RECENT MATHEMATICAL TABLES

Values of the auxiliary numbers arccot 5 and arccot 239 to 2035D are in the possession of the author and also have been deposited in the library of Brown University and the UMT File ¹ of MTAC.

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¹ See MTAC, v. 4, p. 29.

RECENT MATHEMATICAL TABLES


The table gives the first 20 significant figures of \( n! \) for \( n = 1(1) 200 \) together with the exponent of the power of 10 by which the figure should be multiplied to give the approximate value of \( n! \). The author was unaware of a previous table by Uhler ¹ giving the exact values of these factorials.
Professor Uhler reports that a comparison shows that the present tables are quite without error. For references to other tables of large factorials see MTAC v. 1, p. 125, 163, 312, 452, v. 3, p. 205, 340, 355.

1 H. S. Uhler, Exact values of the first 200 factorials, New Haven, 1944. [MTAC, v. 1, p. 312].


This note gives the values of 450!·10⁻¹¹¹ and 448!·10⁻¹⁰⁹. The number 450! has 1001 digits, hence the title. The author gives the frequency of each digit 0–9 in the 890 digits of 450!·10⁻¹¹¹ from which we deduce that the probability of obtaining such a distribution from a wholly random sequence of digits is a little less than 1/5. For other large factorials computed by the author and others see the preceding review.


This paper contains two tables. Table I (p. 136–137) gives ln ln (1/x) for x = 0(.001), 999 to 3D. Table II (p. 138–139) gives 4D values of x + e⁻ˣ, -e⁻ˣ exp (eˣ), e²x/[exp (eˣ) - 1] for x = -5(.1)-2(.05)+1(.1)1.9. The values of -e⁻ˣ exp (eˣ) are nearly all incorrect and appear to have been computed in a very casual manner. For example, x = 0, the value -e is given as -2.7181. Other errors are by no means confined to the last decimal. For example for x = 1.9, the author has -114.9425 instead of -119.8085 and for x = - 4.6 the author has -100.0000 in lieu of -99.4843. This table should not be trusted beyond 3 significant figures.

D. H. L.


On p. 198 are two 4D tables of 10 + log [x/(100 – x)] covering the range x = 0(.01)5(.1)94.9.

R. C. A.


On p. 76 there is a table of the number of k-th power free numbers <250 000 of the form nᵏ + h for k = 3, 4, 5 and h = 1, 2, 3 together with the corresponding values obtained from an approximate formula.


The function j may be defined by

\[ j = x^{-1} \left[ 1 + 240 \sum_{m=1}^{\infty} m^4 x^m (1 - x^m)^{-1} \right] \prod_{n=1}^{\infty} (1 - x^n)^{-24} \]

\[ = x^{-1} + 744 + 196884x + 21493760x^2 + \cdots = \sum_{n=1}^{\infty} c(n)x^n. \]
Although this fundamental function was first investigated by Felix Klein half a century ago, it is only in recent years that some attention has been paid to the properties of the coefficients \( c(n) \). A small table of \( c(n) \) for \( n = 0(1)24 \) has been given by Zuckerman.\(^1\) From this table the author has derived a table (p. 384) showing the highest power of \( p \) dividing \( c(n) \) for \( p = 2, 3, 5, 7, 11 \) and \( n = 1(1)24 \). The additional value

\[
c(25) = 12\,18832\,84330\,42251\,04333\,51500,
\]
given by Lehmer,\(^2\) produces

\[
25 \mid 2, 3, 3, 0, 0
\]
as the 25th line of the table, as noted by the author (p. 386).

D. H. L.


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On p. 247(62) there is a 6D table of \( n \)-th powers of \( \theta = 1.32471795 \), where \( \theta \) is the real root of \( \theta^3 - \theta - 1 = 0 \) for \( n = -7(1)-2(4)4, 5 \).


A table (p. 44–46) is given of right triangles with integral sides arranged according to the radius \( R \) of the inscribed circle for \( R = 1(1)17 \). In addition to the sides of the triangle, the perimeter and the pythagorean generators are given. Of the 74 triangles listed, 31 are primitive. The table is given in duodecimal notation.


