

content of air from the dewpoint temperature, and in finding the proper rate of sampling in dust determinations."

9. PETER L. TEA, "A mechanical integrator for the numerical solution of integral equations," Franklin Institute, *Jn.*, v. 245, May 1948, p. 403-419.

Excellent mathematical discussion and integrator description, together with a table of results obtained by BUCKLEY and HEDEMAN, already referred to by us in reviewing an earlier paper by TEA (*MTAC*, v. 2, p. 41-42).

10. B. C. WILKAS, "Nomograph solves equations for laboratory soil permeability coefficient," *Civil Engineering*, New York, v. 18, June 1948, p. 51-52. 20.9 × 28.6 cm.

The equations in question are $K_T = .0738RFLS'/t'H_{WC}$, and $K_{20} = K_T v_T / v_{20}$.

11. HEINZ WITTKÉ, *Die Rechenmaschine und ihre Rechentechnik. Eine Einführung und ein Übungsbuch mit ausgewählten Anwendungsbeispielen aus der Geodäsie, Geometrie und angewandten Mathematik (Sammlung Wichmann, v. 12)*. Berlin-Grunewald, Herbert Wichmann, 1943, viii, 161 p. 17.1 × 24.9 cm.

NOTES

96. BARTHOLOMÄUS PITISCUS (1561-1613).—It is the purpose of this Note to summarize some information about Pitiscus and his mathematical work, and to give references to the sources where further details may be gleaned.¹⁻¹⁸ We shall particularly try to give comprehensive indications of his activity in connection with the publication of mathematical tables, and their editions. Here certain facts not mentioned in any of the sources below, and others rarely noted, shall be presented.

Very little is known concerning the life of Pitiscus who was born near Grünberg in Silesia. He pursued theological studies in Heidelberg and for more than a score of the last years of his life he was court chaplain and court preacher for Elector FREDERICK IV of the Palatinate. During these latter years he published various editions of a Trigonometry; and Mathematical Tables, and edited and published, just before his death in 1613, the fine sine tables of RHETICUS (1514-1576).

The word Trigonometry is due to Pitiscus¹³ and was first printed in his

1. *Trigonometria: sive De Solutione Triangulorum Tractatus brevis & perspicuus*, 57 p. which was published as the final part (p. 157-213) of the following work by ABRAHAM SCULTETUS⁷ (1566-1625) Professor of theology at the University of Heidelberg: *Sphaericorum Libri Tres Methodicè conscripti & utilibus scholiis expositi*. Heidelberg, 1595, 213 p. This Pitiscus *Tractatus* was developed into the [viii] 371-page volume (2 uncounted white p. between p. [214] and 215),

2. *Trigonometriæ siue De dimensione Triangulorum Libri Quinque. Item Problematum variorum. nempe Geodaeticorum, Altimetricorum, Geographicorum, Gnomonicorum, et Astronomicorum: Libri Decem Trigonometriae Subiuncti, Ad Vsum Eius Demon-Strandum*. Augsburg, 1600. The Trigonometry ends

on p. 122 and the special title-page, 123 (without place or date of publication),

3. *Canon Triangulorum Siue Tabulæ Si-Nvum, Tangentivm Et Secantivm Ad partes radij 100000. & ad scrupula prima Quadrantis*, is followed by the table p. 124–213. The remaining pages (215–370) are occupied with *Problemata Varia*, this section having its own dated title page. The date $\text{C I} \triangleright \text{I} \triangleright \text{I} \triangleright$ seems to have been interpreted as a misprint for $\text{C I} \triangleright \text{I} \triangleright \text{I} \text{C} = 1599$, which agrees with the date given in the astronomical bibliographies of J. F. WEIDLER, and J. DE LALANDE (copied from Weidler). But since the date is here incorrect one may just as well argue that for the second $\text{I} \triangleright$, C was intended, making the date 1600 for p. 215–370, which is much more reasonable following p. 1–214 of the *Trigonometria* and *Canon*, after a title-page dated 1600, and especially since pagination and signatures are continuous, and since in the colophon on p. [371], part of a signature, we find: “Avgvstae Vindelicorvm, | typis Michaëlis Mangeri, | Sumptibus Dominici Custodis Chalchograhi. | M.DC.” Hence the only date properly used in connection with this trigonometry is 1600. Of this work we used the University of Michigan copy which lacks the 4 p. (1 for errata and 3 white) inserted between p. 370 and [371] in the copy used by GRAVELAAR.⁷

In the brief gnomonic part of the *Problemata*, spherical trigonometry is employed in astronomical problems. No single word is written about the motion of the earth which for a theologian of those days was doubtless prudent silence.

