ERRATA TO
"CLASSIFICATION OF THREE-DIMENSIONAL FLIPS"

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This paper makes two corrections to \[\text{[KM92]}\].

Remark 1. The statement \[\text{[KM92], (2.2.3)}\] is false as it stands. \[\text{(2.2.3)}\] comes out of two sources: one from \[\text{(2.13.4)}\] and the other from \[\text{(2.13.10)}\]. The latter \[\text{(2.13.10)}\] is correct and proves that \(m \geq 5\) and the index-two point is of type \(cA/2\). However, the former \[\text{(2.13.4)}\] proves only a weaker assertion that \(m \geq 3\) (not \(m \geq 5\)), the index-two point is of type \(cA/2\), \(cAx/2\), or \(cD/2\) (not just of type \(cA/2\)) and with axial multiplicity \(k \geq 2\) (because the index-one cover is not smooth). Since \(f : X \supset C \rightarrow Y \ni Q\) is divisorial in the case \[\text{(2.13.4)}\] \[\text{[KM92, Prop. (9.3)}\], the following revision \[\text{(2.2.3)}_{\text{rev}}\] should replace \[\text{(2.2.3)}\].

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\text{(2.2.3)}_{\text{rev}} \text{Case for exceptional IA + IA: The two IA points are an ordinary point of odd index } m \text{ (} m \geq 5 \text{ if } f \text{ is isolated (that is, a flipping contraction)); } m \geq 3 \text{ if divisorial) and an index-two point (of type } cA/2 \text{ if } f \text{ is isolated; of type } cA/2, cAx/2 \text{ or } cD/2 \text{ if divisorial) and with axial multiplicity } k \text{ with } 2k + m \geq 7, \text{ and we have } (K_X \cdot C) = -1/(2m). \text{ } (E_Y, Q) \text{ is } D_{2k+m}, \text{ Sing}_X \text{ is } A_{m-1} + D_{2k} \left( A_{m-1} + A_1 + A_1 \text{ if } k = 1 \right) \text{ and } \Delta(E_X \supset C) \text{ is }
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Remark 2. \[\text{[KM92, Lemma (2.12.9)}\] holds under an extra assumption that \(f|_{X \setminus C} : X \setminus C \rightarrow Y \setminus \{Q\}\) is an isomorphism since it is used in the second line of the proof. The following Lemma 3 can be used as a substitute for \[\text{[KM92, Lemma (2.12.9)}\]. The arguments in \[\text{[KM92, (2.12]}\] work after this modification.

Lemma 3. Let \(f : X \rightarrow (Y, Q)\) be a projective bimeromorphic morphism of irreducible normal 3-folds such that \(R^1 f_* \mathcal{O}_X = 0 \text{ and } C = f^{-1}(Q)_{\text{red}}\) is 1-dimensional. Let \(I \subset \mathcal{O}_X\) be a sheaf of ideals such that \(\text{Supp } \mathcal{O}_X/I = C\). For each \(n > 0\), let \(I^{(n)}\) be the sheaf of ideals such that \(I^n \subset I^{(n)} \subset \mathcal{O}_X\) and \(I^{(n)}/I^n\) is the largest subsheaf of \(\mathcal{O}_X/I^n\) with 0-dimensional support. Then \(\chi(\mathcal{O}_X/I^{(n)}) \geq O(n^3)\) as \(n\) grows.

Proof. Since \(f\) is bimeromorphic, let \(E \subset X\) be an effective Cartier divisor with very ample \(\mathcal{O}(-E)\). We note that \(E \supset C\) because \(E \cdot C_i < 0\) for each irreducible

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component $C_i$ of $C$. Since $f^{-1}(Q)$ is 1-dimensional, we can choose a hyperplane section $D \supset Q$ of $Y \ni Q$ such that $D \cap E$ is a curve, where $D = f^*(D)$.

For each subspace $Z \subset X$, let $I_Z \subset \mathcal{O}_X$ denote the sheaf of ideals such that $\mathcal{O}_X/I_Z = \mathcal{O}_Z$. Let $F \subset \mathcal{O}_X$ be any coherent subsheaf such that $\text{Supp} \mathcal{O}_X/F$ is a curve containing $C$. We consider a primary decomposition $F = \bigcap Q_i$ and set

$$F' = \bigcap \{Q_j | \sqrt{Q_j} = I_{C_k} \text{ for some } k\},$$

which does not depend on the choice of $Q_i$‘s. We note $I^{(n)} = (I^n)'$.

We set $J = (I_D + I_E)'$ and first prove the lemma for the case $I = J$.

By the normality of $X$, $J$ is generated by a regular sequence outside a finite set. Hence the natural homomorphism

$$a_1 : (\mathcal{O}/J)(-D) \oplus (\mathcal{O}/J)(-E) \to J/J^{(2)}$$

is injective and $\text{Supp} \text{Coker}(a_1)$ is at most 0-dimensional, and the induced

$$a_2 = S^n(a_1) : \bigoplus_{k=0}^n (\mathcal{O}/J)(-kD - (n-k)E) \to J^{(n)}/J^{(n+1)}$$

has similar properties: $\text{Ker}(a_2) = 0$, $\dim \text{Supp} \text{Coker}(a_2) \leq 0$. By $(-E \cdot C_i) > 0$ and $(D \cdot C_i) = 0$, we have

$$\chi(J^{(n)}/J^{(n+1)}) \geq \sum_{k=0}^n (\chi(\mathcal{O}/J) + (n-k)) \geq n \cdot \chi(\mathcal{O}/J) + n(n+1)/2,$$

and the lemma holds for the case $I = J$.

Let $a$ be a natural number such that $I^{(a)} \subset J$. We note

$$\chi(\mathcal{O}_X/I^{(n)}) = \chi(\mathcal{O}_X/J^{([n/a])}) + \chi(J^{([n/a])}/I^{(n)}),$$

by $I^{(n)} \subset I^{([n/a])} \subset J^{([n/a])}$. Thus it remains to prove $\chi(J^{([n/a])}/I^{(n)}) \geq 0$. Since the cokernel of $S^{([n/a])}(\mathcal{O}(-D) \oplus \mathcal{O}(-E)) \to J^{([n/a])}/I^{(n)}$ is supported on a finite set and $\mathcal{O}(-D) \oplus \mathcal{O}(-E)$ is generated by global sections, we have $H^1(X, J^{([n/a])}/I^{(n)}) = 0$ by the following well-known lemma, and we are done.

[Lemma 4] Let $f : X \to (Y, Q)$ be a proper morphism to a germ such that $R^1 f_* \mathcal{O}_X = 0$ and $C = f^{-1}(Q)_{\text{red}}$ is 1-dimensional. Let $F, G$ be coherent sheaves on $X$ with a homomorphism $a : F \to G$ such that $F$ is generated by global sections and $\text{Supp} \text{Coker}(a)$ is finite over $Y$. Then $R^1 f_* G = 0$.

**Proof.** Since $C$ is 1-dimensional, $R^2 f_* \mathcal{H} = 0$ for each coherent sheaf $\mathcal{H}$ on $X$. Since $F$ is globally generated, there exist an integer $n > 0$ and a surjection $b : \mathcal{O}_X^{\oplus n} \to F$. By $R^2 f_* \text{Ker} b = 0$, we have $R^1 f_* F = 0$. By $R^2 f_* \text{Ker} a = 0$, we have $R^1 f_* \text{Im} a = 0$. Since $R^1 f_* \text{Coker} a = 0$ by the assumption, we have $R^1 f_* G = 0$. \qed
References


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