrix  $M(3A)$  with the possibility of change of signs for rows:

$$\begin{array}{l}
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 \end{array}
 \begin{pmatrix}
 306110016 & 25509168 & 1417176 & 51018336 & 34012224 & 34012224 \\
 1259712 & 1 & 1 & 1 & 1 & 1 \\
 23328 & 54 & 0 & 0 & 27 & -27 & -27 \\
 157464 & 8 & 8 & -1 & 8 & 8 & 8 \\
 34992 & 36 & 9 & 0 & -18 & 18\alpha - 9\bar{\alpha} & -9\alpha + 18\bar{\alpha} \\
 34992 & 36 & 9 & 0 & -18 & -9\alpha + 18\bar{\alpha} & 18\alpha - 9\bar{\alpha} \\
 11664 & 108 & -27 & 0 & 0 & 27 & 27 \\
 & 1 & 12 & 216 & 6 & 4 & 4
 \end{pmatrix}$$

where  $\alpha = (-1 + \sqrt{-3})/2$  and  $\bar{\alpha} = (-1 - \sqrt{-3})/2$ .

Let  $3D, 3E, 3F, 3G, 3H$  and  $3I$  be conjugacy classes of  $\bar{G}$ , obtained by corresponding to this coset. First suppose that the above matrix is the Fischer-Clifford matrix  $M(3A)$  as it is. Then by considering the restriction of  $\psi_2 \in \text{Irr}(Fi'_{24})$  to  $\bar{G}$ , we notice that there will be no fusion from the  $3D$  class of  $\bar{G}$  into  $Fi'_{24}$ . Hence this is not the required Fischer-Clifford matrix and the sign of the rows has to be changed. Now if we multiply rows 4 and 5 by  $\alpha = (-1 + \sqrt{-3})/2$  and  $\bar{\alpha} = (-1 - \sqrt{-3})/2$  respectively, then the resulting character table of  $\bar{G}$  does not satisfy the congruence relations (Lemma 2.8). By checking various other possibilities we deduce that rows 4 and 5 have to be multiplied by  $\bar{\alpha}$  and  $\alpha$  respectively in order to get the proper character table of  $\bar{G}$ . Hence

$$M(3A) = \begin{pmatrix}
 1 & 1 & 1 & 1 & 1 & 1 \\
 54 & 0 & 0 & 27 & -27 & -27 \\
 8 & 8 & -1 & 8 & 8 & 8 \\
 36\bar{\alpha} & 9\bar{\alpha} & 0 & -18\bar{\alpha} & 18 - 9\alpha & -9 + 18\alpha \\
 36\alpha & 9\alpha & 0 & -18\alpha & -9\bar{\alpha} + 18 & 18\bar{\alpha} - 9 \\
 108 & -27 & 0 & 0 & 27 & 27
 \end{pmatrix}.$$

Now consider the coset corresponding to the class  $3B$  of  $O_7(3)$ . This is a nonsplit coset since there is a fusion from  $\alpha$ -irregular class  $3C$  of  $H_2$ . Using Programme C, given in [1], we find the appropriate column weights. We obtained the centralizer orders of the relevant classes by considering these column weights and by using the restriction of irreducible characters of  $Fi'_{24}$  to  $\bar{G}$ . We have the matrix corresponding to this coset in the following shape:

$$\begin{array}{cccc}
 & 7085880 & 2834352 & 708588 & 262440 \\
 174960 & \left( \begin{array}{cccc}
 1 & 1 & 1 & 1 \\
 2 & a & f & m \\
 -30 & b & g & n \\
 45 & c & h & p
 \end{array} \right) \\
 87480 & & & & \\
 2916 & & & & \\
 1944 & & & & \\
 & 2 & 5 & 20 & 54
 \end{array}$$

Notice that the restrictions of irreducible characters of  $Fi'_{24}$  to  $\bar{G}$  determine the entries for the first column. We find the remaining entries of  $M(3B)$  using properties of the Fischer-Clifford matrices discussed in Sections 1 and 3. We have

$$M(3B) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & -1 \\ -30 & 24 & -3 & 0 \\ 45 & 18 & -9 & 0 \end{pmatrix}.$$

We use similar types of arguments to compute all other Fischer-Clifford matrices and conjugacy classes of  $\bar{G}$ . The complete list of Fischer-Clifford matrices and conjugacy classes of  $\bar{G}$  is given in Tables 5 and 6 respectively. Please note that the fusion of elements of  $\bar{G}$  into  $Fi'_{24}$  are given in the last column of Table 6.

