

MATHEMATICAL TABLES—ERRATA

References have been made to Errata in "A new approximation to  $\pi$ "; RMT 360 (Ser), 367 (Ostrogradskii), 368 (Patz), 369 (Cunningham), 370 (Vinogradov), 375 (Fisher & Yates, Croxton & Cowden), 376 (Goncharov), 378 (Hastings & Piedem), 382 (McLachlan), 389 (Stoner), 390 (McLachlan), 392 (N. R. C. Canada), 397 (Aquino); UMT 54 (Br. Standards Inst.), 55 (DeMorgan); QR 28 (Gauss).

99. H. BRANDENBURG, *Sechsstellige trigonometrische Tafel*, Leipzig, 1932, and Ann Arbor, 1945.

This volume was described in *MTAC*, v. 1, p. 387f and errors are listed on p. 388, and in *MTE* 28, v. 1, p. 162. Two further errors have been brought to my notice, both by correspondents in Palestine.

(1) Mr. B. GOUSSINSKY, Superintendent of Surveys, points out that the asterisks in the entries for the tangents of  $0^\circ 27' 40''$  and  $0^\circ 27' 50''$ , p. 32, should be deleted. In the 7-figure tables  $\tan 0^\circ 27' 30''$  is given as 0.007 9996; this has been rounded off (correctly) to 0.008 000 when forming the 6-figure tables, thus changing a leading figure and removing the necessity for asterisks. I have verified that there are no other similar instances of this fault.

(2) Mr. ALEXANDER KATZ, of Jerusalem, finds that on p. 23,  $\cot 2^\circ 46' 32''$ , for 62694 we must read 62684. This error has taken "evasive action" on several occasions. It occurs in both editions of Brandenburg's 7-figure tables. It is marked in my 1923 edition, and was presumably communicated to Brandenburg but does not appear in an extensive list of corrections that he prepared in 1927. It still occurs in the 1931 edition, and is marked in my copy, but not included in the list that I gave in a review in *The Observatory*, v. 54, 1931, p. 301-302. It was, however, communicated to Brandenburg, and is included in a list of errors published by him in 1932. Nevertheless Brandenburg repeats the error in his 6-figure table in 1932, and my proof readers, who examined the table, failed to find it!

L. J. C.

EDITORIAL NOTE: For a reference to still other Brandenburg errors, see *MTAC*, v. 2, p. 46f. In the 1923 edition of Brandenburg's 7-figure table are a two-page "Druckfehler-Verzeichnis," and a leaf headed "Berichtigungsbogen zum Überkleben der Druckfehler." In addition to these there is bound with the Brown University copy of these tables purchased in 1929, a 4-page "Nachtrag zum Druckfehler-Verzeichnis," on differently colored paper, dated 15 March 1927, and listing a very large number of errors, and the names of nine individuals assisting in its compilation.

In 1932 the publishers distributed a one-page "Verzeichnis der Druck- und Formfehler," 18 errors, in the 1931 edition of Brandenburg's 7-figure table. In this sheet, no. 17, for 510 lies 519, read 310 lies 319.

The five errors in this edition of Brandenburg's 7-figure table, listed by L. J. C. in *The Observatory*, are as follows: p. 95, diff. following  $\sin 4^\circ 59' 50''$ , for 485, read 483; p. 111,  $\cot 7^\circ 39' 30''$ , for 8870, read 8871; p. 157,  $\cot 15^\circ 13' 30''$ , for 2744, read 2743; p. 177, diff. following  $\sin 18^\circ 32' 10''$ , for 461, read 459; p. 335,  $\cot 44^\circ 53' 0''$ , for 0808, read 0807. The first is not included in the 1932 "Verzeichnis."

