

#### MATHEMATICAL AMERICAN SOCIETY

### **EDITORS**

Laura G. DeMarco Simon Donaldson Pavel Etingof Michael J. Larsen Sylvia Serfaty Richard Taylor Shmuel Weinberger

# ASSOCIATE EDITORS

Mohammed Abouzaid Ian Agol Benjamin Antieau Jian Ding **Dmitry Dolgopyat** Lawrence Guth Ursula Hamenstadt Richard Kenyon Bruce A. Kleiner Thomas Fun Yau Lam Ciprian Manolescu Lillian Pierce **Anand Pillay** Peter Scholze **Amit Singer** Gigliola Staffilani Benjamin Sudakov Ulrike Tillmann **Burt Totaro** Melanie Matchett Wood



Zhiwei Yun

Providence, Rhode Island USA

#### Journal of the American Mathematical Society

This journal is devoted to research articles of the highest quality in all areas of pure and applied mathematics.

Submission information. See Information for Authors at the end of this issue.

**Publication on the AMS website.** Articles are published on the AMS website individually after proof is returned from authors and before appearing in an issue.

Subscription information. The Journal of the American Mathematical Society is published quarterly and is also accessible electronically from www.ams.org/journals/. Individual subscription prices for Volume 36 (2023) are as follows. For electronic only: non-member, US\$419.00, member, US\$251.40. For paper delivery: non-member, US\$476.00, member, US\$285.60. Add US\$5.00 for delivery within the United States; US\$16 for surface delivery outside the United States. Upon request, subscribers to paper delivery of this journal are also entitled to receive electronic delivery. For information on institutional pricing, please visit https://www.ams.org/publications/journals/subscriberinfo. Subscription renewals are subject to late fees. See www.ams.org/journal-faq for more journal subscription information.

Back number information. For back issues see www.ams.org/backvols.

Subscriptions and orders should be addressed to the American Mathematical Society, P.O. Box 845904, Boston, MA 02284-5904 USA. *All orders must be accompanied by payment*. Other correspondence should be addressed to 201 Charles Street, Providence, RI 02904-2213 USA.

Copying and reprinting. Individual readers of this publication, and nonprofit libraries acting for them, are permitted to make fair use of the material, such as to copy an article for use in teaching or research. Permission is granted to quote brief passages from this publication in reviews, provided the customary acknowledgment of the source is given.

Republication, systematic copying, or multiple reproduction of any material in this publication is permitted only under license from the American Mathematical Society. Requests for permission to reuse portions of AMS publication content are handled by the Copyright Clearance Center. For more information, please visit www.ams.org/publications/pubpermissions.

Excluded from these provisions is material for which the author holds copyright. In such cases, requests for permission to reuse or reprint material should be addressed directly to the author(s). Copyright ownership is indicated in the notice in the lower right-hand corner of the first page of each article.

The Journal of the American Mathematical Society (ISSN 0894-0347 (print); ISSN 1088-6834 (online)) is published quarterly by the American Mathematical Society at 201 Charles Street, Providence, RI 02904-2213 USA and is mailed from Providence, Rhode Island. Periodicals postage is paid at Providence, Rhode Island. Postmaster: Send address changes to Journal of the AMS, American Mathematical Society, 201 Charles Street, Providence, RI 02904-2213 USA.

<sup>© 2023</sup> by the American Mathematical Society. All rights reserved. This journal is indexed in Mathematical Reviews, Zentralblatt MATH, Science Citation Index<sup>®</sup>, Science Citation Index<sup>TM</sup>–Expanded, ISI Alerting Services<sup>SM</sup>, CompuMath Citation Index<sup>®</sup>, and Current Contents<sup>®</sup>/Physical, Chemical & Earth Sciences. This journal is archived in Portico and in CLOCKSS.

Printed in the United States of America.

<sup>⊗</sup> The paper used in this journal is acid-free and falls within the guidelines established to ensure permanence and durability.

# JOURNAL OF THE AMERICAN MATHEMATICAL SOCIETY CONTENTS

Vol. 36, No. 4 October	2023
Daniil Rudenko, On the Goncharov depth conjecture and a formula for	
volumes of orthoschemes	1003
Emanuel Milman and Amir Yehudayoff, Sharp isoperimetric inequalities	
for affine quermassintegrals	1061
Sanchit Chaturvedi, Jonathan Luk, and Toan T. Nguyen, The Vlasov-	
Poisson–Landau system in the weakly collisional regime	1103
Hong Liu and Richard Montgomery, A solution to Erdős and Hajnal's	
odd cycle problem	1191
Roman Bezrukavnikov and Ivan Losev, Dimensions of modular	
irreducible representations of semisimple Lie algebras	1235
B. Bakker, B. Klingler, and J. Tsimerman, Erratum to "Tame topology	
of arithmetic quotients and algebraicity of Hodge loci"	1305

## INDEX TO VOLUME 36 (2023)

Ardila, Federico, Graham Denham, and June Huh. Lagrangian geometry of matroids, 727

Bakker, B., B. Klingler, and J. Tsimerman. Erratum to "Tame topology of arithmetic quotients and algebraicity of Hodge loci", 1305

Bamler, Richard H., and Bruce Kleiner. Ricci flow and diffeomorphism groups of 3-manifolds, 563

Berger, Pierre, and Sébastien Biebler. Emergence of wandering stable components, 397

Bezrukavnikov, Roman, and Ivan Losev. Dimensions of modular irreducible representations of semisimple Lie algebras, 1235

Biebler, Sébastien. See Berger, Pierre

Böhm, Christoph, and Ramiro A. Lafuente. Non-compact Einstein manifolds with symmetry, 591

Briggs, Benjamin, and Srikanth B. Iyengar. Rigidity properties of the cotangent complex, 291

Chaturvedi, Sanchit, Jonathan Luk, and Toan T. Nguyen.  ${\it The~Vlasov-Poisson-Landau~system~in~the~weakly~collisional~regime,~1103}$ 

Christandl, Matthias, Péter Vrana, and Jeroen Zuiddam. *Universal points in the asymptotic spectrum of tensors*, 31

Denham, Graham. See Ardila, Federico

Diao, Hansheng, Kai-Wen Lan, Ruochuan Liu, and Xinwen Zhu. Logarithmic Riemann–Hilbert correspondences for rigid varieties, 483

Eliashberg, Yakov, and Emmy Murphy. Making cobordisms symplectic, 1

Fayad, Bassam. Lyapunov unstable elliptic equilibria, 81

Feyzbakhsh, S., and R. P. Thomas. Rank r DT theory from rank 1, 795

Forey, A. See Sawin, Will

Fournodavlos, Grigorios, Igor Rodnianski, and Jared Speck. Stable Big Bang formation for Einstein's equations: The complete sub-critical regime, 827

Fresán, J. See Sawin, Will

Grodal, Jesper. Endotrivial modules for finite groups via homotopy theory, 177

Hennion, Benjamin, and Mikhail Kapranov. Gel'fand-Fuchs cohomology in algebraic geometry and factorization algebras, 311

Hilburn, Justin, and Sam Raskin. Tate's thesis in the de Rham setting, 917

Huh, June. See Ardila, Federico

Iyengar, Srikanth B. See Briggs, Benjamin

Kapranov, Mikhail. See Hennion, Benjamin

Kleiner, Bruce. See Bamler, Richard H.

Klingler, B. See Bakker, B.

Kowalski, E. See Sawin, Will

Lafuente, Ramiro A. See Böhm, Christoph

Lan, Kai-Wen. See Diao, Hansheng

Liu, Hong, and Richard Montgomery. A solution to Erdős and Hajnal's odd cycle problem, 1191

Liu, Ruochuan. See Diao, Hansheng

Losev, Ivan. See Bezrukavnikov, Roman

Luk, Jonathan. See Chaturvedi, Sanchit

 $\label{eq:manuel} \mbox{Milman, Emanuel, and Amir Yehudayoff.} \quad \textit{Sharp isoperimetric inequalities for affine quermass-integrals}, 1061$ 

Montgomery, Richard. See Liu, Hong

Murphy, Emmy. See Eliashberg, Yakov

Nguyen, Toan T. See Chaturvedi, Sanchit

Quastel, Jeremy, and Sourav Sarkar. Convergence of exclusion processes and the KPZ equation to the KPZ fixed point, 251

Raskin, Sam. See Hilburn, Justin

Rodnianski, Igor. See Fournodavlos, Grigorios

Rudenko, Daniil. On the Goncharov depth conjecture and a formula for volumes of orthoschemes, 1003

Sarkar, Sourav. See Quastel, Jeremy

Sawin, Will, A. Forey, J. Fresán, and E. Kowalski. Quantitative sheaf theory, 653

Speck, Jared. See Fournodavlos, Grigorios

Thomas, R. P. See Feyzbakhsh, S.