TABLE 5. The Fischer-Clifford matrices of  $3^7 \cdot O_7(3)$

$M(g)$	$M(g)$
$M(1A) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 702 & 0 & -27 & 27 \\ 728 & 26 & -1 & -28 \\ 756 & -27 & 27 & 0 \end{pmatrix}$	$M(2A) = \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$
$M(2B) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 90 & -9 & 0 & 9 \\ 80 & 8 & -10 & -1 \\ 72 & 0 & 9 & -9 \end{pmatrix}$	$M(2C) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 6 & 3 & -3 & 0 \\ 8 & -4 & -1 & 2 \\ 12 & 0 & 3 & -3 \end{pmatrix}$
$M(3A) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 54 & 0 & 0 & 27 & -27 & -27 \\ 8 & 8 & -1 & 8 & 8 & 8 \\ 36\bar{\alpha} & 9\bar{\alpha} & 0 & -18\bar{\alpha} & 18 - 9\alpha & -9 + 18\alpha \\ 36\alpha & 9\alpha & 0 & -18\alpha & -9\bar{\alpha} + 18 & 18\bar{\alpha} - 9 \\ 108 & -27 & 0 & 0 & 27 & 27 \end{pmatrix}$	$M(3B) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ -2 & 2 & 2 & -1 \\ -30 & 24 & -3 & 0 \\ 45 & 18 & -9 & 0 \end{pmatrix}$
$M(3C) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 72 & -9 & 0 & 18 & -9 \\ 2 & 2 & -1 & 2 & 2 \\ 96 & 15 & 0 & -12 & -12 \\ 72 & -9 & 0 & -9 & 18 \end{pmatrix}$	$M(3D) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & \alpha & \bar{\alpha} \\ 1 & 1 & \bar{\alpha} & \alpha \\ 24 & -3 & 0 & 0 \end{pmatrix}$
$M(3E) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & \alpha & \bar{\alpha} \\ 1 & 1 & \bar{\alpha} & \alpha \\ 24 & -3 & 0 & 0 \end{pmatrix}$	$M(3F) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 4 & -2 & -2 & 1 \\ 2 & -1 & 2 & -1 \\ 2 & 2 & -1 & -1 \end{pmatrix}$
$M(3G) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 2\bar{\alpha} & -1 & -\alpha & -\bar{\alpha} & 2\alpha & 2 & 2\bar{\alpha} \\ 2\alpha & -1 & -\bar{\alpha} & -\alpha & 2\bar{\alpha} & 2 & 2\alpha \\ \alpha & 1 & \bar{\alpha} & \alpha & \bar{\alpha} & 1 & \alpha \\ \bar{\alpha} & 1 & \alpha & \bar{\alpha} & \alpha & 1 & \bar{\alpha} \\ 2 & -1 & -1 & -1 & 2 & 2 & 2 \\ 18 & 0 & 0 & 0 & 0 & 0 & -9 \end{pmatrix}$	$M(4A) = \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$
$M(4B) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 6 & 3 & -3 & 0 \\ 8 & -4 & -1 & 2 \\ 12 & 0 & 3 & -3 \end{pmatrix}$	$M(4C) = \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$

TABLE 5. The Fischer-Clifford matrices of  $3^7 \cdot O_7(3)$  (continued)