On p. 259 there is a table of 10 integers in the field \( k(\sqrt{7}) \) which are particularly small in absolute value.


On p. 218–224 is a factor table of numbers less than 20000 not divisible by 2, 3, 5, 7, 11.


Four recurring series are involved in this note \([MTAC, v. 3, p. 519]\)

\[
\begin{align*}
U_n &= U_{n-2} + U_{n-3} & V_n &= V_{n-2} + V_{n-3} \\
\bar{U}_n &= -\bar{U}_{n-2} + \bar{U}_{n-3} & \bar{V}_n &= -\bar{V}_{n-2} + \bar{V}_{n-3}
\end{align*}
\]
with initial conditions
\[(U_0, U_1, U_2) = (U_0, U_1, U_2) = (0, 0, 1)\]
\[(V_0, V_1, V_2) = (V_0, V_1, V_2) = (3, 0, 2).\]

The tables give for each series the values of \(n \pmod{P}\) for which the \(n\)-th term of the series is divisible by \(p\) together with the number of such \(n < P\). The primes \(p\) considered are those \(\leq 31\).


The matrix representations of the elements of the symmetric groups of degrees 3, 4, and 5 are set forth in an abbreviated tabular form.


If \(G_1(g), G_2(g), \cdots\) are assigned polynomials, and if
\[x = g + \sum_{n=1}^{\infty} G_n(g) y^n/n!,\]
defines \(x\) in terms of \(g\) and a parameter \(y\), then
\[g = x + \sum_{n=1}^{\infty} X_n(x) y^n/n!,\]
where
\[-X_n = Y_n(aG_1(x), aG_2(x), \cdots, aG_n(x)),\]
\(Y_n\) being the multivariate polynomial of the reviewer\(^1\) in the variables \(G_1(x)\) to \(G_n(x)\) and the symbolic variable \(a\) which is such that
\[a^i = a_i = (-d/dx)^{i-1},\]
with differentiations on all products \(G_1(x)\) to \(G_n(x)\) associated with it in the polynomial. This is the author’s first inversion formula. Table 1 gives the explicit forms of \(Y_n(f_{g_1}, f_{g_2}, \cdots, f_{g_n})\) for \(n = 1(1)8\).

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The author considers two quadrature formulas
\[(1) \quad \int_0^1 xf(x)dx = \frac{1}{2n} \sum_{i=1}^{n} f(x_{i,n}) + R_n^{(1)}(f)\]
\[(2) \quad \int_{-1}^{1} xf(x) = k_n \sum_{i=1}^{n} [f(y_{i,n}) - f(y_{i+n,n})] + R_n^{(2)}(f)\]
suggested by Chebyshev\textsuperscript{1} and of interest because the equal coefficients on the right minimize the probable error in the “observed” values of $f$.

As in the case of the unweighted formula

$$\int_{-1}^{1} f(x) dx = \frac{2}{n} \sum_{i=1}^{n} f(x_i, n)$$

of Chebyshev\textsuperscript{1} the optimal quantities $x_i, n, y_i, n$ are algebraic numbers which have, as $n$ increases, the unpleasant tendency of leaving the interval of integration thus rendering the proposed quadrature useless\textsuperscript{1} $MTAC$, v. 3, p. 97. This phenomenon occurs in the case of (1) and (2) for $n \geq 4$, whereas in the case of (3) it occurs for $n = 8, 10, 11, \ldots$.

The present paper gives $x_i, n$ to 8D for $n = 1, 2, 3$, and $k, n, y_i, n$ for $n \leq 4$. For $n = 2, 3$ there are two possible values of $k, n$ and two sets of $y$'s; for $n = 4$, there are four values of $k, n$ and four sets of $y$'s. All results are to 8D, except for $n = 4$, when only 7D are given. The formula (2) for $n = 3$ is illustrated in two cases for $f(x) = e^x$ and compared with (3) for $n = 6$ and the Newton-Cotes formula with 7 ordinates. The results speak well for (2).