4. The second enlarged edition of 2, containing viii, 334, 219, 3 p. errata mostly in the *Trigonometria*, was published at Augsburg in 1609 (p. 334), not 1608, although 1608 is the date on the title-page. The trigonometry now occupies p. 1–172 and is followed by the *Problemata Varia*, p. 173–333.

5. The largely expanded tables, *Canon Triangulorum Emendatissimus* (219 p.), are separately paged (at the end of the volume) and have their own title-page, dated 1608.

6. Still another arrangement occurs in the third edition of 2, appearing at Frankfort in 1612. There are 3 sections, each with its own title page. The first section viii, 183, 2 p. is the *Trigonometria*, and the third section, separately paged, 270 p., is devoted to the *Problemata Varia*.

7. The second section is the *Canon*, unpagged [219 p.] and with a quite different type-display from 5, and is followed by 3 p. of errata in the *Canon*.

In the dedicatory epistle to Elector Frederick IV, in nos. 2, 4, 6, the following passage occurs: “Good God! How great and how rare an ornament is affability among theologians! And how thoroughly desirable would it be in this age that all theologians be mathematicians, that is, that they be reasonable and gentle men.”

We shall here pause to describe the contents of the tables, which all give the natural values of all six of the trigonometric functions. No. 3, containing [91] p., is a 5–6D table at interval 1’.

In nos. 5 and 7 the intervals are $0(1'')1'(2'')10'(10'')1^\circ(1')45^\circ$, with PP 10’’. It is a 7D table for sin, cos; 7–8D table for tan, cot; 8–9D table for sec, csc.

Many writers have declared that Pitiscus used the decimal point in his *Trigonometry and Tables* but CAJORI has shown¹¹ that such writers are mistaken; Pitiscus did not use the decimal point.

8. There were three English editions¹⁵⁻¹⁸ of the Trigonometry, alone, of Pitiscus: (a) 1614, [xi], 176, 33, [2] p.; (b) 1630 [x], 210 p.; and (c) [1631], [viii], 208 p. This translation was made from no. 6, p. 1-183. The title of (b) is as follows: *Trigonometrie | or | The Doctrine Of | Triangles. | First written in Latine, by | Bartholomev Pitiscvs | of Grunberg in Silesia, and now | Translated into English, | By Ra: Handson. | Whereunto is added (for the Marri- | ners vse) certaine Nauticall Questions, toge- | ther with the finding of the Variation of | the Compasse. All performed Arith- | metically, without Map, Sphære, | Globe, or Astrolabe, | by the said | R. H. [London] Printed by B. A. and T. | Fawcit for J. Tap. [1630].* The *Trigonometrie* occupies p. 1-176. On p. 210 is London. | Printed by B. Alsop and T. Favvcit for Iohn Tap, and | are to be sold at his shop at St. Magnus Corner. | 1630. | It will now be desirable to indicate also the title-page for 8(c) given by SAMPSON¹⁰: *Trigonometry: | Or, The | Doctrine | Of | Triangles. | First written in Latine, by | Bartholomew Pitiscvs | of Grunberg in Silesia, and now | Translated into English, | by Ra: Handson. | Whereunto is added (for the Mariners | use) certaine Nauticall Questions, to- | gether with the finding of the Variation of | the Compasse. All performed Arith- | metically, without Map, Sphære, Globe, or Astrolabe, | by the said R. H. | Printed by T. P.[urfoot] for G. Hurlock | neare Magnus Corner | s. l., s. a. size 5½ × 7¼ inches.*