$M(g)$	$M(g)$
$M(4D) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 12 & 0 & 3 & -3 \\ 8 & -4 & -1 & 2 \\ 6 & 3 & -3 & 0 \end{pmatrix}$	$M(5A) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 12 & 0 & 3 & -3 \\ 8 & -4 & -1 & 2 \\ 6 & 3 & -3 & 0 \end{pmatrix}$
$M(6A) = \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$	$M(6B) = ( 1 )$
$M(6C) = \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$	$M(6D) = \begin{pmatrix} 1 & 1 & 1 \\ 18 & -9 & 0 \\ 8 & 8 & -1 \end{pmatrix}$
$M(6E) = \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$	$M(6F) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 12 & -6 & 3 & 0 \\ 2 & 2 & 2 & -1 \\ 12 & 3 & -6 & 0 \end{pmatrix}$
$M(6G) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 6 & -3 & -3 & 3 & 0 \\ 4\bar{\alpha} & -1+2\alpha & 2-\alpha & -2\bar{\alpha} & \bar{\alpha} \\ 4\alpha & 2\bar{\alpha}-1 & -\bar{\alpha}+2 & -2\alpha & \alpha \\ 12 & 3 & 3 & 0 & -3 \end{pmatrix}$	$M(6H) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 6 & 6 & -3 & -3 & 0 \\ 2 & 2 & 2 & 2 & -1 \\ 12 & -6 & -6 & 3 & 0 \\ 6 & -3 & 6 & -3 & 0 \end{pmatrix}$
$M(6I) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 6 & -3 & -3 & 3 & 0 \\ 4\bar{\alpha} & -1+2\alpha & 2-\alpha & -2\bar{\alpha} & \bar{\alpha} \\ 4\alpha & 2\bar{\alpha}-1 & -\bar{\alpha}+2 & -2\alpha & \alpha \\ 12 & 3 & 3 & 0 & -3 \end{pmatrix}$	$M(6J) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 6 & 3 & -3 & 0 \\ 8 & -4 & -1 & 2 \\ 12 & 0 & 3 & -3 \end{pmatrix}$
$M(6K) = ( 1 )$	$M(6L) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & \alpha & \bar{\alpha} \\ 1 & \bar{\alpha} & \alpha \end{pmatrix}$
$M(6M) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & \alpha & \bar{\alpha} \\ 1 & \bar{\alpha} & \alpha \end{pmatrix}$	$M(6N) = \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$
$M(6O) = \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$	$M(6P) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & \alpha & \bar{\alpha} \\ 1 & \bar{\alpha} & \alpha \end{pmatrix}$
$M(7A) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & \alpha & \bar{\alpha} \\ 1 & \bar{\alpha} & \alpha \end{pmatrix}$	$M(8A) = \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$
$M(8B) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & \alpha & \bar{\alpha} \\ 1 & \bar{\alpha} & \alpha \end{pmatrix}$	$M(9A) = \begin{pmatrix} 1 & 1 & 1 \\ -1 & -1 & 2 \\ 6 & -3 & 0 \end{pmatrix}$
$M(9B) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 3\alpha & 3\bar{\alpha} & 3 & 0 & 3\bar{\alpha} & 3 & 3\alpha \\ 3\bar{\alpha} & 3\alpha & 3 & 0 & 3\alpha & 3 & 3\bar{\alpha} \\ 2 & 2 & 2 & -1 & 2 & 2 & 2 \\ 6\alpha & -3 & -3\bar{\alpha} & 0 & 6 & 6\bar{\alpha} & -3\alpha \\ 6\bar{\alpha} & -3 & -3\alpha & 0 & 6 & 6\alpha & -3\bar{\alpha} \\ 6 & -3 & -3 & 0 & 6 & 6 & -3 \end{pmatrix}$	$M(9C) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & \alpha & \bar{\alpha} \\ 1 & \bar{\alpha} & \alpha \end{pmatrix}$
$M(9D) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & \alpha & \bar{\alpha} \\ 1 & \bar{\alpha} & \alpha \end{pmatrix}$	$M(10A) = \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$
$M(10B) = \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$	$M(12A) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & \alpha & \bar{\alpha} \\ 1 & \bar{\alpha} & \alpha \end{pmatrix}$

TABLE 5. The Fischer-Clifford matrices of  $3^7 \cdot O_7(3)$  (continued)