To conclude the references in *MTAC* to all known errors in the 1931 and 1932 Brandenburg tables we may note that the value of  $e$  in each of these tables is given to 30D as, " $e = 2.718\ 281\ 828\ 459\ 045\ 235\ 339\ 784\ 490\ 662\ \dots$ " which is correct to only 19D. The value is obtained as the sum to 30D of terms  $1/n!$  plus unity,  $n = 1(1)20$ . Of the values of these 20 terms 4 are not rounded off, and 5 are so rounded. Underneath Brandenburg's incorrect value of  $e$  is given Euler's correct value to 23D, taken from his *Introductio in Analysin Infinitorum*, v. 1, § 122. Quite astonishingly the author seems to suggest that he, rather than Euler, is correct.

100. FMR, *Index*. See *MTAC*, v. 2, p. 13-18, 136, 178-181, 219-220.

The notes made by S. A. J. and myself in *MTE* 89, p. 178-181, were mainly a budget of suggestions on minor questions for consideration in a possible new edition. Doctors Fletcher and Miller have communicated to us certain statements which should have equal publicity.

- P. 35, 3.14 we suggested "27 dec. Thoman" instead of "20 dec. Thoman." Dr. F. writes: "As its title implies, Thoman's book mostly gives 27 dec. But log factorials are to 20 dec.—I have the book in front of me. One or two other tables in it are also to less than 27 dec."
- P. 51, 4.41, for 4.412, read 4.4121; for 4.413, read 4.4132. Dr. M. writes "Delete this item. Such an alteration would destroy the effect intended.  $A_n$  and  $S_n$  are essentially the same function, and the special relations for functions of these types make  $2x + 1$  and  $x - \frac{1}{2}$  or  $2x - 1$  essentially the same, or closely related, arguments. Thus, I regard 4.4122 as being correctly included in the general heading  $S_n$ . Likewise 4.4131 and 4.4132, since  $x^2 + x = (x + 1)^2 - (x + 1)$ , and  $S_n$  and  $V_n$  are related."
- P. 144. I had written, "One wonders at the omission in 8.4 of a reference to Legendre's table of  $\log \tan (45^\circ + \frac{1}{2}x)$ ." But there is no such table by Legendre; the table is of  $\ln \tan (45^\circ + \frac{1}{2}x)$ , which FMR carefully list on p. 184, as Dr. F. has pointed out. Here is a case where an index of names in Part I of the *Index* would have saved me from this slip. While my admiration for the *Index* has increased rather than diminished since I wrote RMT 233, I have more than once found difficulty in determining what printed or ms. tables by a given author are referred to in the volume. Hence I have started the preparation of a complete card catalogue of names mentioned up to p. 373 of the *Index*. It will only be after its completion that I can feel that all the resources of the volume are at my command. Perhaps these remarks may suggest to the authors some amplification in a second edition.
- P. 200, We wrote "7 dec. Brownlee 1923 (Russell, which one?)," since two Russells are mentioned in Part II. Dr. M. comments: "We do not pretend to give all *computers* of tables, listed under other authorship in Part II. The answer to 'which one?' here is, the one mentioned in Brownlee 1923." This remark concerning FMR editorial policy would also make unnecessary the reference to L. Brockway under Sherman 1933, p. 433.
- P. 377. I raised the question of a possible edition of Bertrand's *Calcul d. Prob.* in 1888. Dr. F. reports that such an edition is in the library of the University of Liverpool.
- P. 420, I noted, "The title of Newton's work is *Trigonometria Britanica* (not *Britannica*)."  
Dr. F. writes: Spelling 'Britannica' was intentional as (i) it is the correct spelling of the Latin adjective in question, (ii) Newton (as Mr. Cosens pointed out to me) used it in headings, etc. There seems to be one  $n$  in the title (as you say) because there was no room for two. I dislike departing from the title page, but on this occasion decided, rightly or wrongly, to take a broad view."
- P. 431. I had noted that of Schrön 1860, there was an Italian edition in 1867 and a French edition in 1891. Dr. F. remarks that at the University of Liverpool there is a Swedish edition, ed. by F. W. Hultman, Braunschweig, Vieweg, and Stockholm, Bonnier, 1868.
- P. 434. I raised the question of listing the 1819 edition of WILLIAM SPENCE, *Mathematical Essays*, either in addition to, or in place of this the listed 1820 edition. Dr. F. writes: "We have seen only Spence 1809. Very interested in your 1819 copy. I had four references to 1820: (i) *English Catalogue of Books*, 'c. 1820'; (ii) *Edinburgh Univ. Lib. Cat.*; (iii) *Univ. Coll.; London, Lib. Cat.*; (iv) De Morgan, *The Differential and Integral Calculus*, London, 1842, p. 658-659. (ii)-(iv) give 1820 without 'circa.'" I find that the *Cat. of the Astor Library* in New York also lists a copy of the 1820 edition. Thus four libraries having the 1820 edition at the times their catalogues were published, are listed. On the other hand the Boston Public Library and the Harvard University Library have copies dated 1819, "London, Printed for Thomas and George Underwood." The *Essays* were edited by Sir John F. W. HERSCHEL<sup>1</sup> as a young man, and his "Preface to the *Essays*" is dated "Slough Dec. 10, 1818"; hence one would expect this volume to have been published in 1819. Since the above was written Dr. F. reported, "We have now seen the Royal Astronomical Society copy of that date."