D. H. L.


In a previous paper\textsuperscript{1} \textit{MTAC}, v. 3, p. 107 the author has given a table of coefficients for the repeated integration with forward and backward differences. In the present note the coefficients are based on central differences and were obtained by repeated integration of Everet’s interpolation formula. The table extends from the case of 2-fold integration to 6-fold integration. In the important case of 2-fold integration the first 25 pairs of coefficients are given, the first 11 exactly and the others to 16D. For $k$-fold integration $k = 3(1) 6$ the coefficients, which are all small, are given to 8 or 9S.

\textsuperscript{1} H. E. Salzer, “Table of coefficients for repeated integration with differences,” \textit{Phil. Mag.}, s. 7, v. 38, 1947, p. 331–338.


The function $\beta_p$ under tabulation is the $p$-th moment of $W(\beta)$, where

$$W(\beta) = 2\beta^{-1} \int_{0}^{\infty} e^{-uy} \sin \beta y dy, \quad u = y^{1/n}.$$  

The function $\beta_p$ is given explicitly by

$$\beta_p = 2\pi^{-1}(p + 1)\Gamma(p)\Gamma(1 - np/3) \sin \frac{1}{2}np$$
and is tabulated to 5S for

\[
\begin{align*}
n &= 1.6, \ p = .25(.25)1.75, 1.80, 1.85 \\
n &= 2, \ p = .25(.25)1.25(.05)1.45, 1.475 \\
n &= 3, \ p = .2(.2).8, .9, .95, .975 \\
n &= 4, \ p = .1(.1).5, .55, .575 \\
n &= 6, \ p = .1(.1).4, .45, .475 \\
n &= 8, \ p = .1(.1).3, .325, .35, .36 \\
n &= 10, \ p = .1, .2, .25, .275, .280
\end{align*}
\]

On p. 222, there is a 5S table of

\[
\left[\frac{3}{2}x(n + 3)^{-1}(3/n) \sin \left(\frac{3}{2}x - 1\right)\right]^n^{1/2}
\]
for \( n = 1.51, 1.52(.02)1.6(.1)2(.5)4(2)10(5)25. \)


Let \( \chi(i, n) \) be the \( i \)-th largest in a sample of \( n \) from the normal population with density function \((2\pi)^{-1}e^{-x^2/2}\). Hastings et al.\(^1\) gave (a) the expectations and standard deviations of \( \chi(i, n) \) to 5D, and (b) the covariances to 2D, for \( n = 1(1)10 \) and all \( i \). Jones\(^2\) obtained some of these values explicitly for \( n = 4 \). In the present paper more exact numerical integration is employed to improve the accuracy of tables in footnote 1, giving (a) to 7D and (b) to 5D. Correlations are given in a 4D table. The author also extends the results of Jones,\(^2\) providing 26 new explicit values, for \( 4 \leq n \leq 6 \).

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Let \( P(c, n, p) = \sum_{k=0}^{c} (\frac{c}{k})p^k(1 - p)^{c-k} \), and define \( p_1 \) and \( p_2 \) by \( P(c, n, p_1) = 0.95 \) and \( P(c, n, p_2) = 0.1 \). Interpolating to about 3S in published tables\(^1\) of the beta and \( F \) distributions, the author tables \( p_1 \) and \( p_2 \) for \( c = 0(1)9, n = 1(1)150 \). A corresponding single entry table based on the Poisson approximation is also given. The tables are intended to aid in selecting sample size and acceptance number in sampling inspection plans having 5 per cent producer’s risk and 10 per cent consumer’s risk.

J. L. Hodges, Jr.


The author considers the functions

\[ \psi(x) = e^{-x} \int_{-\infty}^{\infty} \exp (-e^t - e^{-t-x}) dt \]

and

\[ \Psi(x) = \int_{-\infty}^{\infty} \exp (t - e^t - e^{-t-x}) dt. \]

Table 1, p. 193-196, gives \( \Psi(x) \), \( \Delta \Psi \), \( \psi(x) \) to 5D for \( x = -3(,5)10.5 \).