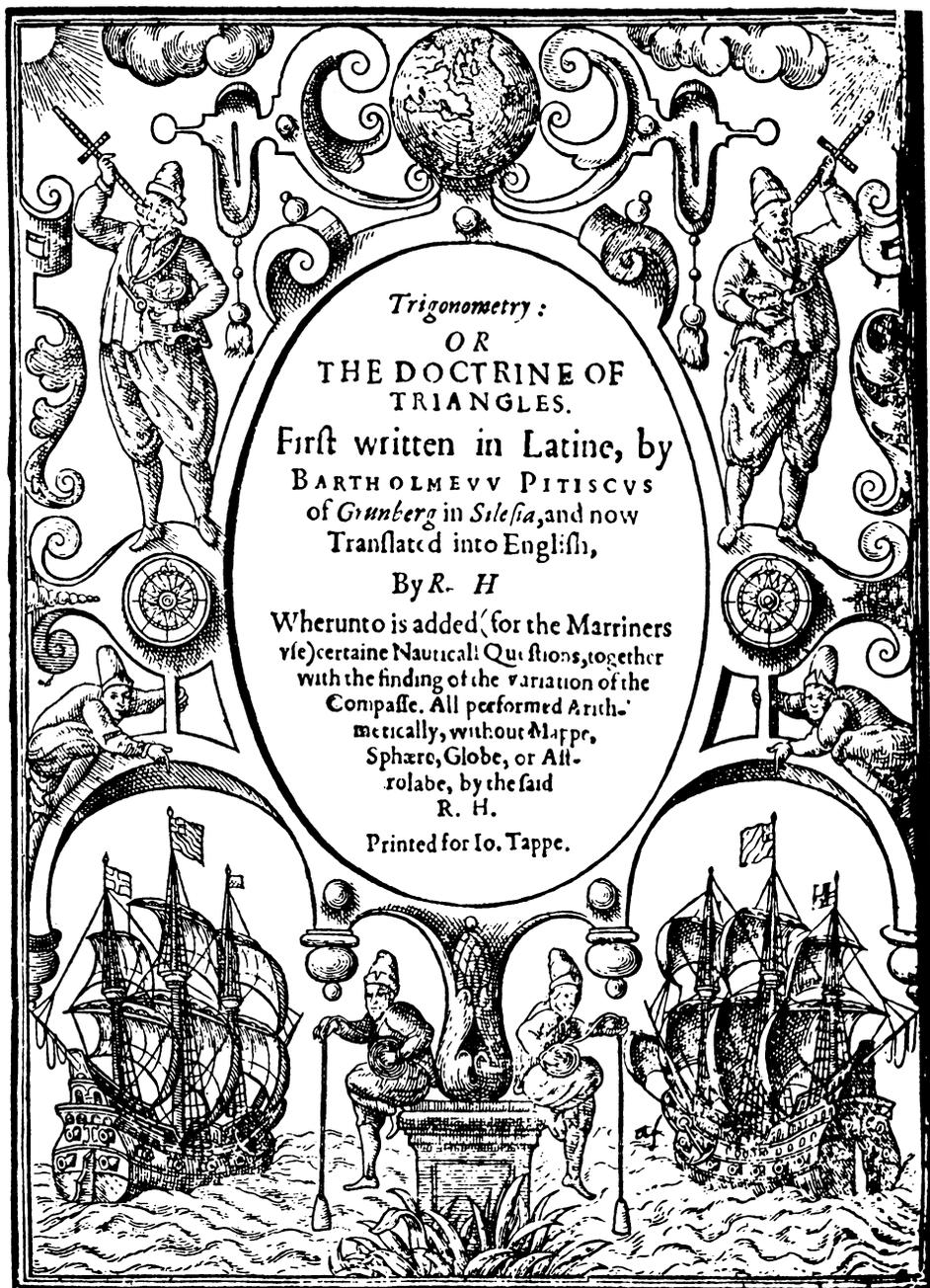
The differences of the title-pages of 8(b) and 8(c) are thus evident. It is therefore clear not only that SAMPSON was incorrect in stating that the undated 8(c) of the Crawford Library was the 1630 edition, but also that the Crawford Library *Catalogue*¹² may now have [1631] added to its entry, and that owners of *STC* should list the Crawford Library under 19968a.