Tsimerman, J. See Bakker, B.

### INDEX TO VOLUME 36 (2023)

Vrana, Péter. See Christandl, Matthias Yehudayoff, Amir. See Milman, Emanuel

Yuan, Allen. Integral models for spaces via the higher Frobenius, 107 Zhu, Xinwen. See Diao, Hansheng Zuiddam, Jeroen. See Christandl, Matthias

VOLUME 36 2023



### A M E R I C A N M A T H E M A T I C A L S O C I E T Y

### **EDITORS**

Laura G. DeMarco Simon Donaldson Pavel Etingof Michael J. Larsen Sylvia Serfaty Richard Taylor Shmuel Weinberger

#### ASSOCIATE EDITORS

Mohammed Abouzaid Ian Agol Benjamin Antieau Jian Ding Dmitry Dolgopyat Lawrence Guth Ursula Hamenstadt Richard Kenyon Bruce A. Kleiner Thomas Fun Yau Lam Ciprian Manolescu Lillian Pierce Anand Pillay Peter Scholze **Amit Singer** Gigliola Staffilani Benjamin Sudakov Ulrike Tillmann **Burt Totaro** Melanie Matchett Wood



Zhiwei Yun

Providence, Rhode Island USA

#### Journal of the American Mathematical Society

This journal is devoted to research articles of the highest quality in all areas of pure and applied mathematics.

Submission information. See Information for Authors at the end of this issue.

**Publication on the AMS website.** Articles are published on the AMS website individually after proof is returned from authors and before appearing in an issue.

Subscription information. The Journal of the American Mathematical Society is published quarterly and is also accessible electronically from www.ams.org/journals/. Individual subscription prices for Volume 36 (2023) are as follows. For electronic only: non-member, US\$419.00, member, US\$251.40. For paper delivery: non-member, US\$476.00, member, US\$285.60. Add US\$5.00 for delivery within the United States; US\$16 for surface delivery outside the United States. Upon request, subscribers to paper delivery of this journal are also entitled to receive electronic delivery. For information on institutional pricing, please visit https://www.ams.org/publications/journals/subscriberinfo. Subscription renewals are subject to late fees. See www.ams.org/journal-faq for more journal subscription information.

Back number information. For back issues see www.ams.org/backvols.

Subscriptions and orders should be addressed to the American Mathematical Society, P.O. Box 845904, Boston, MA 02284-5904 USA. *All orders must be accompanied by payment*. Other correspondence should be addressed to 201 Charles Street, Providence, RI 02904-2213 USA.

Copying and reprinting. Individual readers of this publication, and nonprofit libraries acting for them, are permitted to make fair use of the material, such as to copy an article for use in teaching or research. Permission is granted to quote brief passages from this publication in reviews, provided the customary acknowledgment of the source is given.

Republication, systematic copying, or multiple reproduction of any material in this publication is permitted only under license from the American Mathematical Society. Requests for permission to reuse portions of AMS publication content are handled by the Copyright Clearance Center. For more information, please visit www.ams.org/publications/pubpermissions.

Excluded from these provisions is material for which the author holds copyright. In such cases, requests for permission to reuse or reprint material should be addressed directly to the author(s). Copyright ownership is indicated in the notice in the lower right-hand corner of the first page of each article.

The Journal of the American Mathematical Society (ISSN 0894-0347 (print); ISSN 1088-6834 (online)) is published quarterly by the American Mathematical Society at 201 Charles Street, Providence, RI 02904-2213 USA and is mailed from Providence, Rhode Island. Periodicals postage is paid at Providence, Rhode Island. Postmaster: Send address changes to Journal of the AMS, American Mathematical Society, 201 Charles Street, Providence, RI 02904-2213 USA.