Table 1A, p. 397, gives the inverse function \( \Psi^{\prime}(x) \) to 2D for \( \Psi(x) = .0002(.0001).001(.001).01(.01).1(.1).9(.01).99(.001).999(.0001).9997. \)


This volume, by a professor at the École Supérieure d’Électricité and a "Lieutenant-Colonel des Transmissions," contains tables and rather rough graphs which may be noted even if the tables contain nothing new. In no case has the originator of any table been definitely indicated.

P. 333-334: Graphs of sinh\( x \), cosh\( x \), tanh\( x \) and 5 or 6S tables of \( e^x \), \( e^{-x} \), sinh\( x \), \( \sinh x \) for \( x = 0(.2)6 \).

P. 336-339: \( \sin(x) = - \int_{-\infty}^{\infty} t^2 \sin t \, dt \), \( \sinh(x) = \frac{1}{2} \pi + \sin(x) \). There are tables of \( \sin(x) \), \( \cos(x) \) for \( x = 0(.01)1(.1)6(1)15(5)100(10)200(100)1000, 10^4, 10^4, 10^4, 4 \infty \), \( 4-7D \), mostly 4D. There are also 5-6D tables of the maxima and minima of \( \cos(x) \) for \( x/\pi = 0(,5)15.5 \) and of \( \sin(x) \) for \( x/\pi = 1(1)15 \), as well as graphs of \( \cos(x) \) and \( \sin(x) \).

P. 342: Graphs and 5D table of \( \theta(x) = 2\pi^{-1} \int_0^x e^{-t^2} dt \) with \( \Delta \) for \( x = .05(.05)2 \).

P. 349-350: Graph and table of \( \Gamma(1 + x) \), for \( x = [0(.01)2; 4D] \), \( x = [2(.01)3.99; 4-5S] \). Apparently reprinted from JAHNIKKE & EMDE.

P. 375-376: Graphs of \( I_n(x) \), \( n = 0(1)11 \); \( K_n(x) \), \( n = 0, 1 \); those of \( I_n(x) \) apparently copied from Jahnke & Emde.

P. 380: Graphs of \( \text{ber}(x) \), \( \text{bei}(x) \), \( M_0(x) \), \( \theta_0(x) \).

P. 403-407: Tables of \( J_0(x) \), \( J_1(x) \), \( Y_0(x) \), \( Y_1(x) \) for \( x = [0(.1)16; 4D] \).

P. 408-409: Tables to 4D of \( J_n(x) \), \( n = 2(1)9, x = 0(1)24; n = 10(1)17, x = 4(1)29 \).

P. 410: First to ninth roots (4D) of \( J_n(x) = 0, n = 0(1)22 \). Also first to eighth root (4-5D) of \( J_n(x) = 0, n = 0(1)19 \).

P. 411-412: Tables of \( J_{n/2}(x) \), \( n = 1(2)13, x = [0(.1)24; 4D] \).

P. 413-415: Tables of \( \text{ber}x \), \( \text{ber}'x \), \( \text{bei}x \), \( \text{ker}x \), \( \text{ker}'x \), \( \text{kei}x \), \( \text{kei}'x \) for \( x = 0(.1)10 \) mostly 4S.

P. 416-417. Tables of \( M_0(x) \), \( \theta_0(x) \), \( M_1(x) \), \( \theta_1(x) \), \( x = 0(.05)1.7(1).3(2)5(.5)6(1)12(2)20(5)45 \).

P. 442-444: Tables of LEGENDRE polynomials \( P_n(x) \), \( n=1(1)7, x=[0(.01)1; 4D] \), apparently reprinted from Jahnke & Emde.
P. 445–446. Graphs of the associated Legendre functions of the first kind. Apparently taken from Jahnke & Emde, figs. 60–63.


R. C. A.

711[L].—HARVARD UNIVERSITY, COMPUTATION LABORATORY, Annals, v. 12:

Sixteen previously published volumes of the Annals have been reviewed in MTAC, v. 2, p. 176–177, 185–187, 261–262, 344, 368, v. 3, p. 41, 102, 185–186, 311–314, 367, 432–440, 474–475, 517–518. These volumes on publication listed at $10.00 each, are now listed at $8.00 each, which is more reasonable for offset-printed volumes.