Through the kindness of Mr. HORBLIT, who bought a copy of 8(a) in 1948, we were allowed to reproduce its title page, in which the name of the translator has been almost wholly erased. From the Huntington Library we learn that the original of this mutilated line was

By Ra: Handson.

Colophon: LONDON. | Printed by *Edw: Alde* for *Iohn Tap*, and are to be sold at his | shop at *St. Magnus* corner. | 1614. |

9. With 8(a) and (b) were published English editions of 3: 9(a) *A Canon of Triangles: or The Tables, of Sines, Tangents & Secants, the Radius assumed to be 100000.* [91] p. In the 1614 edition there was nothing else on the title-page. But in the second English edition, 9(b), entirely reset, one finds "London, Printed for Iohn Tap. 1630." In the case of known copies of 8(c), 9(b) is bound in. The Brown Univ. copy of 9(a) is untrimmed and of size 14.3 × 18.9 cm. No. 9(a) is in this Note identified in print for the first time, although the *STC* gave the date, 17 Jan. 1614, of its entry in the Stationers' Companys' Register. Furthermore, that 9(a), (b) are English editions, with corrections, of 3 is here indicated for the first time. To illustrate corrections: the value of $\tan 1^{\circ}44'$ is given in 3 as .03926 which is entered correctly in 9(a) as .03026; in the column of values of tangents under 8° , the errors in 3 of 5 to 10 units in the fifth decimal place for 33', 36' and 55' are similarly corrected. The *form* of title-page for 9(a) is practically



Title-Page of the Pitiscus *Trigonometry*, 1614

identical with that for 3. In the *Canon* each page is headed

A Table of
Sines || Tangents || Secants

but under each of these headings are two columns of figures; under Sines (for sines and cosines), under Tangents (for tangents and cotangents), under Secants (for secants and cosecants). Compare 17.

10. From manuscript material collected by Dr. JAMES HENDERSON of the University of London, author of *Bibliotheca Tabularum Mathematicarum*, part 1(1926), I learn of the following French edition of 3, which he had evidently inspected: *Canon Manuel des Sinus, touchantes et coupantes. Supputé par B. Pitiscus, & corrigé en ceste édition en laquelle sont additionnées toutes les choses principales & nécessaires à la Trigonometrie, extraites des traictez de la doctrine des triangles, tant rectilignes, que spheriques, faits par D. Henrion*. Paris, Abraham Pacard, 1619. According to the *Catalogue* of the Bibliothèque Nationale¹⁴ it possesses two copies of 11, a 1623 edition of no. 10, published by M. Mondière in Paris, 288 p. Of course the original may have been 5,

12. The *Catalogue* of the Bibliothèque Nationale¹⁴ lists also another work of 1623, in Latin, which seems to be yet another edition of no. 3 (or 5): *Sinum, Tangentium et secantium Canon Manualis, Supputatus a B. Pitisco et emendatus in hac editione . . . Desiderii Henrioni*. Paris, apud M. Mondière, 1623, 323 p.

13. We now turn back chronologically to 1607, when the second table of Pitiscus was published. RICHE DE PRONY,^{3a} DELAMBRE³ and DEMORGAN⁶ seem to be the only writers who give definite accurate details regarding this publication, which, with the last publication of Pitiscus referred to above, are more intimately linked with RHETICUS, to whose work we must now briefly refer. In 1551 RHETICUS, that is, GEORG JOACHIM of Rhetia, published in Leipzig his *Canon Doctrinae Triangulorum*⁴ which gave 7D tables (14 p.) of the six trigonometric functions, at interval 10', with differences; he here initiated our semi-quadrantal arrangement. The interval was probably chosen because of the reported statement of COPERNICUS, with whom Rheticus was intimately associated, that such tables at interval 10' would be of value in astronomy. This is the first table in which sines, tangents and secants are joined together. VIÈTE in 1579 gave a 7D canon for every minute.

