ERRATUM TO “REAL BOUNDS, ERGODICITY AND NEGATIVE SCHWARZIAN FOR MULTIMODAL MAPS”

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In Part 1 of Theorem C of the paper Real bounds, ergodicity and negative Schwarzian for multimodal maps, see [1], the assumption that $V$ is nice was, by mistake, omitted. We would like to thank Weixiao Shen for pointing this out. The correct version of Theorem C(1) is as follows:

**Theorem C(1) (Improved Macroscopic Koebe Principle).** Assume that $f : M \to M$ is contained in $A^{1+\text{Zygmund}}$. Then for each $\xi > 0$, there exists $\xi' > 0$ such that if $I$ is a nice interval, $V$ is nice and $\xi$-well-inside $I$ and $x \in I$, $f^k(x) \in V$ (with $k \geq 1$ not necessarily minimal), then the pullback of $V$ along $\{x, \ldots, f^k(x)\}$ is $\xi'$-well-inside the return domain to $I$ containing $x$.

Here, as before, we define an open interval $K$ to be nice if no iterate of $\partial K$ enters $K$. This implies that if $K_1$ and $K_2$ are pullbacks of $K$, then they are either disjoint or nested.

In Lemma 9 (page 762) it was implicitly assumed that $V$ is disjoint from $J_n$. It is for this reason that the proof of Theorem C(1) does not work unless we assume $V$ is nice (or something similar). The proof of Theorem C(1) as stated above is essentially the same as before, using Lemma 6 below instead of Lemma 6; then in Lemma 9 (page 762) we do not need to require that $k_n+1$ is a jump time provided we assume that $V$ is nice. Making the additional assumption that $V$ is nice, Proposition 1 (and its proof) and the rest of the paper go through unchanged.

**Lemma 6’.** For each $\rho > 0$ sufficiently small, there exists $\delta_3 > 0$ such that if $I$ is a $\rho$-scaled neighbourhood of a nice interval $V \subset I$, then $J$ is a $\delta_3$-scaled neighbourhood of any component $A$ of $\phi_j^{-k}(V)$ (where $k \geq 1$ is arbitrary).

**Proof.** Let $V_i$, $i = 0, \ldots, k$ be the component of $\phi_j^{-k}(V)$ containing $\phi_j(A)$. Of course, we may assume that $k$ is large and that $V_0, \ldots, V_k$ are disjoint.

**Claim.** There exists $\alpha > 0$ such that if $0 \leq j < k$ and $V_{j+1}$ is contained in a neighbourhood of $V_j$ of size $(1 + \alpha)|V_j|$, then $V_j$ is $\alpha$-well-inside $I$. Similarly, if $0 \leq j < k - 1$ and $V_j$ lies between $V_{j+1}$ and $V_{j+2}$, then $V_j$ is $\alpha$-well-inside $I$.

**Proof of Claim.** If $V_{j+1}$ is contained in a neighbourhood of $V_j$ of size $(1 + \alpha)|V_j|$ and $\alpha$ is small enough, then $\phi'$ is close to zero on a definite neighbourhood of $V_j$. So $V_j$ is contained in the basin of an attracting fixed point with multiplier close to zero. Since $V_k$ is nice and $\delta_3$-well-inside $I$, we easily get that $V_j$ is $\delta_3$-well-inside $I$. 

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I, proving the first part of the claim. The second part of the claim follows in the same way by first applying part 1 to $V_j, V_{j+1}$ and then applying it to $V_j, V_{j+2}$ while considering $\phi^2$ instead of $\phi$, completing the proof of the claim.

If both sides of $J$ are small, then $|\phi'|$ is bounded on $J$. There are three possibilities.

(a) $V_1$ lies between $V_2$ and $V_3$, in which case, by the second part of the Claim, $V_1$ is well-inside $I$.

(b) $V_2$ lies between $V_1$ and $V_3$; in this case since $|V_1|/|V_2|$ is not small and by the first part of the Claim, $V_2$ is well-inside $I$.

(c) $V_3$ lies between $V_1$ and $V_2$; then, because $|V_1|/|V_2|$ and $|V_2|/|V_3|$ are not small, $V_3$ is well-inside $I$. In all cases, we get that $V_0$ is well-inside $J$.

So assume one of the sides, say the right side, of $J$ is not small. Let $1 \leq j \leq k$ be the largest integer so that $V_1, \ldots, V_j$ are all not $\alpha$-well-inside $I$. By taking $\alpha > 0$ small, we may assume $j \leq k - 2$. Since $V_{j+1}$ is $\alpha$-well-inside $I$, we may assume that $j \geq 1$ and that there exists $\alpha' > 0$ so that $V_j, V_{j-1}$ are $\alpha'$-well-inside $J \subset I$. By the claim, for each $i = 1, \ldots, j$, $V_i$ has an $\alpha$-small and an $\alpha$-big side, and $V_{i+1}$ is contained in the $\alpha$-big side. Since the right side of $J$ is not small, $V_1, \ldots, V_{j+1}$ lie therefore ordered from left to right. If for each $i = 1, \ldots, j-1$, $V_{i-1}$ is contained in a $\beta$-scaled neighbourhood of $V_i$, then $V_j$ is in a $(\beta + \beta^2 + \cdots + \beta^j)$-scaled neighbourhood of $V_{j-1}$. So taking $\beta \in (0, 1)$ so small that $\beta/(1 - \beta) < \alpha'/2$, then, because $V_{j-1}$ is $\alpha'$-well-inside $I$, the left component of $I \setminus V_1$ has at least size $\frac{\alpha'}{\beta^j} |V_{j-1}| > \frac{\alpha'}{\beta} |V_1|$, i.e., $V_1$ is well-inside $I$, and $V_0$ is well-inside $J$. Hence we may assume there exists $i \in \{1, \ldots, j-1\}$ so that $V_{i-1}$ is not contained in a $\beta$-scaled neighbourhood of $V_i$. This and the first part of the Claim imply that $V_i$ is well-inside the convex hull $H_i := [V_{i-1}, V_{i+1}]$ of $V_{i-1}$ and $V_{i+1}$. Because the intervals $V_1, \ldots, V_{j+1}$ lie ordered, it follows that the pullback of $H_i$ along $V_1, \ldots, V_i$ has intersection multiplicity at most 4 and therefore that $V_1$ is well-inside $I$. This again gives that $V_0$ is well-inside $J$. (This method of proof can also be used to provide a slightly shorter proof of Lemma 5.) \[ \square \]

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