interest lies in comparing statistics of such expansions with known distributions of these statistics over random numbers. For example, KHINTCHINE\(^1\) has shown that over random numbers \(x\) uniformly distributed between 0 and 1, the sum \(S_n(x)\) of the first \(n\) partial quotients of \(x\) is equivalent in the sense of Bernoulli to \(Z_n = n \log n / \log 2\).

The first result is the computation of more than 2000 partial quotients of \(2^{1/2}\). The table below shows \(S_n(2^{1/2})\) for \(n = 100(100)2000\), with \(Z_n\) given for comparison. It appears that \(S_n(2^{1/2})\) oscillates considerably in relation to \(Z_n\), being most of the time larger, up to a factor of about 2. We do not know whether this deviation is significant, since oscillations of \(S_n(x)\) of this type occur for almost all \(x\). Expansions of additional numbers, as well as more detailed statistics, will follow.

The code depends on subroutines which do the necessary algebra on polynomials whose coefficients are \(p\)-tuples of computer words, for arbitrary and variable \(p\). At present it handles cubic polynomials; a generalization to \(n\)th degree polynomials is planned. The methods and results will be reported later at greater length.

<table>
<thead>
<tr>
<th>(n)</th>
<th>(n \log n / \log 2)</th>
<th>(S_n(2^{1/2}))</th>
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<td>100</td>
<td>664.4</td>
<td>1384</td>
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<td>2000</td>
<td>21931.6</td>
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The Values of \(\Gamma (\frac{1}{2})\) and \(\Gamma (\frac{3}{2})\) and their Logarithms
Accurate to 28 Decimals

The values of \(\Gamma (\frac{1}{2}), \Gamma (\frac{3}{2}), \log \Gamma (\frac{1}{2}), \log \Gamma (\frac{3}{2})\) were computed to 28 decimals using the series

\[
\log \Gamma (2 + x) = C_1 x + C_2 x^2 - C_3 x^3 + C_4 x^4 - C_5 x^5 + \cdots + (-1)^r C_r x^r + \cdots
\]
where \(C_1 = 1 - \gamma\), \(\gamma = \lim \left(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} - \log n\right)\): Euler’s constant

\[
C_r = \frac{1}{r} \left(\frac{1}{2^r} + \frac{1}{3^r} + \frac{1}{4^r} + \cdots\right) \quad (r = 2, 3, \ldots).
\]

The values of \(S_r = \frac{1}{1^r} + \frac{1}{2^r} + \frac{1}{3^r} + \cdots\) and \(\gamma\) were taken from Stieltjes’ table.¹

Part of the calculation was done with the assistance of Mr. E. V. HANKAM on an IBM (602-A type) calculating punch. Uhler’s radix table was used for getting the antilog of \(\log \Gamma\). The values \(\Gamma(\frac{1}{2})\) and \(\Gamma(\frac{3}{2})\) were required for calculating the power series coefficients of Bessel functions of order \(\frac{1}{2}\) and of functions related to them.

The values were checked by the identity

\[
\sqrt{3} \Gamma(\frac{1}{2}) \Gamma(\frac{3}{2}) = 2\pi
\]

\[
\Gamma(\frac{1}{2}) = 2.67893 \quad 85347 \quad 07747 \quad 63365 \quad 56929 \quad 410
\]

\[
\Gamma(\frac{3}{2}) = 1.35411 \quad 79394 \quad 26400 \quad 41694 \quad 52880 \quad 282
\]

\[
\log \Gamma(\frac{1}{2}) = .98542 \quad 06469 \quad 27767 \quad 06918 \quad 71740 \quad 370
\]

\[
\log \Gamma(\frac{3}{2}) = .30315 \quad 02751 \quad 47523 \quad 56867 \quad 58628 \quad 174
\]

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**Modification of a Method for Calculating Inverse Trigonometric Functions**

The 605 programming that I gave recently¹ fails for arguments near \(2^{-4}\). The reason for this failure is that the double angle formulations used multiply round-off errors until they are intolerably large. These formulations were originally introduced to assure that \(\cos 2\theta\) depend on both \(\sin \theta\) and \(\cos \theta\). Upon closer examination it was found that it is only necessary that \(\cos 2\theta\) depend on \(\sin \theta\), hence we may use

\[
\cos 2\theta = 1 - 2 \sin^2 \theta.
\]

The use of the above formula and

\[
\sin 2\theta = 2 \sin \theta \cos \theta
\]

avoids the errors mentioned and is just as easily programmed for the 605.

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