

In the asymptotic expansion

$$I_0(x) = \frac{e^x}{(2\pi x)^{\frac{1}{2}}} \left\{ 1 + \frac{1^2}{1!8x} + \frac{1^2 \cdot 3^2}{2!(8x)^2} + \frac{1^2 \cdot 3^2 \cdot 5^2}{3!(8x)^3} + \dots \right\},$$

upon setting  $x = 30$ , and evaluating 25 terms of the series, each to 22 decimal places, I found the sum of the terms in the braces to be

$$1.00424\ 76530\ 20713\ 59155(8).$$

This calculation was performed twice,—first by means of a calculating machine, the second time long-hand. The results agreed perfectly.

The value of  $e^{30}$  was found in the W.P.A. *Tables of the Exponential Function  $e^x$*  (1939), p. 533, to 19S. Subsequently I checked and extended this approximation to 38S. I have unpublished values of  $\pi^{\frac{1}{2}}$  and  $1/\pi^{\frac{1}{2}}$  to 317 and 310 decimal places respectively. These were calculated from  $\pi$  and  $1/\pi$ , respectively, and the product of the square roots was formed with the assistance of a machine, and was found to consist of a sequence of 309 consecutive 9's. Consequently I have great confidence in the accuracy of these roots. The value of  $15^{\frac{1}{2}}$  was determined to 40D. By multiplication I found  $e^{30}15^{\frac{1}{2}}/30\pi^{\frac{1}{2}} = 7.78366\ 06884\ 04464\ 04193\ 55906\ 75 \times 10^{11}$ ; and finally,  $I_0(30) = 7.81672\ 29782\ 39774\ 8972 \times 10^{11}$ , correct to 20S.

Hence the value of Wrinch and Wrinch is nearly correct, the error being in the last significant figure which they give. The value of Colwell and Hardy is entirely incorrect.

J. W. WRENCH, Jr.

4211 Second St., N.W.  
Washington 16, D. C.

## UNPUBLISHED MATHEMATICAL TABLES

References have been made to Unpublished Mathematical Tables in RMT 138 (DAVIS, MILLS), and RMT 157 (DAVIS, MATH. TABLES PROJECT).

21[B].—*Tables of Fractional Powers*, Mss. prepared by, and in possession of, the MATHEMATICAL TABLES PROJECT, 50 Church St., New York City.

Of these 12 tables 6 are of  $A^x$ , and 6 of  $x^a$ , as follows:

I:  $A = 2(1)9$ , and  $x = [0.001(0.001)0.01(0.01)0.99; 15D]$ .

II-IV:  $A = 10, \pi$ , and Euler constant,  $x = [0.001(0.001)1.000; 15D]$ .

V:  $A = 0.01(0.01)0.99$ , and  $x = [0.001(0.001)0.01(0.01)0.99; 15D]$ .

VI:  $A = p10^{-3}$ ,  $p =$  the primes between 101 and 997, and  $x = [0.001(0.001)0.01(0.01)0.99; 15D]$ .

VII-XI:  $x = [0.01(0.01)9.99; 15D]$  and  $a = \pm\frac{1}{4}, \pm\frac{1}{2}, \pm\frac{3}{4}, \pm\frac{1}{3}, \pm\frac{2}{3}$ .

XII:  $a = 0.01(0.01)0.99$ . For any  $a$  the values of  $x^a$  were computed at the interval 0.001 in  $x$  approximately up to that value of  $x$  for which the derivative of the function (which is  $\infty$  at  $x = 0$ ) is in the neighbourhood of unity. All entries are to 7D.

A. N. LOWAN

22[D, H].—*Roots of the equation  $\tan x + ax = 0$* ,  $a = 3/(8\pi)$ ,  $3/(12\pi)$ ,  $3/(16\pi)$ . Ms. in possession of the Department of Aeronautical Engineering, University of Michigan, Ann Arbor, Mich. Compare Q. 8.

The first three roots of each of these equations were calculated to 5S, and the next five roots to 3S. First, we obtained an approximate value  $x_n^0$ , graphically, and then applied

a correction

$$\epsilon_n = -(\tan x_n^0 + ax_n^0)/(\sec^2 x_n^0 + a).$$

The corrected values  $x_n^0 + \epsilon_n$  are about as accurate as we need them.

OTTO LAPORTE

University of Michigan

23[L].—HANSRAJ GUPTA, *Table of Liouville's function and its sum*. Ms. in possession of the author, Government College, Hoshiarpur, Punjab, India.

Liouville's function  $\lambda(n)$  may be defined for the positive integer argument  $n = p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_t^{\alpha_t}$  to be +1 or -1 according as  $m = \alpha_1 + \alpha_2 + \cdots + \alpha_t$  is even or odd. This table gives  $\lambda(n)$  and also the sum

$$L(n) = \lambda(1) + \cdots + \lambda(n)$$

for  $n \leq 20\,000$ . The function  $L(n)$  is clearly the excess of the number of those integers  $n$  which have an even number of prime factors over the number of integers having an odd number of prime factors. The reason for computing this table is to verify or refute an important conjecture of PÓLYA (*Deutsche Math.-Ver. Jahresb.*, v. 28, 1919, p. 38) to the effect that this "excess" is really negative or zero for  $n > 1$ . The table shows that the conjecture is correct as far as  $n = 20\,000$  but does not show that the conjecture has safely passed its worst trials. In fact one finds that  $L(48\,512) = -2$ . Data, taken from this table, on the behavior of  $L(n)$  have been published by the author, *Indian Acad. Sci., Proc.*, v. 12A, 1940, p. 407-409. Current interest in Pólya's conjecture (which implies the Riemann Hypothesis) has been heightened by the recent results of A. E. INGHAM, *Amer. J. Math.*, v. 64, 1942, p. 313-319. The corresponding problem for the companion function  $\mu(n)$  of MÖBIUS and its sum is also unsolved and has been the subject of much tabulation; see *Guide to Tables in the Theory of Numbers*, Nat. Res. Council, *Bull.* no. 105, Washington, D. C., 1941, p. 9.

D. H. L.

## NOTES

17. DESIRABLE MATHEMATICAL TABLES FOR REPUBLICATION.—The Office of Alien Property Custodian has licensed, during the past several months, the reprinting of scientific and technical books, of enemy origin, which are not available in a quantity sufficient to meet the demands of the wartime operations of science and industry. In RMT 79, 113, 128, references have been made to three volumes of this kind. Before definite decision can be made regarding the licensing of additional Mathematical Tables for republication, it is necessary for the Custodian to be informed about the extent of the need of such tables and to receive suggestions of specific titles for consideration. This can be accomplished if suggestions of specific significant tables, or any inquiries, are sent by individuals to the undersigned at Division of Patent Administration, Office of Alien Property Custodian, Washington, D. C.

H. H. SARGEANT, Chief

18. *Phil. Mag.* TABLES, SUPPL. 1 (see *MTAC*, p. 135-141).—In H. C. PLUMMER, "The numerical solution of a type of equation," s. 7, v. 32, 1941, p. 505-512, roots are found of the following transcendental equations, among others, of the type  $\tan x = xf(x)$ , where  $f(x)$  is a single-valued function: