\[
\begin{align*}
    f_1(3/x) & \approx .79788456 + .00000156 (3/x) + .01659667 (3/x)^2 \\
    & + .0017105 (3/x)^3 - .00249511 (3/x)^4 + .00113653 (3/x)^5 \\
    & - .00020033 (3/x)^6 \\
    |\epsilon|_{\max} & = 8 \times 10^{-8} \\
    \phi_1(3/x) & \approx .78539816 - .12499612 (3/x) - .00005650 (3/x)^2 \\
    & + .00637879 (3/x)^3 - .00074348 (3/x)^4 - .00079824 (3/x)^5 \\
    & - .00029166 (3/x)^6 \\
\end{align*}
\]

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170.—A SIEVE PROBLEM ON "PSEUDO-SQUARES." The following problem originated by Kraitchik,\(^1\) and extended by Lehmer\(^2\) by a special sieve, has recently been further extended by the SWAC. Let \(p\) be a prime. Let \(N_p\) be the least positive non-square integer of the form \(8x + 1\) that is a quadratic residue of all primes \(\leq p\). In this definition, zero is not counted as a quadratic residue so that \(N_p\) is not allowed to be divisible by any primes \(\leq p\). Since squares are quadratic residues of any prime, the numbers \(N_p\) behave like squares and may be called pseudo-squares. This fact makes the problem of discovering pseudo-squares not only a sifting problem but also one of rejecting squares. Thus the problem is unsuitable for a high speed sieve alone since the output would be mostly squares, each of which would have to be tested by a more elaborate arithmetic unit. The problem is therefore one for an all-purpose computer programmed for sieving.\(^3\)

Since for \(p > 3\), \(N_p\) must be of the form \(24x + 1\). One may proceed to exclude values of \(x\) using prime moduli between 5 and \(p\) inclusive. For every value of \(x\) not excluded the machine is programmed to extract the square root of \(24x + 1\). If this is a perfect square, the machine returns to the sifting program for the next value of \(x\). Fortunately the early part of the program, where the squares come thick and fast, had already been carried\(^4\) as far as \(N_{41} = 48473881\) in 1928 so that when programmed for the SWAC the routine spends most of its time sifting. Actually, for the record, the SWAC was instructed to print out every 64th square it produced. The complete table of \(N_p\) for \(p < 83\) is as follows.

\[
\begin{array}{cccc}
\hline
p & N_p & p & N_p & p & N_p \\
\hline
2 & 17 & 19 & 53881 & 47 & 9257329 \\
3 & 73 & 23 & 87481 & 53 & 22000801 \\
5 & 241 & 29 & 117049 & 59 & 48473881 \\
7 & 1009 & 31 & 515761 & 61 & 48473881 \\
11 & 2641 & 37 & 1083289 & 67 & 175244281 \\
13 & 8089 & 41 & 3206641 & 71 & 427733329 \\
17 & 18001 & 43 & 3818929 & 73 & 427733329 \\
\hline
\end{array}
\]

All \(N_p\) above are primes except for

\[
\begin{align*}
    N_{11} & = 19 \cdot 139 \\
    N_{17} & = 47 \cdot 383 \\
    N_{29} & = 67 \cdot 1747 \\
    N_{41} & = 643 \cdot 4987
\end{align*}
\]
The interest in pseudo-squares is heightened by the fact that they may be used in tests for primality, as shown by Marshall Hall. The operation of the SWAC and the reduction and checking of the output data was done by John Selfridge.

D. H. L.


171.—L. F. Richardson (1881–1953). This English mathematician made several notable contributions to numerical analysis. A brief account of his life by P. A. Sheppard appears in Nature (v. 172, 1953, p. 1127–8). His work on numerical analysis (apart from that appearing incidentally in his book!) was mainly contained in three long papers:


There were minor contributions in


An introduction to some of the material of A, B appeared in


and one to some of the material of C in


His work is highly individualistic and his language and symbolism picturesque. For instance he introduced the terms "marching" problem, for initial value problems of the form

\[ y'' = ky, \quad y(0), \quad y'(0) \text{ given}, \]

and "jury" problem for a problem of the form

\[ y^i - 3y^i + 3y'' - y = \lambda y, \quad y = y'' = y''' = 0 \text{ for } x = \pm 1. \]