using five terms agree to 5 places of decimals, the value of the fifth term in the series being 10^{-5}.

The values of the roots are given below

	$\tan x = -x$	$\tan x = -2x$
x_1	2.02876	1.83660
x_2	4.91318	4.81584
x_3	7.97867	7.91705
x_4	11.08554	11.04083
x_5	14.20744	14.17243
x_6	17.33638	17.30764
x_7	20.46917	20.44480
x_8	23.60428	23.58314
x_9	26.74092	26.72225
x_{10}	29.87859	29.86187
x_{11}	33.01700	

The calculations were carried to 6D and rounded off to 5D. It is not believed that in any case the last figure is in error by more than one unit.

It has been brought to the author's attention by R. P. EDDY, of the Naval Ordnance Laboratory, that in LOTHAR COLLATZ, *Eigenwertprobleme und ihre numerische Behandlung*, Leipzig, 1945, p. 145, are given 4D values of the first 3 roots of $\tan x = -x$, the first 2 roots of $\tan x = \pm 2x$, and the first 4 roots of $\tan x = x$.

It might also be noted that the first 7 roots, 6–10D, of the equations (i) $\cot x + x = 0$, or $J_{-\frac{3}{2}}(x) = 0$, (ii) $\tan x - x = 0$, or $J_{\frac{3}{2}}(x) = 0$, (iii) $\tan x - 3x/(3 - x^2) = 0$, or $J_{\frac{5}{2}}(x) = 0$, (iv) $\tan x + (3 - x^2)/x$, or $J_{-\frac{5}{2}}(x) = 0$, are to be found in NBSMTP, *Tables of Spherical Bessel Functions*, v. 2, 1947, p. 318–319.

L. G. POOLER

U. S. Navy Dept., Bureau of Ordnance

[1] *MTAC*, v. 1, p. 203; see also p. 336, 459 and v. 2, p. 95.—EDITOR.

[2] RAYLEIGH, *Theory of Sound*, second ed., rev. and enl. by R. B. LINDSAY. New York, 1945, v. 1, p. 334.

105. NOTE ON THE FACTORS OF $2^n + 1$.—I have established the primality of

$$\begin{aligned} N &= (2^{92} + 1)/17 \\ &= 29\ 12800\ 09243\ 61888\ 82115\ 58641. \end{aligned}$$

This is the fifth largest prime known, the four largest ones being

$2^{127} - 1$	(LUCAS (?) 1876, FAUQUEMBERGUE 1914)
$2^{107} - 1$	(POWERS, Fauquembergue 1914)
$(10^{31} + 1)/11$	(D. H. LEHMER 1927)
$2^{89} - 1$	(Powers 1911, Fauquembergue 1912)

My work is in four steps and is based on the converse of FERMAT's theorem as modified by Lehmer, and may be described briefly as follows.

In step I, the sequence $3, 3^2, 3^4, 3^8, \cdots$ was computed (mod N) by successive squaring. It was found that

$$3^{2^{90}} \equiv -81 \equiv -3^4 \pmod{N}.$$

Hence

$$3 \cdot 3^{2^{92}} = 3^{17N} \equiv 3^{17} \pmod{N}.$$

That is, N "behaves like a prime."

In step II, by combining previously computed values of $3^k \pmod N$ in the appropriate way it was found that $3^{N-1} \equiv 1 \pmod N$.

In step III the following theorem of Lehmer was used. Let p be a prime factor of $N - 1$ and let $N - 1 = mp = hp^a$. Then if N divides $b^{N-1} - 1$ but is prime to $b^m - 1$, all the divisors of N are of the form $p^a x + 1$. Since, in our case

$$N - 1 = 2^4(2^{88} - 1)/17 = 2^4 \cdot 3 \cdot 5 \cdot 23 \cdot 89 \cdot 353 \cdot 397 \cdot 683 \cdot 2113 \cdot 2931542417,$$

the best value of p is 2931542417. It was found that

$$3^m - 1 \equiv 16\ 79443\ 67320\ 76409\ 93695\ 68642 \pmod N,$$

a number prime to N. Hence the theorem applies with $b = 3$ and $a = 1$. The factors of N (if any) are of the form $2931542417x + 1$. It is well known that these factors are also of the form $184x + 1$, and hence of the combined form

$$539403804728x + 1.$$

In step IV the 30 numbers of this form not exceeding the square root of N were examined as possible factors of N. All but 8 of these have prime factors under 100 and hence cannot be primes. None of the 8 others divides N; hence N is a prime.

I have also investigated $N_k = (2^k + 1)/3$ for $k = 67$ and 71. Both are composite since

$$(1) \qquad \begin{aligned} 3^{2^{67}} &= 24486\ 86690\ 62763\ 73758 \pmod{N_{67}} \\ 3^{2^{71}} &= 6\ 00827\ 62146\ 43042\ 03171 \pmod{N_{71}} \end{aligned}$$

I discovered the factors of

$$N_{67} = 7327657 \cdot 671\ 31031\ 82899$$

later (see *MTAC*, v. 3, p. 451). The factors of N_{71} are unknown.

A. Ferrier

Collège de Cusset
Allier, France

Editorial Notes: The four steps of Mr. Ferrier give a valid and positive proof of the primality of $(2^{92} + 1)/17$. However steps II and IV may be shown to be unnecessary. Step IV may be obviated by the following simple reasoning: If N were composite we would have

$$N = (mr + 1)(nr + 1) = A^2 - B^2,$$

where $r = 539403804728$, $m \geqslant n > 0$ and $2A = (m + n)r + 2$. Since the least prime factor of N exceeds r,

$$2A < r + N/r < 10^{14} < r^2.$$

Now

$$N + 1 = 1 + (mr + 1)(nr + 1) \equiv 2 + (m + n)r \equiv 2A \pmod{r^2}.$$

Hence the remainder on division of $N + 1$ by r^2 must be less than 10^{14}. A very rough calculation shows it to be about $3 \cdot 26 \cdot 10^{22}$ however.

The elimination of the more lengthy step II requires a little more reasoning. By the same method of proof the theorem used in step III may be modified to the following: Let p be a prime factor of $N - 1 = mp = hp^a$, and let k be prime to p ($k = 17$ in the above example). Then if N divides $b^{kN} - b^k$ but is prime to $b(b^m - 1)$, all the factors of N are of the form $p^a x + 1$. Thus step II is unnecessary.

Mr. Ferrier's result (1) was obtained also by D. H. L. in October 1946. This comforting agreement, though no longer of much importance in the presence of the factors of N_{67}, adds extra strength to Mr. Ferrier's assertion that N_{71} is composite.