The volume under review is the tenth in the Harvard series of tables of Bessel functions of the first kind, which, after three more volumes have been published, will contain tables, to 10D at least, for \( J_n(x) \), for \( n = 0(1)100 \), and \( x = 0.(01)100 \); for \( n = 0(1)3 \), \( x = [0.(001)25(.01)99.99; 18D] \), and \( n = 4(1)15 \), \( x = [0.(001)25(.01)99.99; 10D] \), but beginning with order 16 the argument interval of the tables is constantly .01. The values of \( J_n(100) \), \( n = 0(1)100 \) are to be given in the final volume 15. In the first two volumes only two orders were tabulated, while there are twelve in the current volume in which the first significant values .00000 00001 occur in connection with \( J_{14}(27.53) \) and finally \( J_{61}(36.34) \).

The tables in this volume are wholly new. The Harvard Computation Laboratory is at present the outstanding center in the world for the computation and publication of mathematical tables. In five years not only have there been 15 volumes of this kind in the Annals series, but also other tables which have been reviewed in MTAC, v. 2, p. 218, 300, 307.

R. C. A.

712[L].—MARIETTE LAURENT, "Table de la fonction elliptique de Dixon pour l'intervalle 0–0.1030," Acad. r. de Belgique, classe des sciences, Bull., s. 5, v. 35, 1949, p. 439–450, 15.8 × 25.1 cm.

On p. 441–445 is a table of values of \( sm \, u \) for \( u = [0.(001).103; 10D] \), \( \Delta^4 \), where \( x = sm \, u \) and \( u = \int_0^x(1 - \nu^2)^{-1/2}d\nu \), and on p. 446–450 is a table of \( u \) with argument \( sm \, u \ = [0.(001).103; 10D] \), \( \Delta^4 \). The author states that 11 decimals were used in the calculations, the 9-th decimal corresponding to the precision of a centimeter in geodesic applications, and that there may be unit errors in the tenth decimal place.

For previous tables see MTAC, v. 3, p. 249, and A. C. DIXON, Quart. Jn. Math., v. 24, 1890, p. 167–233. The connection between the Dixon function \( sm \, u \) and the equianharmonic Weierstrass function is given by \( sm \, u = [2\sqrt{3}\wp(u/\sqrt{3})]/[\sqrt{3} - \wp'(u/\sqrt{3})] \).

R. C. A.

On p. 184–190 are 6S tables for $z = .5(.5)8$, and $\alpha = .001, .01, .05, 1, .2, .25, .3(.1)1$ of

$$M(\alpha; \gamma; z) = \sum_{n=0}^{\infty} \frac{\Gamma(\gamma)\Gamma(\alpha + n)z^n}{\Gamma(\alpha)\Gamma(\gamma + n)n!}$$

for $\gamma = \frac{1}{2}(-1/2)^2$, and of the logarithmic solution for $\gamma = 1, 2, 3$.

R. C. A.


If $f(x)$ stands for one of the functions $Ai(x)$, $Bi(x)$ or $aAi(x) + bBi(x)$, then the derivatives of $f(x)$ can be expressed as

$$f^{(2n)}(x) = P_n f(x) + Q_n f'(x)$$
$$f^{(2n+1)}(x) = R_n f(x) + S_n f'(x),$$

where $P_n, Q_n, R_n,$ and $S_n$ are polynomials in $x$. These polynomials are tabulated on p. 37–38 for $n = 1(1)15$.

R. C. A.


Tables on p. 43–126 are as follows:

A. Complete normal elliptic integrals.

(i) 5D values of $k^2 = \sin^2 \alpha$, $K$ and $E$ for $\alpha = 0(1')70'(30')80'(12')-89'(6')90'$.

(ii) 5D values of $K$, $K'/K$, $K/K'$, $\log q$, $\log q'$ for $k^2 = 0(.01)5$.

(iii) 5D values of $K$, $K'$, $K'/K$, $K/K'$ for $k^2 = .000001(.000001).00001$, .0001(.0001).003.

B. Tables of normal elliptic integrals of the first kind; values of $F(k, \phi)$, $k = \sin \alpha$ for $\alpha = 5'(5')90'$ and $\phi = [1'(1')90'; 5D]$ values for $\phi \leq 5'$, 4D values for $\phi > 5'$.