<sup>© 2023</sup> by the American Mathematical Society. All rights reserved. This journal is indexed in Mathematical Reviews, Zentralblatt MATH, Science Citation Index<sup>®</sup>, Science Citation Index<sup>TM</sup>–Expanded, ISI Alerting Services<sup>SM</sup>, CompuMath Citation Index<sup>®</sup>, and Current Contents<sup>®</sup>/Physical, Chemical & Earth Sciences. This journal is archived in Portico and in CLOCKSS.

Printed in the United States of America.

<sup>⊗</sup> The paper used in this journal is acid-free and falls within the guidelines established to ensure permanence and durability.

# JOURNAL OF THE AMERICAN MATHEMATICAL SOCIETY CONTENTS

Vol. 36, No. 1	January 2023
Yakov Eliashberg and Emmy Murphy, Making cobor Matthias Christandl, Péter Vrana, and Jeroen Zu	iddam, Universal
points in the asymptotic spectrum of tensors	
Bassam Fayad, Lyapunov unstable elliptic equilibria	
Allen Yuan, Integral models for spaces via the higher Fro	
Jesper Grodal, Endotrivial modules for finite groups via Jeremy Quastel and Sourav Sarkar, Convergence of e	exclusion processes
and the KPZ equation to the KPZ fixed point  Benjamin Briggs and Srikanth B. Iyengar, Rigidity cotangent complex	properties of the
Vol. 36, No. 2	April 2023
Benjamin Hennion and Mikhail Kapranov, Gel'fand- in algebraic geometry and factorization algebras Pierre Berger and Sébastien Biebler, Emergence of	311
components	~
Hansheng Diao, Kai-Wen Lan, Ruochuan Liu, ar Logarithmic Riemann-Hilbert correspondences for ri	nd Xinwen Zhu, gid varieties 483
Richard H. Bamler and Bruce Kleiner, Ricci flow a groups of 3-manifolds	
Vol. 36, No. 3	July 2023
Christoph Böhm and Ramiro A. Lafuente, Non-manifolds with symmetry	
Federico Ardila, Graham Denham, and June geometry of matroids	Huh, Lagrangian
S. Feyzbakhsh and R. P. Thomas, Rank r DT theory Grigorios Fournodavlos, Igor Rodnianski, and Jar Big Bang formation for Einstein's equations: The corregime	ed Speck, Stable mplete sub-critical
Justin Hilburn and Sam Raskin. Tate's thesis in the o	de Rham setting 917

Vol. 36, No. 4 October 2023

Daniil Rudenko, On the Goncharov depth conjecture and a formula for	
volumes of orthoschemes	1003
Emanuel Milman and Amir Yehudayoff, Sharp isoperimetric inequalities	
for affine quermassintegrals	1061
Sanchit Chaturvedi, Jonathan Luk, and Toan T. Nguyen, The Vlasov-	
Poisson–Landau system in the weakly collisional regime	1103
Hong Liu and Richard Montgomery, A solution to Erdős and Hajnal's	
odd cycle problem	1191
Roman Bezrukavnikov and Ivan Losev, Dimensions of modular	
irreducible representations of semisimple Lie algebras	1235
B. Bakker, B. Klingler, and J. Tsimerman, Erratum to "Tame topology	
of arithmetic quotients and algebraicity of Hodge loci"	1305

#### **Editors**

Laura G. DeMarco
Department of Mathematics
Northwestern University
Evanston, IL 60208-2730 USA
demarco@northwestern.edu

Pavel Etingof
Department of Mathematics
Massachusetts Institute of Technology
77 Massachusetts Avenue
Cambridge, MA 02139 USA
etingof@math.mit.edu

Sylvia Serfaty
New York University-Courant Institute
251 Mercer Street
New York, NY 10012-1185 USA
serfaty@cims.nyu.edu

Simon Donaldson Simons Center for Geometry and Physics Stony Brook University Stony Brook, NY 11794 USA sdonaldson@scgp.stonybrook.edu

Michael J. Larsen Indiana University Department of Mathematics 831 E 3rd Street Bloomington, IN 47405 USA mjlarsen@indiana.edu

Richard Taylor Stanford University Building 380 450 Jane Stanford Way Stanford, CA 94305 USA rltaylor@stanford.edu

