"A decimal counting tube"  
"New improvements and applications in Remtron counter tube"

**Tube Manufacture and Crystal Diode Experience**

"Design and manufacture of electron tubes for electronic computer service"  
"Problems in the manufacture of special tubes for computer usage"  
"Development of the 7AK7"

"Some problems involved in the manufacture of germanium diodes"  
"Experience with germanium diodes in the SEAC program"  
"Crystal diode life experience in the Whirlwind Computer circuits"

**Williams Type Storage**

"Tube experience in the SWAC"  
"The SEAC memory using Williams storage"  
"The selection of cathode-ray tubes for Williams storage"  
"Methods of testing cathode-ray tubes for service in Williams storage systems"  
"Theory of storage in cathode-ray tubes"  
"Progress report on the electron mechanism on the Williams storage technique"  
"Space charge effects in Williams storage tube"

T. R. Kohler, Philips Laboratories  
Frank J. Cooke, Remington Rand Corp.  
J. G. Brainerd, University of Pennsylvania, Chairman  
R. E. Higgo and H. E. Stumman, RCA  
R. L. McCormack, Raytheon Manufacturing Co.  
R. W. Slinkman, Sylvania Electric Products, Inc.  
N. DeWolff, General Electric Co.  
H. Wright, NBS  
H. B. Frost, MIT  
J. H. Bigelow, Institute for Advanced Study, Chairman  
H. D. Huskey, NBS  
W. W. Davis, NBS  
J. H. Pomerene, Institute for Advanced Study  
D. Friedman, NBS  
J. Kates, University of Toronto  
A. W. Holt, NBS  
L. Brillouin, IBM

**OTHER AIDS TO COMPUTATION**

**A New Differentiating Machine**

The first step in the design of a differentiating machine is the selection of a mechanical analogue for the derivative of a plotted function. The incorporation of this mechanical differential ratio into a machine that will draw the derived curve of a plotted function will follow with the use of the proper linkages, gears, etc., that will transfer the relative value of this mechanical $\frac{dy}{dx}$ to a writing pen for tracing the derived curve.

The most common type of differentiating machine that has been built uses a tangent line analogue based on the geometric concept of the derivative. The differentiating machine built in 1904 by J. E. Murray employs this tangent line analogue in the form of two dots on a celluloid plate. So long as the dots remain on the curve to be differentiated, the chord connecting them is approximately parallel to the tangent to the curve at the mid point between the dots, then by a system of connecting linkages, a writing pen is caused to draw the derived curve.
Armin Elmendorf's differentiating machine\textsuperscript{2} employs a tangent line analogue in the form of a sharp edged wheel which is rolled along the curve to be differentiated. A pulley system of parallel linkages connects the wheel to a writing pen. Both Murray's and Elmendorf's machines can be adapted to use a "normal mirror" for finding the slope of a curve. A line perpendicular to a mirror held normal to a curve will be a tangent line.

F. E. Myard\textsuperscript{3} based his machine on a different analogue from those already mentioned. His machine develops a differential ratio when a single point is made to trace a curve. This is done by an adding unit (a set of differential gears) and a crown wheel (integrating wheel) which together form the differential ratio $\frac{dy}{dx}$. This ratio is plotted as a curve.

No attempt is made in this article to evaluate the above mentioned machines as differentiating devices.

A new differentiator has been designed and built by Cyril P. Atkinson (Fig. 1). It is a "tangent line analogue" type of differentiating machine using a "tracer bar" to measure the slope of the tangent line. The relative value of the tangent of the angle of slope of the tracer bar is transferred to the writing pen by a pair of synchro-transmitters (selsyn motors) in such a way that the motor attached to the writing pen will cause it to describe the derived curve to a scale determined by the constants of the system.

It had been hoped that the differentiator could be operated continuously in the differentiation of a curve and thus produce a continuous derivative. This plan had to be abandoned since an operator is unable to move the tracer bar with such finesse to cause the writing pen to draw a smooth curve. In the process of differentiation the machine is moved from interval to interval...
across the original curve by means of the motor and drive shaft. At each interval a slope is determined by the tracer bar and a point is tapped by the writing pen, so that a series of points describes the derivative. A smooth curve drawn through these points is the derived curve. The units of the derived curve can be obtained from the scale factor of the machine. If the units of the abscissa of the original curve are equal to the distance between the centerline of the writing pen M of Fig. 1 and the center of rotation of the shaft of selsyn motor N, the units of the ordinates of the derived curve are equal to the ordinate units of the original curve divided by the units of the abscissa of the original curve. The abscissa units are the same for both curves.

Various curves have been differentiated by the machine and the accuracy was determined to be from 2.5 to 5%. Fig. 1 shows the cosine curve under the writing pen of the differentiator. This cosine curve was produced by differentiating the sine curve shown under the tracer bar. Harmonic curves can be differentiated most easily when they are plotted with the units of the abscissa equal to distance between the centerline of the writing pen and the selsyn motor. When the slope of the tracer bar must exceed 45 degrees to be tangent to a curve as in the differentiation of a circular arc of 90 degrees, a technique that is employed is to determine the normal to the curve by holding the tracer bar across the curve. By observing the reflection of the curve in the polished surface of the tracer bar and causing the reflection to be continuous with the original curve a very accurate normal can be obtained. Since a plot of the slope of the normal would be a curve of \(-\frac{dx}{dy}\) it is only necessary to take the negative reciprocal of this curve to obtain the derivative.

Some of the defects of the earlier differentiators were kept in mind in the development of the present differentiator. For example, the tracer bar, the counterpart of which was obscured in both Murray’s and Elmendorf’s machines, is placed immediately before the operator and can be viewed from above and held tangent or normal to the curve without interference from other parts of the mechanism. Also, the writing pen, which is driven by a synchro-transmitter, can be placed in any position the designer desires, even in an adjoining room if need be. It was placed so the operator would have an unimpeded view of his work, both in the differentiation of the original curve and in the plotting of the derivative.

The defects of the differentiator are (1) the point by point method of producing the derived curve; (2) the inaccuracies due to (a) the approximation of the derivative by a chord, (b) selsyn motor inaccuracies, (c) mechanical inaccuracies.

The machine could be improved by increased precision in the dimensions of the moving parts. The use of larger and more accurate selsyn motors would allow a derived curve to be drawn to a larger scale. The incorporation of photo tubes into the “tracer bar” to replace the points which are used to trace the original curve would make the machine automatic and cause it to draw a continuous derived curve.
The comparison of the present machine with the machines mentioned earlier in this paper as to cost, performance, etc., and a more detailed account of the theory of differentiating machines awaits a more general article.

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A. S. Levens

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2 Armin Elmendorf, "Mechanical differentiation," Franklin Institute, Jn., v. 185, 1918, p. 119-130.
5 J. Lipka, Graphical and Mechanical Computations. New York, 1918.

Bibliography Z–XV


A device is described for solving the equation \( \frac{dx}{dt} = -\lambda x + f(t) \), where \( t \) is real time. The unknown function \( x \) is the rotation of a shaft driven by a servomotor whose rate of rotation \( \frac{dx}{dt} \) is regulated to equal a voltage \( -\lambda x + f(t) \). This voltage is measured by a tachometer and the difference between it and \( -\lambda x + f(t) \) is used as an input for a servo-system of the type with a "memory circuit" described by Williams & Uttley. A helical potentiometer on the output shaft yields the voltage \( \lambda x \) and \( f(t) \) is introduced by a manual follower. Various applications to atomic pile calculations are described.

F. J. M.


The system of equations to be solved are realized by means of Kirchoff's laws. The coefficients are represented by impedances either directly or by reflection through one-to-one transformers. This last device is used to remove the restrictions which normally apply to real transfer impedances.

F. J. M.

The complex Fourier transform $F(\omega)$ of the real function $f(\tau)$ is expressed as

$$F(\omega) = \int_{-\infty}^{+\infty} f(\tau) e^{-i\omega \tau} d\tau.$$ 

$F(\omega)$ may be evaluated on analog equipment by a solution of the linear integrodifferential equation

$$y' + \omega^2 \int y dt = f(t); \quad y(0) = \int_0^t y dt = 0.$$ 

From a solution of a similar system of equations one may obtain the real inverse Fourier transform of $F(\omega)$. An illustrative example and an analog machine program are given. The author has reported several omissions and errata that do not affect the results.