14. After 1551, for at least a dozen years before his death in 1576 Rheticus and a corps of computers carried on colossal computations in preparing the manuscript for his monumental *Opus Palatinum de Triangulis*, published 20 years after his death, at Neustadt in the Palatinate, 1596; it is so called in honor of the Elector of the Palatinate, Frederick IV, who bore the expense of publication. There are about 1440 folio pages in the whole work, tables occupying one half. There is the remarkable, complete trigonometric canon (with errata, 554 p.) to 10D, at interval 10''; to 15D with the first and last degree at interval 1''; and to 15D in tables of tangents and secants, at interval 1'. Following this great table was a 7D table of the trigonometric canon, much less accurate, and at interval 10'', evidently a preparatory ms. of Rheticus; its publication in this volume by the editor VALENTINE OTHO (1550?-1605) a pupil whom Rheticus had engaged to assist him in the year before he died, was difficult to understand. Otho became professor of mathematics at the Univ. of Wittenberg. The great canon

was all but complete at the time of Rheticus' death. Otho added about two-thirds (460 p.) of the text to one third prepared by Rheticus, and the publication of the work was a great advance in the development of trigonometry.

Shortly after the *Opus Palatinum* was published it was found "that the tangents and secants towards the end of the quadrant became more and more erroneous and at the extreme end were very erroneous indeed." Pitiscus was engaged to correct the tables. Rheticus seems to have realized that a sine or cosine table to more than 10D would be necessary for such correction. Finally after the death of Otho, Pitiscus found such a Rheticus ms. of sines and cosines to 15D, Δ^3 , at interval $10''$; and also a supplementary 15D, Δ^2 , sine table for the first and last degrees of the quadrant, at interval $1''$. Pitiscus then made two publications.

15. In the first he corrected all that part of the great table in which the tangents and secants are sensibly erroneous, being the first 86 pages. These he reprinted, and joined to the remaining pages of the great table, making 540 pages since the errata pages disappeared. He then cut away some of the text material and added a short description or *commonefactio*, as he calls it. The whole was issued with a special title page¹⁰ *Bartholomaei Pitisci Grunbergensis Silesii Brevis Et Perspicua commonefactio De Fabrica Et Vsu Magni Canonis doctrinae Triangulorum Georgii Ioachimi Rhetici. Neostadii, Typis Nicolai Schrammii*. MDCVII. DeMorgan writes⁵ that the 86 pages of reprint are easily distinguishable by the inferiority of paper and type. Riche de Prony states^{3a} that he knew of only two existing copies of the 86 pages by Pitiscus, one copy which he had himself acquired, and the other listed in *Catalogue des Livres de la Bibliothèque du Conseil d'État*. Paris, v. 1, 1803, nos. 2781–2782. But this library was destroyed by the Commune in 1871.

16. In the second publication, to the Rheticus mss. which we have described above, Pitiscus added the sines of every 10th, 30th, 50th second, in the first 35 minutes, to 22D and published the whole at Frankfurt in 1613 in a folio volume with a long descriptive title beginning *Thesaurus Mathematicus*. Pitiscus died very shortly after its publication. The manuscript of the great table in the *Thesaurus* must have been extraordinarily accurate. Only three results of examination of the printed table appear to have been published. By comparing the 14th and 15th decimal places of 400 entries of the *Thesaurus* with the 14th–20th decimal places in the 25D *Table du Cadastre*, at interval $0^{\circ}.01$ (see *MTAC*, v. 1, p. 34), Riche de Prony gives a table to show^{3a} that the Rheticus 14D is always exact; that 15D is often in error by 1 unit, sometimes 2 units, and rarely 3 units—but never more. He pointed out also that in the supplementary Rheticus table, the first significant figure in the value for $\sin 1''$ is misprinted 2 (instead of 4). A. GERNERTH tested the first 8 of the 15 decimal places of the 32400 entries of the sine-cosine table ("Bemerkungen über ältere und neuere mathematischen Tafeln," *Z.f.d. österr. Gymn.*, Vienna, Heft 6, 1863, p. 426–428) and found 122 errors, two of them in first differences. Of the 120 remaining errors, two of them were digit transpositions, and 118 were single digit errors. In every case the first differences given corresponded to the corrected result. This suggests, therefore, that there was not a single error in the original ms. in the first 8 places of 32400 entries. Gernerth did not list 22 of the above-mentioned 120 errors, because they had been already listed by G.