$M(g)$	$M(g)$
$M(12B) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 6 & -3 & -3 & 3 & 0 \\ 4\bar{\alpha} & -1+2\alpha & 2-\alpha & -2\bar{\alpha} & \bar{\alpha} \\ 4\alpha & 2\bar{\alpha}-1 & -\bar{\alpha}+2 & -2\alpha & \alpha \\ 12 & 3 & 3 & 0 & -3 \end{pmatrix}$	$M(12C) = \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$
$M(12D) = \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$	$M(12E) = \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$
$M(12F) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & \alpha & \bar{\alpha} \\ 1 & \bar{\alpha} & \alpha \end{pmatrix}$	$M(12G) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & \alpha & \bar{\alpha} \\ 1 & \bar{\alpha} & \alpha \end{pmatrix}$
$M(12H) = \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$	$M(13A) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & \alpha & \bar{\alpha} \\ 1 & \bar{\alpha} & \alpha \end{pmatrix}$
$M(13B) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & \alpha & \bar{\alpha} \\ 1 & \bar{\alpha} & \alpha \end{pmatrix}$	$M(14A) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & \alpha & \bar{\alpha} \\ 1 & \bar{\alpha} & \alpha \end{pmatrix}$
$M(15A) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & \alpha & \bar{\alpha} \\ 1 & \bar{\alpha} & \alpha \end{pmatrix}$	$M(18A) = ( 1 )$
$M(18B) = ( 1 )$	$M(18C) = \begin{pmatrix} 1 & 1 & 1 \\ -1 & -1 & 2 \\ 6 & -3 & 0 \end{pmatrix}$
$M(18D) = \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$	$M(20A) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & \alpha & \bar{\alpha} \\ 1 & \bar{\alpha} & \alpha \end{pmatrix}$

$$\alpha = (-1 + \sqrt{-3})/2, \quad \bar{\alpha} = (-1 - \sqrt{-3})/2$$

TABLE 6. The conjugacy classes of  $3^7 \cdot O_7(3)$

$[g]_{O_7(3)}$	$[x]_{3^7 \cdot O_7(3)}$	$ C_{3^7 \cdot O_7(3)}(x) $	$\longrightarrow$ $Fi'_{24}$
1A	1A	10028164124160	1A
	3A	13264767360	3A
	3B	13774950720	3B
	3C	14285134080	3C
2A	2A	39191040	2B
	6A	19595520	6B
2B	2B	50388480	2A
	6B	559872	6F
	6C	699840	6A
	6D	629856	6D
2C	2C	373248	2B
	6E	62208	6I
	6F	46656	6E
	6G	31104	6D

TABLE 6. The conjugacy classes of  $3^7 \cdot O_7(3)$  (continued)

$[g]_{O_7(3)}$	$[x]_{3^7 \cdot O_7(3)}$	$ C_{3^7 \cdot O_7(3)}(x) $	$\rightarrow F_{24}^i$
3A	3D	153055008	3D
	3E	12754584	3C
	3F	708588	3D
	3G	25509168	3B
	3H	38263752	3D
	3I	38263752	3A
3B	3J	7085880	3B
	3K	2834352	3C
	3L	708588	3D
	9A	262440	9B
3C	3M	34012224	3A
	3N	1062882	3D
	9B	209952	9C
	3O	1417176	3C
	3P	1417176	3D
3D	9C	472392	9A
	9D	59049	9D
	9E	52488	9C
	3Q	52488	3E
3E	9F	472392	9A
	9G	59049	9D
	9H	52488	9C
	3R	52488	3E
3F	3S	26244	3E
	9I	13122	9E
	9J	13122	9E
	9K	6561	9D
3G	9L	39366	9A
	9M	6561	9C
	9N	6561	9E
	9O	6561	9B
	3T	13122	3D
	9P	13122	9E
	9Q	19683	9D
4A	4A	8640	4B
	12A	4320	12D
4B	4B	31104	4A
	12B	5184	12E
	12C	3888	12B
	12D	2592	12A
4C	4C	1152	4C
	12E	576	12K

TABLE 6. The conjugacy classes of  $3^7 \cdot O_7(3)$  (continued)

