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ERRATUM TO "THE STRUCTURE OF RATIONAL AND RULED SYMPLECTIC 4-MANIFOLDS"

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Francois Lalonde pointed out that Theorem 1.3 in [2] about the structure of symplectic S^2-bundles needs an extra hypothesis. The argument which proves uniqueness works only for a restricted range of cohomology classes, and the condition $a^2(V) > (a(F))^2$ need not hold when M has genus > 0. The mistake occurred in Lemma 4.15 where I failed to realise that the integral of the form ρ over the section Γ is not zero in general, but depends on the homology class of Γ. Thus this Lemma holds only under certain restrictions on $[\omega]$.

The methods developed in [2] suffice to prove the following theorems which replace Theorem 1.3, Corollary 1.5(iii), and Proposition 4.17: for complete details, see [3]. We will say that two symplectic forms ω_0 and ω_1 are pseudo-isotopic if they may be joined by a family of not necessarily cohomologous symplectic forms.

Theorem 1. *Let ω be a symplectic form on $M \times S^2$ which is compatible with the projection onto the Riemann surface M.*

(i) *If M is S^2 or T^2, ω is isotopic to a split form.*

(ii) *If M has genus $g > 1$, the statement in* (i) *holds provided that $\mu_1 > q\mu_2$, where $\mu_1 = \omega(M \times \mathrm{pt})$, $\mu_2 = \omega(\mathrm{pt} \times S^2)$ and $q = [\frac{g}{2}]$.*

(iii) *ω is always pseudo-isotopic to a split form. Moreover, it is isotopic to a split form iff it has a symplectic section in class $[M \times \mathrm{pt}]$.*

In the case of the nontrivial S^2-bundle V_M over M, we write $\{b_-, b_+\}$ for the basis of $H^2(M)$ dual to the homology basis $\{[M_-], [M_+]\}$ where M_- (resp. M_+) is a section with self-intersection -1 (resp. $+1$).

Theorem 2. (i) *When $M = S^2$, the class $a = \mu_+ b_+ + \mu_- b_-$ may be represented by a compatible symplectic form on V_M only if $\mu_+ > \mu_- > 0$. Moreover, up to isotopy there is a unique such form in each class.*

(ii) *If M has genus $g > 0$, every class with $\mu_+ > |\mu_-| > 0$ has a compatible symplectic representative. There is a unique form up to isotopy in each class such that $q\mu_- > (q-1)\mu_+$, where $q = [\frac{g+1}{2}]$.*

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(iii) *All these forms are pseudo-isotopic.*

Other inaccuracies:

1. In Lemma 4.9 one must assume that $\omega_0 = \omega_1$ at the point $S_L \cap F$. Similarly in Proposition 4.18. A corrected version of Proposition 4.18 is given in [3].

2. The proof of Lemma 4.16 is not quite right because the complex structure J_1' has too many cusp-curves. However, this is easy to rectify: one just has to replace J_1' by a generic integrable J. This is discussed in detail in [3].

3. The proof of (5.5) is inadequate because when I was considering blowing down I did not allow for the possibility that the intersection number $C \cdot \Sigma$ might be ≥ 2, so that the blow down of C would no longer be embedded. The given arguments prove that the category under consideration is closed under pseudo-isotopy, under blowing up, and under blowing down when $C \cdot \Sigma \leq 1$. When $C \cdot \Sigma \geq 2$, we reduce to the corresponding result in the integrable case as follows. Let $(\overline{V}, \overline{\omega})$ be a minimal reduction of (V, ω) obtained by blowing down a family of exceptional spheres Σ_i which includes Σ. Then C descends to an immersed J-holomorphic sphere in $\overline{V}$ which is not embedded. If $\overline{V}$ were an S^2-bundle over a Riemann surface M of genus > 0 the projection of C onto M would be a map of positive degree, which is impossible. Therefore, $(\overline{V}, \overline{\omega})$ is rational, and hence Kähler. Furthermore, there is a Kähler form ω' on V which is pseudo-isotopic to ω. (Take ω' so that its integral over the Σ_i is small.) By deforming ω' further, we may assume that the Kähler manifold (V, ω') is a blow-up of $\mathbb{C}P^2$ rather than $S^2 \times S^2$. By Lemma 3.1, the homology class $[\Sigma]$ is represented by a J-holomorphic curve or cusp-curve, for each ω'-tame J. If we choose J to be integrable and good and generic in the sense of [1, Ch. III], then $[\Sigma]$ must be represented by a curve Σ_J. Then Σ_J is isotopic to Σ, and, because our category is closed under pseudo-isotopy, it suffices to show that the complex surface obtained from V by blowing down Σ_J contains a suitable embedded rational curve. But this follows from the Castelnuovo criterion.

References

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