VEGA, in his *Logarithmische, Trigonometrische, und andere zum Gebrauche der Mathematik eingerichtete Tafeln und Formeln*. Vienna, 1783, p. VI.

Copies of the Pitiscus edition of the *Thesaurus* which belonged in succession to LALANDE, DELAMBRE, and BABBAGE, and also of the PITISCUS *Opus Palatinum*, belonging to BABBAGE, are in the Crawford Library.¹² According to Dr. Hohenemser's *Katalog der mathematischen Abteilung* (1909) of the City of Frankfurt Library, copies of 14, 15 are to be found there; but the Librarian informed me that these were destroyed during World War II. Nos. 1, 5, 6, 14, are in the Bibliothèque Nationale; 1, 2, 4-7, 13, 14, are in the British Museum; nos. 2 (film), 3 (film), 4-7, 14 (film), 15 (film), 16, 17 (film) are at Brown University; 4, 5 are in the Lincoln Cathedral; 2, 6, 7, are at Columbia University; 2 is in Edinburgh Univ.; 14 is in Library of Congress; 2, 3, are at the University of Michigan; 16, is in the Greenwich Observatory Lib., and 6, 7, are in the New York Public Lib. and Yale University Lib. This record of Library copies of Pitiscus' tables here listed makes no pretence at completeness. See further below¹⁸ where the list is as complete as it could be made.

17. After galley proof of this article had arrived I received from the University of Cambridge Library information concerning their copy of a Pitiscus volume, which appears to be excessively rare, and unlisted in any of the ordinary bibliographical or historical sources. This volume is entitled: *Sinum, tangentium et secantium Canon Manualis Accomodatus ad trigonometriam. Bartholomæi. Pitisci Grünbergensis Silesij. Heidelbergæ*. Typis Iohan Lancelloti, Acad. Typo. Impensis Ionæ Rosæ. MDCXIII. Signatures: A-H¹², I⁴. [200 p.] It contains the same tables as 3, but differently arranged, each page for sin, tan, sec, being opposite a page for cos, cot, csc, although still headed sin, tan, sec. In addition there are a 2-page Explicatio numerorum huius canonis, and an 8-page De erratis huius canonis. Hence 9a, 9b may have been English editions of 3, taking account of the corrected 17. Just as the page-proof arrived I discovered that the University of Illinois Library also has a copy of 17, which it acquired when in 1948 it purchased the collection of fables belonging to the late Egon, Ritter von Oppolzer (1869-1907). Nos. 2, 3, 6, 12, 16 are also in this collection.

R. C. A.

¹ GASS, *Allg. Deutsche Biographie*. Leipzig, v. 26, 1888.

² A. G. KÄSTNER, *Geschichte der Mathematik*. Göttingen, v. 1, 1796, p. 564-565, 581-590, 612-626; v. 2, 1797, p. 743-746.

³ DELAMBRE, *Histoire de l'Astronomie Moderne*, v. 2. Paris, 1821, p. 26-35.

⁴ G. C. F. M. RICHE DE PRONY, "Eclaircissemens sur un point de l'histoire des tables trigonométriques," *Mémoires de l'institut Nat. d. Sci. et Arts, Sci. Math. et Phys.*, Paris, v. 5, 1804, p. 67-93.

⁵ DEMORGAN, "On the almost total disappearance of the earliest trigonometrical canon," *RAS, Mo. Not.*, v. 6, 1845, p. 221-228; reprinted with an addition in *Phil. Mag.*, s. 3, v. 26, 1845, p. 517-526. Dealing chiefly with the Rheticus table of 1551, no. 13.

⁶ A. DEMORGAN, "Table," in *The English Cyclopaedia, Arts and Science Sect.*, London, v. 7, 1861, cols., 987-990.

⁷ J. W. L. GLAISHER, BAASMT, *BAAS Report 1873*, 1873, p. 1-175 "Rheticus" and "Pitiscus," p. 43-45, 158.

⁸ N. L. W. A. GRAVELAAR, "Pitiscus' Trigonometria," *Nieuw Archief voor Wiskunde*, s. 2,

v. 3, p. 253–278, 1898. This valuable article contains complete titles, signatures, sub-headings and titles for the four Pitiscus Trigonometries, nos. 1, 2, 4, 6.