R. C. A.

<sup>1</sup> HERSCHEL, CHARLES BABBAGE, and GEORGE PEACOCK while still undergraduates formed in 1812 an Analytical Society, which through varied publications did important work in bringing about reforms in mathematical notation; see W. W. R. BALL, *A History of the Study of Mathematics at Cambridge*, Cambridge, 1889. "The Analytical School," p. 117f.

101. NBSMTP, *Table of Circular and Hyperbolic Tangents and Cotangents for Radian Argument*, 1943. See *MTAC*, v. 1, p. 178f.

For the value of  $\tan 1.5708$  read  $- 272241.80841$ , not  $- 27224.18084$ .

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102. D. H. LEHMER, "On the converse of Fermat's theorem," *Amer. Math. Mo.*, v. 43, 1936, p. 347-354.

This paper contains a table (p. 349-351) of composite solutions  $n$  of the congruence  $2^n \equiv 2 \pmod{n}$  having prime factors exceeding 313. A recent recomputation of this table by POULET reveals the following complete list of errata:

Delete,	68462551	5851	76839733	1019.
Insert,	44070841	2113	74874869	3533
	70541099	4643	92438581	3331
	71079661	3187	96135601	881.
	74705401	3529		

A relatively unimportant error on p. 351 may be cited. Line 13 purports to give the product of all primes  $p$  where  $17 \leq p \leq 101$ . This number contains the factor  $79^2$  and so should be replaced by

7754324487462449580421688873809769.

D. H. L.

103. J. T. PETERS, *Sechstellige Tafel der trigonometrischen Funktionen . . . von zehn zu zehn Bogensekunden . . .*, Berlin, 1929; there was a second edition in 1939. See *MTAC*, v. 1, p. 121, 162.

The following additional errors have come to light when preparing copy for the new Chambers' six-figure tables. All are on p. 8 of the 1929 edition.

Cot  $0^\circ 27' 3''$  for 127.086, read .086  
4 for 126.008, read 127.008  
5 for .930, read 126.930.

In other words the integers 127 and 126 each need to be lowered one line.

L. J. C.

104. J. T. PETERS, *Zehnstellige Logarithmentafel. Erster Band: Zehnstellige Logarithmen der Zahlen von 1 bis 100 000*. Berlin, 1922. See *MTAC*, v. 1, p. 57-59.

The proofs of a new six-figure table now being prepared for Messrs. Chambers were compared with this table, revealing, to our great surprise, two errors:

P. 406 log 69731 for 843 4358 934, read 843 4258 934;  
p. 566 log 93748 for 974 9620 114, read 971 9620 114.

This comparison would not reveal errors in decimals beyond the sixth.