Shmuel Weinberger Department of Mathematics University of Chicago 5734 S University Avenue Chicago, IL 60637 USA shmuel@math.uchicago.edu

#### **Associate Editors**

Mohammed Abouzaid, Columbia University Ian Agol, University of California, Berkeley Benjamin Antieau, Northwestern University Jian Ding, Peking University Dmitry Dolgopyat, University of Maryland Lawrence Guth, Massachusetts Institute of Technology Ursula Hamenstadt, University of Bonn Richard Kenyon, Yale University Bruce A. Kleiner, New York University-Courant Institute Thomas Fun Yau Lam, University of Michigan Ciprian Manolescu, Stanford University Lillian Pierce, Duke University Anand Pillay, University of Notre Dame Peter Scholze, Universität Bonn Amit Singer, Princeton University Gigliola Staffilani, Massachusetts Institute of Technology Benjamin Sudakov, ETH Zürich Ulrike Tillmann, University of Oxford Burt Totaro, University of California, Los Angeles Melanie Matchett Wood, Harvard University Zhiwei Yun, Massachusetts Institute of Technology

#### **Editorial Information**

Information on the backlog for this journal can be found on the AMS website starting from http://www.ams.org/jams.

In an effort to make articles available as quickly as possible, articles are electronically published on the AMS website individually after proof is returned from authors and before appearing in an issue.

A Consent to Publish is required before we can begin processing your paper. After a paper is accepted for publication, the Providence office will send a Consent to Publish and Copyright Agreement to all authors of the paper. By submitting a paper to this journal, authors certify that the manuscript has not been submitted to nor is it under consideration for publication by another journal, conference proceedings, or similar publication.

#### Information for Authors

Initial submission. All articles submitted to this journal are peer-reviewed. The AMS has a single blind peer-review process in which the reviewers know who the authors of the manuscript are, but the authors do not have access to the information on who the peer reviewers are. The AMS uses Centralized Manuscript Processing for initial submission. Authors should submit a PDF file using the Initial Manuscript Submission form found at www.ams.org/submission/jams or should send one copy of the manuscript to the following address: Centralized Manuscript Processing, JOURNAL OF THE AMS, 201 Charles Street, Providence, RI 02904-2213 USA. If a paper copy is being forwarded to the AMS, indicate that it is for the *Journal of the AMS* and include the name of the corresponding author, contact information such as email address or mailing address, and the name of an appropriate Editor to review the paper (see the list of Editors above).

The first page must contain a descriptive title that is short, but informative; useless or vague phrases such as "some remarks about" or "concerning" should be avoided. Although an abstract is not required upon initial submission, upon acceptance authors will be requested to supply an abstract for the electronic version of this journal. The AMS offers free worldwide access to the electronic abstracts. An abstract should be at least one complete sentence and at most 300 words. No abstracts appear in the printed journal starting in 1998. Included with the footnotes to the paper should be the 2020 Mathematics Subject Classification representing the primary and secondary subjects of the article. The classifications are accessible from www.ams.org/msc/. The Mathematics Subject Classification footnote may be followed by a list of key words and phrases describing the subject matter of the article and taken from it. Journal abbreviations used in bibliographies are listed in the latest Mathematical Reviews annual index. The series abbreviations are also accessible from www.ams.org/msnhtml/serials.pdf. To help in preparing and verifying references, the AMS offers MR Lookup, a Reference Tool for Linking, at www.ams.org/mrlookup/.

Electronically prepared manuscripts. Manuscripts should be electronically prepared in  $\mathcal{A}_{\mathcal{M}}\mathcal{S}$ -LaTeX. To this end, the Society has prepared  $\mathcal{A}_{\mathcal{M}}\mathcal{S}$ -LaTeX author packages for each AMS publication. Author packages include instructions for preparing electronic manuscripts, samples, and a style file that generates the particular design specifications of that publication series. Articles properly prepared using the  $\mathcal{A}_{\mathcal{M}}\mathcal{S}$ -LaTeX style file and the \label and \ref commands automatically enable extensive intra-document linking to the bibliography and other elements of the article for searching electronically on the Web.