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The author points out the usefulness of a double electronic resolver which will give the three spherical coordinates of a vector from its three cartesian coordinates expressed as voltages. Such a spherical resolver is readily obtained by combining two ordinary resolvers of the two dimensional type. An electronic resolver which yields $(e_1^2 + e_2^2)^{1/2}$ and the arctan $e_2/e_1$ is described. The input voltages $e_1$ and $e_2$ are used to modulate two sinusoidal voltages of the same frequency, 90° out of phase. These modulated voltages are added and the amplitude is detected to yield $(e_1^2 + e_2^2)^{1/2}$. The phase of this sum voltage is also detected by a pulse technique. A pulse is emitted when the reference voltage passes through zero and also one when the sum voltage passes through zero. The difference in time between these is the required angle arctan $e_2/e_1$. Special provision is made to yield a zero angle for zero amplitude and for obtaining the base line for amplitude measurements. A proposal is also made for displaying the results in the three dimensional case.

F. J. M.


This article describes a large analog computer located in the Aerial Measurements Laboratory at Northwestern Technological Institute. This device “now consists of the following types of equipment: inductors, resistors, and capacitors; special amplifiers; special synchronous switches and relays to provide repetition of the transients and to re-establish initial conditions between such repetitions; various function generators; single and dual gun cathode ray oscilloscopes to provide visual representation of out-
puts together with cameras for making permanent records; and various
standard electrical devices for calibration and testing. Circuit connections
are made by busses with selector switches and by plugs and jacks. . . ."

The cost was about $250,000.

F. J. M.


The equation $\varphi'' - [E - V(t)]\varphi = 0$ is solved by means of feed back
amplifier integrators. The potential $V(t)$ is obtained by a manual follower.
The design considerations for a simple and relatively inexpensive instrument
are given at some length and also its use for the solution of the appropriate
characteristic value problems.

F. J. M.


A polynomial $f(z) = \sum_{i=1}^{n} a_i z^i$ with real $a_i$ can be represented as a function
of $R$ and $\theta$ when $z = R \exp i\theta$, i.e., $f(x) = \sum_{i=1}^{n} a_i R^i \cos j\theta + i \sum_{i=1}^{n} a_i R^i \sin j\theta$.

If $R$ is entered manually as a shaft rotation, it is possible to obtain d.c.
voltages corresponding to the quantities $a_i R^i$ by a suitable arrangement of
potentiometers. These d.c. potentials are applied to commutators to give a
square wave voltage of frequency $60/\pi$ per second. All but the fundamental of
this voltage is filtered out and added to yield the real and imaginary parts
of $f(z)$. These are used as the horizontal and vertical deflection voltages of
an oscilloscope. $R$ is varied until a zero is observed. The instrument handles
polynomials of the tenth degree and the output voltages have three figure
accuracy under suitable scaling.

F. J. M.

18. François-Henri Raymond, "Sur un type général de machines mathé-

The author considers three particular mathematical machines of the
continuous type. One is a machine for solving systems of linear algebraic
equations and associated problems (e.g., inversion of a matrix, characteristic
values of a matrix). This will be called the "algebraic machine." The other
two machines are for solving systems of differential equations. One machine
is capable of handling larger systems than the other. However, the theory
of both is the same and either machine will be referred to as a "differential
analyzer." After some introductory remarks the author briefly describes the
operation and applications of the machines. The next portion of the paper,
which is of greatest interest to mathematicians, is a discussion of the sta-
bility and precision of the devices. These phases will be elucidated below.

Consider the system of $n$ linear equations (in matrix form) $Ax = b$ where
$A = \|a_{ij}\|$ is a square matrix and $x$ and $b$ are column vectors. It is assumed
that by a preliminary manipulation all the coefficients of $A$ have been reduced (by a scale factor) such that $|a_{ij}| \leq 1$. It is also assumed that to each coefficient $a_{ij}$ there is attached an error $\alpha_{ij}$ whose upper bound is a certain percentage of $a_{ij}$. Similarly for the $b_i$ (components of the vector $b$). Hence the machine actually solves the equation $(A + \alpha)x' = b''$. The problem is to estimate $|x'_i - x_i|$, that is the difference between the observed solutions $x'_i$ and the actual solutions $x_i$. By using the Redheffer formula the author obtains the following result:

$$
\Delta \leq \epsilon D^{-1}(L + n^3X_M)(T/(n - 1))^{(n-1)/2}
$$

where $\Delta = \max |x'_i - x_i|$, $\epsilon =$ percentage of the coefficient $a_{ij}$ which forms the error $\alpha_{ij}$; i.e., $\alpha_{ij} = \epsilon a_{ij}$ and $|b''_i - b_i| = \epsilon |b_i|$, $D =$ the modulus of the determinant of $A$, $T =$ the trace of $AA^t$ ($A^t =$ transpose of $A$), $L =$ the length of the vector $b = \left[\sum b_i\right]^t$, $X_M = \max |x'_i|$, $n =$ number of equations in the system.