About 1924 I noted that Peters, in spite of the great pains he took to ensure complete accuracy in the tenth decimal (see his Einleitung, p. vii), and his list of cases in which 15 or 16 decimals were necessary for this purpose, had missed one such case,

P. 16 log 11275 for 506, read 505;

the differences are also affected. This error was pointed out as long ago as 1872 by GLAISHER

(R. A. S., *Mo. Not.*, v. 32, p. 258—misprinted 358), who gives the mantissa as .05211 65505-49998 14... In the following year Glaisher quoted a letter from the then owner of MICHAEL TAYLOR's copy of VLACQ (*ibid.*, v. 33, p. 452) saying that some previous owner (GARDINER is suggested) had corrected log 11275 by hand.

Glaisher's remarks on end-figure errors are quoted in N 72. They were prompted by this particular "error."

L. J. C.

### UNPUBLISHED MATHEMATICAL TABLES

53[A, B].—NBSMTP, *Tables of Circumferences and Areas of Circles*. Tables prepared for the U. S. Bureau of Ordnance, Navy Department. Compare, *MTAC*, v. 2, p. 86–87.

These tables are for circles with diameters ranging [.001(.001)10; 6D]. The computations were made with IBM equipment, and a manuscript was prepared on the tabulator.

NBSMTP

54[L, M].—CARL HAMMER, *Table of selected values of  $Li(x) = \int_0^x dt/\ln t$ , and  $\int_2^x dt/\ln t$* , mss. in possession of the author at 304 West 105th St., New York City; and in the Library at Brown University.

These tables are for  $x = [2(1)10(10)100(100)1000(1000)10\ 000(10\ 000)100\ 000; 8S]$ . The values for  $Li(x)$  were previously given by J. VON SOLDNER, *Théorie et Tables d'une Nouvelle Fonction Transcendante*, Munich, 1809, p. 43–49, for  $x = [0(.01)1(.1)2(.5)3(1)20; 7D]$ , [22(2)40(5)80(10)160(20)320(40)640(80)1280; 8S],  $\Delta^3$  to .8. This table was reprinted (without  $\Delta^3$ , and with a misprint of 1220 for 1280) in A. DEMORGAN, *The Differential and Integral Calculus* . . ., London, 1842, p. 662–663. Thus of 90 values given in the ms., 22 were Soldner's values. Three other values of  $Li(x)$ , for  $x = 10^3, 10^4, 10^6$  were taken from F. W. BESSEL, "Untersuchung der durch das Integral  $\int dx/\ln x$  ausgedrückten transcendenten Function," *Königsberger Archiv f. Naturw. u. Math.*, v. 1, 1811, p. 31, and F. W. BESSEL, *Abhandlungen*, Leipzig, v. 2, 1876, p. 339. The other 65 values were computed by means of NBSMTP, (a) *Table of Natural Logarithms*, v. 2, 1941; (b) *Table of Sine and Cosine Integrals* . . ., 1942; (c) *Table of Sine, Cosine and Exponential Integrals*, 2 v., 1940; (d) *Tables of Lagrangian Interpolation Coefficients*, 1944, using five points for  $x = 500$  to 9000, and seven points for  $x = 20\ 000$  to 90 000.

C. HAMMER

55[P].—SIDNEY JOHNSTON, *Roller Chain Transmission Basic Exact Centre Distance Tables*. Ms., iii + 10 sheets typed on one side. 20.3 × 32 cm. Original in possession of the author at 81 Fountain St., Manchester 2, England; carbon copy in the Library of Brown University. Among the "References" in the ms. are the following: (a) K. B. JACOB, "Driving chains and theory application to power transmission," Engineering and Shipbuilding Draughtsmen's Assoc., *Trans.*, 1928–29 (also as a pamphlet, T. 5, centre distance tables, p. 47–54); (b) *Machinery's Handbook*, New York, Industrial Press, twelfth ed., 1943, p. 861–2.

These tables are intended to serve the mechanical engineer in solving the bothersome problem of the design of roller chain transmission. Suppose that a roller chain of  $N$  links