⁸ M. CANTOR, *Vorlesungen über Geschichte der Mathematik*. Leipzig, v. 2, second ed., 1900, Pitiscus, p. 603–604, 619, 642, 646–647, etc.

⁹ A. VON BRAUNMÜHL, *Vorlesungen über Geschichte der Trigonometrie, Erster Teil*. Leipzig, 1900, Rheticus and Pitiscus, p. 144–148, 212–226; Teil 2, 1903, various references to Pitiscus.

¹⁰ *Napier Tercentenary Memorial Volume*, ed. by C. G. KNOTT. Publ. for the R. Soc. Edinburgh, 1915; “The great tables preceding the discovery of logarithms,” p. 213–218, by R. A. SAMPSON.

¹¹ F. CAJORI, *A History of Mathematical Notations*, v. 1, Chicago, 1928, “Did Pitiscus use the decimal point?” p. 317–323.

¹² *Catalogue of the Crawford Library of the Royal Observatory Edinburgh*. Edinburgh, 1890. Pitiscus and Rheticus entries are here of interest to us.

¹³ Our words tangent and secant are also due to a sixteenth century writer, THOMAS FINCKE (1561–1656), a Dane, in his *Geometriae Rotundi Libri XIII ad Fridericum secundum*. Basle, 1583; second ed. 1591.

¹⁴ BIBLIOTHÈQUE NATIONALE, *Catalogue Général*, Paris, v. 138, 1936, entries under Pitiscus.

¹⁵ A. W. POLLARD & G. R. REDGRAVE, *A Short-Title Catalogue of Books Printed . . . 1475–1640*. . . . London, 1926. *STC*. Two errors in Pitiscus listings.

¹⁶ W. W. BISHOP, *A Checklist of American Copies of “Short-Title Catalogue” Books*. Ann Arbor, Mich., 1944. *STCA*.

¹⁷ C. K. EDMONDS, *Huntington Library Supplement to the Short Title Catalogue*, *Huntington Lib. Bull.*, no. 4, 1933. *STCH*.

¹⁸ English Pitiscus Entries^{15–17} Revised

[19966a]. *A Canon of Triangles*, [1614], entered in the Stationers’ Company Register 17 Jan. 1614. No place of publication, no printer’s name, no date. No entry in *STC*; here identified for the first time. Signatures A-L⁴, M².

Library Copies: Brown Univ., Mr. HARRISON D. HORBLIT of New York, Huntington Lib.

19966. [Anr. ed.] 1630. 4to. T. Purfoot for J. Tapp, 1630.

Library Copies: Boston Public Lib., British Museum (omitted in *STC*), Univ. Cambridge, Crawford Lib., Huntington Lib., Univ. Michigan, Yale Univ.

19967. *Trigonometry*. Tr. Ra: Handson, 1614.

Library Copies: Mr. Harrison D. Horblit, Huntington Lib., Lincoln Cathedral. Not in the British Museum as stated in *STC*.

19968. [Anr. ed.], 1630.

Library Copies: Boston Public Lib., British Museum (omitted in *STC*), Huntington Lib., Yale Univ.

19968a. [Anr. ed.], [1631]. Entered in the Stationers’ Companys’ Register 1 Aug. 1631.

Library Copies: Univ. Cambridge, Crawford Lib., Univ. Illinois, Univ. Michigan. Not in the Huntington Lib. (*STCH*), as stated in *STC*.

97. FRITZ EMDE.—Few names are more familiar to the mathematician than those of JAHNKE & EMDE, to various editions of whose *Tables of Higher Functions*, since the first in 1909, we have frequently made reference. Indeed the last German edition, surveyed elsewhere in this issue, appeared shortly after Professor Emde’s seventy-fifth birthday, July 13, 1948. Since Jahnke died in 1921, to Professor Emde’s genius belong the greatly enlarged and improved editions of the *Tables* after the first. In 1912 Professor Emde was appointed Professor of Electrotechnics and Director of the Electrotechnic Institute in the Technische Hochschule, Stuttgart. And now as professor emeritus (nominally since 1938 but actually since 1943) he still keeps in