Authors may retrieve an author package for Journal of the AMS starting from www.ams.org/jams/jamsauthorpac.html. The AMS Author Handbook is available in PDF format from the author package link. The author package can also be obtained free of charge by sending email to tech-support@ams.org or from the Publication Division, American Mathematical Society, 201 Charles Street, Providence, RI 02904-2213 USA. When requesting an author package, please specify the publication in which your paper will appear. Please be sure to include your complete email address.

After acceptance. The source files for the final version of the electronic manuscript should be sent to the Providence office immediately after the paper has been accepted for publication. The author should also submit a PDF of the final version of the paper to the

Editor, who will forward a copy to the Providence office. Accepted electronically prepared manuscripts can be submitted via the Web at www.ams.org/submit-book-journal/, sent via email to pub-submit@ams.org, or sent on CD to the Electronic Prepress Department, American Mathematical Society, 201 Charles Street, Providence, RI 02904-2213 USA. When sending a manuscript electronically via email or CD, please be sure to include a message indicating in which publication the paper has been accepted. Complete instructions on how to send files are included in the author package.

Electronic graphics. Comprehensive instructions on preparing graphics are available from www.ams.org/authors/journals.html. A few of the major requirements are given here.

Submit files for graphics as EPS (Encapsulated PostScript) files. This includes graphics originated via a graphics application as well as scanned photographs or other computer-generated images. If this is not possible, TIFF files are acceptable as long as they can be opened in Adobe Photoshop or Illustrator.

Authors using graphics packages for the creation of electronic art should also avoid the use of any lines thinner than 0.5 points in width. Many graphics packages allow the user to specify a "hairline" for a very thin line. Hairlines often look acceptable when proofed on a typical laser printer. However, when produced on a high-resolution laser imagesetter, hairlines become nearly invisible and will be lost entirely in the final printing process.

Screens should be set to values between 15% and 85%. Screens which fall outside of this range are too light or too dark to print correctly. Variations of screens within a graphic should be no less than 10%.

Any graphics created in color will be rendered in grayscale for the printed version unless color printing is authorized by the Managing Editor and the Publisher. In general, color graphics will appear in color in the online version.

AMS policy on making changes to articles after publication. Articles are published on the AMS website individually after proof is returned from authors and before appearing in an issue. To preserve the integrity of electronically published articles, once an article is individually published to the AMS website, changes cannot be made in place in the paper. The AMS does not keep author-related information such as affiliation, current address, and email address up to date after a paper is electronically published.

Corrections of critical errors may be made to the paper by submitting an errata article to the Editor. The errata article will be published electronically, will appear in a future print issue, and will link back and forth on the Web with the original article.

Secure manuscript tracking on the Web. Authors can track their manuscripts through the AMS journal production process using the personal AMS ID and Article ID printed in the upper right-hand corner of the Consent to Publish form sent to each author who publishes in AMS journals. Access to the tracking system is available from www.ams.org/mstrack/. An explanation of each production step is provided on the Web through links from the manuscript tracking screen. Questions can be sent to jams-query@ams.org.

Inquiries. Any inquiries concerning a paper that has been accepted for publication that cannot be answered via the manuscript tracking system mentioned above should be sent to jams-query@ams.org or directly to the Electronic Prepress Department, American Mathematical Society, 201 Charles Street, Providence, RI 02904-2213 USA.

# JOURNAL OF THE AMERICAN MATHEMATICAL SOCIETY CONTENTS

Vol. 36, No. 4 October	2023
Daniil Rudenko, On the Goncharov depth conjecture and a formula for	
volumes of orthoschemes	1003
Emanuel Milman and Amir Yehudayoff, Sharp isoperimetric inequalities	
for affine quermassintegrals	1061
Sanchit Chaturvedi, Jonathan Luk, and Toan T. Nguyen, The Vlasov-	
Poisson–Landau system in the weakly collisional regime	1103
Hong Liu and Richard Montgomery, A solution to Erdős and Hajnal's	
odd cycle problem	1191
Roman Bezrukavnikov and Ivan Losev, Dimensions of modular	
irreducible representations of semisimple Lie algebras	1235
B. Bakker, B. Klingler, and J. Tsimerman, Erratum to "Tame topology	
of arithmetic quotients and algebraicity of Hodge loci"	1305