The quantity $\Delta$ measures the percentage error. However it appears to the reviewer that the scaling operations are not considered in sufficient detail; for while the percentage error may be small, the actual error may be large if a large scale factor has been used. Also the justification for the assumption that $\alpha_{ij} = \epsilon a_{ij}$ is not obvious. Noise is not considered.

The algebraic machine is essentially of the Goldberg-Brown type; and the stability of the system depends on the characteristic roots of the matrix $A$. In writing the equilibrium equations for the machine, the following matrix form is obtained:

$$
[A - ((n + 1)/G)I]x - b = 0
$$

where $G(p)$ is the gain characteristic (in operational form) of the amplifiers used, $(p = d/dt$ is the Heaviside operator). If $\mu_v, v = 1, 2, \cdots, n$ are the characteristic roots of $A$, then $\mu_v = (n + 1)/G(p)$. If the amplifiers are so constructed that $\Re G(p) < 0$ when $\Re p > 0$ then $\Re p > 0$ if $\Re \mu_v > 0$ and the machine is stable. [$\Re =$ "real part of." ]

In the differential analyzer the equations to be solved are (in matrix form)

$$
g(t) + [A\dot{x} + Br + Cp + D]y = 0
$$

where $A, B, C, and D$ are square matrices with constant coefficients, $g(t)$ and $Y$ are column vectors and $p = d/dt$. In a manner similar to that used in deriving (1),

$$
\{[A - ((n + 1)/G)I] + B\alpha + C\alpha^2 + D\alpha^3\}V + g = 0
$$

is obtained. [In the ideal case, $G = \infty$ (infinite gain amplifier) and $\alpha(p) = -p^{-1}$.] As in the algebraic case, everything depends on the characteristic roots of the matrix

$$
A + B\alpha + C\alpha^2 + D\alpha^3.
$$

In the ideal case, the characteristic frequencies are the roots of the determinantal equation

$$
\det |A - Bp^{-1} - Cp^{-2} - Dp^{-3}| = 0.
$$
Call them $p_k$. The characteristic roots of the matrix (3) will differ from the $p_k$ by small amounts, $\delta p_k$ which the author computes later (using the first few terms of a Taylor series expansion). However this matrix (3) may have extraneous roots, $q_\nu, \nu = 1, 2, \ldots$

Under the assumptions $|G(q)| < |G(0)|$ and $|\alpha(p)|$ is small ($\sim 10^{-2}$) it can be shown that the $q_\nu$ depend only on the matrix $A$ and not on $B$. The author states that if this is not the case his results are not valid and that a general discussion appears difficult and is outside the scope of the present study.

Making certain (practical) assumptions regarding the physical system, for example, assuming $G(p) = -G_0/(1 + pT_0)$ where $G_0 = |G(0)|$ and also assuming (for the present) that $A$ is symmetric, the author obtains the results that two sufficient conditions for the correct functioning of the differential analyzer are (a) $|p_k| < 2\pi\Delta f_0$, (b) $|q_m| > 2\pi\Delta f_0$ where $|q_m| = \min |q_\nu|$ and $\Delta f_0$ is the band-width of the amplifier $(2\pi\Delta f_0T_0 = 1)$. Using the Redheffer formula again, it is also shown that

$$ q_m \geq -[1 + G_0/(n + 1)||A||(n - 1)/T]^{n-1}T_0^{-1} $$

where $||A|| = |\det A|$ and $T$ is the trace of $A$.

The author remarks that (a) can always be satisfied by an appropriate change of scale of the independent variable in the original system of equations. Since $G_0$ is always large ($\sim 10^4$) in a well designed amplifier, he concludes that the critical parameter, from the point of view of stability and precision, is $||A||/T^{n-1}$. Errors in the matrices $A$, $B$, $C$, $D$ are not considered (neither steady state nor random).

Certain results are obtained if the condition that the matrix $A$ be symmetric is relaxed.

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A Coradi intergraph is modified by replacing the pen by a stand holding a slide rule glass slide. The method of operation is carefully described in this note. An accuracy of two percent in the permissible range of slopes was obtained.