C. Tables of normal elliptic integrals of the second kind, 4D values of $E(k, \phi)$, $k = \sin \alpha$, for $\alpha = 5'(5')90'$ and $\phi = 1'(1')90'$.

R. C. A.


On leaf 4 is a table of $a_k = \frac{3}{2}k(k + 1)(2k + 1) = \frac{3}{2}B_5(k + 1)$, where $B_5$ is the third Bernoulli polynomial, for $k = 1(1)100$.

R. C. A.
This work contains two tables of "Fresnel functions." The main table gives

\[ A(x) = x^{-1/4} \int_0^\infty t^{-1} (1 + t^2)^{-1} \exp \left( -\frac{1}{2} \pi t^2 \right) dt \]

\[ = \left\{ \frac{1}{2} - S(x) \right\} \cos \frac{1}{2} \pi x^2 - \left\{ \frac{1}{2} - C(x) \right\} \sin \frac{1}{2} \pi x^2 \]

and

\[ B(x) = x^{-1/4} \int_0^\infty t^4 (1 + t^2)^{-1} \exp \left( -\frac{1}{2} \pi t^2 \right) dt \]

\[ = \left\{ \frac{1}{2} - S(x) \right\} \sin \frac{1}{2} \pi x^2 + \left\{ \frac{1}{2} - C(x) \right\} \cos \frac{1}{2} \pi x^2 \]

and

\[ Z(x) = \pi \int_0^\infty A(u) du, \]

where \( S(x) \) and \( C(x) \) are the Fresnel integrals

\[ C(x) = \int_0^x \cos \frac{1}{2} \pi u^2 du, \quad S(x) = \int_0^x \sin \frac{1}{2} \pi u^2 du. \]

These are tabulated to 4D for \( x = 0(.01)1(.05)1.5(.1)7(.2)10(.5)15 \) with first differences, and in the case of \( Z(x) \), second differences. For \( x \geq 1 \) the table gives also

\[ A_1(x) = (\pi x)^{-1} - A(x) \]

\[ Z_1(x) = Z(x) - \ln x. \]

The second table gives 4D values of the integrals

\[ A^*(x) = \int_x^\infty A(u) du/u \]

\[ B^*(x) = \int_x^\infty B(u) du/u \]

for \( x = 0(.1)5 \) with first and second differences. For \( x < 1 \), \( A^*(x) + \ln x \) and \( B^*(x) + \ln x \) are also given.

D. H. L.
$M$ being the confluent hypergeometric function. Thus

$$L_4(t) = t^4 - 16t^3 + 72t^2 - 96t + 24.$$  

The range of $x$ and $n$ is

$$x = 0(0.01)1(1)18(2)20(5)21(1)26(2)30; \quad n = 0(1)5.$$  

The table is given mostly to 5S, but in some cases only to 3S. A spot check reveals a number of last digit errors and the following gross error

$$x = 4.7, n = 4, \quad \text{for } 88260 \quad \text{read } 89267.$$  

No statement is made as to the method of construction of the table. Apparently no really good tables of $L_n(t)$ have been published. See MTAC, v. 2, p. 267, FMR Index, p. 337.

D. H. L.

719[Q].—V. Krat & S. Petrov, "Tablifsy vspomogatelnich funktsfî i $\chi$ dlîa opredeleniia elementov sistem zatmennykh peremenykh" (Tables of auxiliary functions $\psi$ and $\chi$ for determination of the elements of systems of eclipsing variables) II, Central Astronomical Observatory, Pulkovo, Izvestia, v. 17, No. 5, 1947, p. 117.

This paper consists, in essence, of three tables. Tables 1 and 2 contain numerical values of Russell's well-known $\psi(k, \alpha = n)$ function which is needed for the computation of the ratio $k$ of the radii of components of an eclipsing binary system from an analysis of light curves due to total eclipses of a star which is completely darkened at the limb. A definition of this auxiliary function in terms of the basic $p$-functions was first given by Russell (Astrophys. Jn., v. 35, 1912, p. 315); it is repeated in the introduction to the tables under review.