$[g]_{O_7(3)}$	$[x]_{3^7 \cdot O_7(3)}$	$ C_{3^7 \cdot O_7(3)}(x) $	$\rightarrow F_{24}'$
4D	4D	5184	4B
	12F	648	12D
	12G	864	12G
	12H	432	12L
5A	5A	3240	5A
	15A	540	15A
	15B	405	15B
	15C	270	15C
6A	6H	69984	6H
	6I	34992	6E
6B	6J	3888	6I
6C	6K	11664	6D
	6L	5832	6J
6D	6M	69984	6G
	6N	34992	6C
	6O	2916	6G
6E	6P	5184	6D
	18A	2592	18A
6F	6Q	23328	6A
	6R	5832	6F
	6S	5832	6G
	18B	1296	18B
6G	6T	23328	6J
	6U	5832	6D
	6V	5832	6J
	6W	3888	6E
	6X	1944	6I
6H	6Y	11664	6F
	6Z	5832	6C
	6AA	5832	6F
	6AB	2916	6G
	18C	648	18D
6I	6AC	11664	6B
	6AD	2916	6E
	6AE	2916	6H
	6AF	1944	6I
	6AG	972	6J
6J	6AH	11664	6D
	6AI	1944	6H
	6AJ	1458	6J
	6AK	972	6I
6K	6AL	324	6K

TABLE 6. The conjugacy classes of  $3^7 \cdot O_7(3)$  (continued)

$[g]_{O_7(3)}$	$[x]_{3^7 \cdot O_7(3)}$	$ C_{3^7 \cdot O_7(3)}(x) $	$\rightarrow F_{24}^i$
6L	18D	648	18C
	18E	648	18A
	6AM	648	6K
6M	18F	648	18C
	18G	648	18A
	6AN	648	6K
6N	6AO	324	6K
	18H	162	18E
6O	6AP	324	6K
	18I	162	18E
6P	18J	162	18E
	18K	162	18C
	6AQ	162	6J
7A	7A	42	7B
	21A	42	21C
	21B	42	21D
8A	8A	48	8A
	24A	24	24A
8B	8B	48	8C
	24B	48	24F
	24C	48	24G
9A	9R	1458	9B
	9S	729	9E
	9T	972	9F
9B	9U	4374	9E
	9V	2187	9C
	9W	2187	9D
	9X	243	9F
	9Y	4374	9C
	9Z	4374	9A
	9AA	2187	9E
9C	27A	81	27B
	27B	81	27C
	27C	81	27A
9D	27D	81	27C
	27E	81	27B
	27F	81	27A
10A	10A	120	10A
	30A	60	30A
10B	10B	60	10B
	30B	30	30B

TABLE 6. The conjugacy classes of  $3^7 \cdot O_7(3)$  (continued)

$[g]_{O_7(3)}$	$[x]_{3^7 \cdot O_7(3)}$	$ C_{3^7 \cdot O_7(3)}(x) $	$\rightarrow F_{24}'$
12A	12I 36A 36B	432 432 432	12C 36A 36B
12B	12J 12K 12L 12M 12N	3888 972 972 648 324	12F 12A 12F 12B 12E
12C	12O 12P	216 108	12D 12L
12D	12Q 12R	216 108	12L 12G
12E	12S 12T	144 72	12M 12H
12F	36C 36D 12U	108 108 108	36C 36B 12I
12G	36E 36F 12V	108 108 108	36C 36A 12J
12H	12W 36G	72 36	12L 36D
13A	13A 39A 39B	39 39 39	13A 39A 39A
13B	13B 39C 39D	39 39 39	13A 39B 39B
14A	14A 42A 42B	42 42 42	14B 42B 42C
15A	15D 45A 45B	45 45 45	15B 45A 45B
18A	18L	108	18G
18B	18M	108	18H
18C	18N 18O 18P	162 162 162	18C 18E 18A
18D	18Q 18R	108 54	18F 18D
20A	20A 60A 60B	60 60 60	20A 60A 60A

The character table of  $\bar{G} = 3^7 \cdot O_7(3)$  can be obtained by using the Fischer-Clifford matrices (Table 5), the projective characters of the inertia factors  $H_2 = 2U_4(3)$  with the factor set  $\alpha^{-1}$  and the ordinary character tables of  $H_3 = 3^5 : U_4(2)$  and  $H_4 = L_4(3)$  together with the fusions of  $H_2$ ,  $H_3$  and  $H_4$  into  $O_7(3)$  (Tables 3, 4 and 5). The full character table of  $3^7 \cdot O_7(3)$  which was also calculated independently by Alexander Hulpke, is available in GAP [29].

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