F. J. M.


The device evaluates

$$ \int_a^b f_1(t)f_2(t)dt $$
by means of two ball cage integrators, $f_1$ and $f_2$ are entered by manual
followers from graphs. $f_2$ is utilized as the displacement of the cage of the
first integrator whose disk is driven at a constant rate. The output of the
first integrator drives the disk of the second and the cage displacement of
the second integrator is proportional to $f_1(t)$. The output of the second
integrator is registered on a counter and is the required answer when the
interval of integration has been covered. The accuracy of the device depends
on the slope of the functions $f$.

F. J. M.

thesizing two-dimensional Fourier series in the determination of crystal

The stators of a pair of selsyns are connected so that a voltage across
the rotor of the first induces a voltage in the rotor of the second proportional
to the cosine of the sum of the angles of rotation of each rotor from a refer-
ence position. Thus if the first is rotated an amount $nx$ and the second an
amount $my$ a term $a_{nm} \cos (nx + my)$ can be represented. These voltages can
be immediately added to produce $F(x, y) = \sum a_{nm} \cos (nx + my)$. The co-
ordinates $x$ and $y$ are entered as the rotation of two shafts and the rotations
$nx$ and $my$ are taken from these by a suitable arrangement of gears. In the
present device $n$ and $m$ each have a range of 8 but it is planned to extend
this range to 16.

F. J. M.

22. F. C. Snowden & H. T. Page, "An electronic circuit which extracts

This circuit is based on the use of a 6SK7-GT as an inverted triode. The
grid current is then proportional to the antilogarithm of the applied plate
voltage for a range of 15 volts and for a range of grid current from .33 to
1 ma.

F. J. M.

23. Robert R. Reid & Du Ray E. Stromback, "Mechanical computing
mechanisms," Product Engineering, v. 20, 1949, August no. 8, p. 131–135,

In the first article of this series of four, specifications for the input and
output of mechanical computers are described. Three types of errors are
described: Class A errors due to inaccuracies in components. Class B errors
due to mathematical incompleteness in the setup and Class C or personal
errors. It seems to be desirable to accept certain Class B errors which are
known in order to minimize Class A and Class C errors. Designs questions
in relation to scaling are discussed. In the second article, cams, resolvers,
trigometric function generators and differentials are discussed relative to
available scales and error characteristics, in the above classification. The
third article deals with multiplication, division, integration differentiation
and link mechanisms. The last article describes a number of applications.

F. J. M.

A proposed device is described for presenting

\[ \int_a^b K(x, t)f(t)dt \]

in the form of a graph of a function of x on a cathode ray tube. \( K(x, t) \) is to be obtained by scanning a photographic plate whose opacity corresponds to the value of \( K \) at the point \( x, t \). The multiplication of \( K \) and \( f \) and the value of \( f(t) \) itself are to be obtained by using the components due to Macnee.\(^1\)

The author shows how the persistence of the image on the screen of the cathode ray tube can be utilized to construct an iterate of the transform and describes also how non-linear transforms in the form

\[ \int_a^b K(x, t)k(t, f(t))dt \]

can be obtained.

The paper also lists a variety of applications of such a device. The case in which \( K(x, t) = \cos xt \) yields the impulse response of an electrical network. Such a device would readily yield the coefficients of the orthogonal expansion of a function, the Hilbert transform and the convolution integral.

Another set of applications is concerned with the solution of the integral equations. Special cases treated include simultaneous linear algebraic equations, the Volterra equation, the use of Liouville-Neumann series, Fredholm’s integral equation of the second kind, the Dirichlet problem for a plane potential and a non-linear problem for a pendulum.

F. J. M.


**NOTES**

124. **Leslie John Comrie** (1893–1950).—This great table maker and pioneer in the art of mechanical computation was born in New Zealand in 1893. He received his early training and a M.A. degree at the University of New Zealand. He saw active service during the first world war with the New Zealand Expeditionary Forces and after the armistice went to University College, London and Cambridge University, where he received his Ph.D. in Astronomy in 1923. After 3 years teaching in the United States at Swarthmore College and Northwestern University he returned to England and the Royal Greenwich Observatory as Deputy Superintendent of H.M. Nautical Almanac Office. He became Superintendent in 1930 and held that post until 1936. Here he introduced modern computing methods which did much to increase the efficiency and productivity of the office. It is this work which brought out his genius for the organization and keen analysis of computing and table preparation for which he later became so famous. He also served brilliantly as secretary of the BAASMT during 1929–36 and was much concerned with the production of the committee’s first six volumes. In 1937 he left the Observatory to devote his entire energy to the development of the Scientific Computing Service, the first enterprise of its kind. The history of this organization is one of lasting achievement and pioneering