Of these, Table 1 contains 3D values of $\psi(k, n)$ appropriate for the partial phase of a total eclipse, while Table 2 gives values of the same function appropriate for the annular phase of a transit. The arguments of tabulation are $k = .1(.1)1$, $n = 0(.1)1$ for Table 1, and $k = 2(.1)1$, $n = 0(.1)1$ for Table 2. The intervals of tabulation in both arguments are too large to make the tables easy of interpolation. Both tables are not original, but are revised versions of earlier tables of the same functions published by Russell & Shapley (Astrophys. Jn., v. 36, 1912, p. 239; Table IIx on p. 245 corresponds to Krat and Petrov's Table 1; while Table IIy on p. 391 of the same volume of the Astrophys. Jn. corresponds to Krat and Petrov's Table 2). A comparison of the corresponding entries of the new and old tables reveals discrepancies attaining the second significant place, and due no doubt to the inferior accuracy of the old tables which were based on inaccurate $p$-functions. The new Russian tables are based on the extensive and accurate 5D tables of $p(k, n)$ which were published in 1939 by Tsesevich [MTAC, v. 3, p. 191–194].

Table 3—the main feature of the paper under review—contains a set of 4D tables of Krat's auxiliary functions $\psi(k, n, \alpha_0)$ and $\chi(k, \alpha_0)$ appropriate for partial eclipses of stars exhibiting uniformly bright disks. The arguments of tabulation are $k = .1(.1)1$, $\alpha_0 = .1(.1)1$, and $n = 0(.1)9$. The reader is cautioned to notice that Krat's functions $\psi$ and $\chi$ are not identical with
Russell’s well-known functions denoted currently by the same symbols; for the definition of Krat’s functions tabulated in the paper under review cf. Russian Astronomical Jn., v. 11, 1934, p. 412 (Russian, with English summary).

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Contains tables of the probable errors in photogrammetric control extension by nadir point triangulation. The tables, p. 82–96, are based on a triangulation net of fifteen rhombuses (nadir numbers, \(N_0\) to \(N_{15}\)), in which are located two fixed points. Fifteen error-tables are given, in which \(N_0\) is the first fixed point, and the second fixed point is placed successively at \(N_1, N_2, \ldots, N_{15}\).

On p. 28 is a table giving the coefficients in the expansion of Lucas \(U_n = (a^n - b^n)/(a - b)\) and \(V_n = a^n + b^n\) as polynomials in \(a + b\) and \(ab\) for \(n = 0(1)16\), together with numerical values of \(U_n\) and \(V_n\) in the cases \(ab = 1, a + b = 3, 4\). These are used to compute tables of weighting coefficients p. 34–46.

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On p. 19 there are two tables of the function \(G_\theta(Y, \tau) = \frac{2}{\pi} \tan^{-1} (\tau/Y)\) for \(\tau = 0(2)3.8, Y = 2.2(2)3.8\) and for \(\tau = 0(2)1.8, Y = 2.2(2)3.8\).

On p. 20, there are tables of

\[
f_1(Y, \tau) = G_\theta(Y, \tau + 1) - G_\theta(Y, 1) - \frac{1}{\pi} \int_0^{\tau/2} \frac{Y^4 G_\theta(x, \tau - 2x)}{(1 + x)^4 (1 + x + Y)} \, dx
\]

for \(\tau = 0(2)2.8, Y = .1(.1)1.0\) and for \(\tau = 0(2)1.8, Y = 1.2(2)3.0\) and a table of

\[
f_2(Y, \tau) = \frac{1}{\pi} \int_0^{\tau/2} \frac{Y^4 f_1(x, \tau - 2x)}{(1 + x)^4 (1 + x + Y)} \, dx
\]

for \(\tau = 0(2)1.8, Y = .2(2)2.0\). All tables are to 4D.


On p. 350 there is a table of \([Si(k) - (1 - \cos k)/k]/\pi\) for \(k = .1(.1)\cdot .4(2)2(.5)5(1)16, 20(10)50\), and for \(k = n\pi\) with \(n = 1(1)5\) to 4S.