Then application of the recurrence relation (6) shows that

\[ f(x) = b_0p_0(x) + b_1[p_1(x) + a_0p_0(x)]. \]

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2 NBS Applied Mathematics Series 9, Tables of Chebyshev Polynomials \( S_n(x) \) and \( C_n(x) \). U. S. Govt. Printing Office, Washington, 1952.

**Conjectures Concerning the Mersenne Numbers**

Conjectures concerning the Mersenne numbers are appropriate since they were inaugurated with one. A conjecture [1] that seems likely to be false, but unlikely to be proved false, is that all numbers \( p_n \) are prime \( (n = 1, 2, 3, \ldots) \), where, for example, \( p_4 \) is

\[
\begin{align*}
2 &-1 \\
2 &-1 \\
2 &-1 \\
2 &-1
\end{align*}
\]

Recursively, \( p_1 = 3 \), \( p_{n+1} = 2^{p_n} - 1 \). The first four are 3, 7, 127 and \( 2^{127} - 1 \), all known to be prime. Any factor of \( p_6 \) is congruent to 1 modulo \( p_4 \), so \( p_6 \) certainly has no factor less than \( 2^{127} \). Similarly

\[ 2^{2^{231}} - 1 \]

is not divisible by any known prime, if \( 2^{2^{31}} - 1 \) is still the largest known prime [2]. One can try to argue about the probability that a number of the form \( 2^p - 1 \) is prime, when \( p \) is known to be prime. The probability that a whole number \( x \) is prime is about \( 1/\log x \), and is close to

\[
\frac{1}{2} e^{\gamma(1 - \frac{1}{2})(1 - \frac{1}{3})(1 - \frac{1}{5}) \cdots \left(1 - \frac{1}{q}\right)}
\]

where \( q \doteq \sqrt{x} \), so the factors \( (1 - \frac{1}{2}), (1 - \frac{1}{3}), \) etc., can be regarded as probabilities that are not far from independent. But if \( x = 2^p - 1 \), only every \( p \)th factor of (1) should be taken, and the probability apparently ought to be about the \( p \)th root of \( 1/p \cdot \log 2 \), which is approximately 1 when \( p \) is large. But this argument is also invalid, as we may see from the statistics of Mersenne primes [2]. We may see from these statistics (assuming them to contain no gaps), that, if \( m_n \) denotes the \( n \)th Mersenne prime \( (m_1 = 3) \), then

\[
2.18 \log \log m_n < n < 2.72 \log \log m_n \quad (3 \leq n \leq 17)
\]

while

\[
2.31 \log \log m_{17} = 17.
\]
It is reasonable to suppose that the number of Mersenne primes less than \( x \), when \( x \) is large, is about \( 2.3 \log \log x \). This conjecture may be shown to be equivalent to the assertion that the probability of \( 2^p - 1 \) being prime, when \( p \) is known to be prime and is large, is about \( 1.6(\log p)/p \), and is perhaps asymptotically \( (\log_2 p)/p \). If so, the probability that \( p \) is prime is negligible, and we should be able to say with confidence that our original conjecture was the exact opposite of the truth.

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REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS


Hamartiexéresis appears to be a technical term in theology, meaning the absolute removal of sin.

This paper contains in tabular form, the exponents of the prime factors (2, 3, ..., 997) in the product \((1!)(2!)(3!)...(1000!)\).

This table was used to check the first thousand entries in the table of F. J. Duarte [2]. Two errors were found:

- \( \log 99! \): the seventh quartet should read 8029 instead of 8929.
- \( \log 266! \): the eighth quartet should read 1897 instead of 1987.

Later calculations indicate no (non-cancelling) errors in the range from \( n = 1001 \) to \( n = 1200 \).

J. T.

1 See also Nat. Acad. Sei., Proc., v. 41, 1955, p. 183, for errata.
2 F. J. Duarte, Nouvelles tables de \( \log n! \) à 33 décimales, depuis \( n = 1 \) jusqu’à \( n = 3000 \). Geneva and Paris, 1927.


This book contains rational approximations of the following functions with approximate precision as indicated (there are several approximations to each function and the approximate precision of each is shown):

\[
\log_{10} x, \quad 10^{-4} \leq x \leq 10^4, \quad 3D, \quad 5D, \quad 6D, \quad 7D; \quad \varphi(x) = (1 - e^{-x})/x, \quad 0 \leq x < \infty, \quad 3D, \quad 4D, \quad 5D; \quad \arctan x, \quad -1 \leq x \leq 1, \quad 3D, \quad 4D, \quad 5D, \quad 6D, \quad 7D, \quad 8D; \quad \sin \frac{1}{4} \pi x, \quad -1 \leq x \leq 1, \quad 4S, \quad 6S, \quad 8S; \quad 10^x, \quad 0 \leq x \leq 1, \quad 4S, \quad 6S, \quad 7S, \quad 9S; \quad W(x) = e^{-x}/(1 + e^{-x}), \quad -\infty < x < \infty, \quad 3D, \quad 4D, \quad 5D; \quad E^1(x) = e^{-x/4} / \sqrt{2\pi}, \quad -\infty < x < \infty, \quad 3D, \quad 3D, \quad 4D; \quad K(n) = (n - 2n^2 - 2n^3) \ln (1 + 2/n) + (2n + 18n^2 + 16n^3 + 4n^4)(2 + m)^{-2}, \quad 0 \leq n < \infty, \quad 3D; \quad \Gamma(1 + x), \quad 0 \leq x \leq 1, \quad 5D, \quad 5D, \quad 6D, \quad 7D; \quad \Psi(x) = (\pi/2 - \arcsin x)(1 - x)^{-1}, \quad 0 \leq x \leq 1, \quad 4D, \quad 5D, \quad 6D, \quad 7D, \quad 8D; \quad \log_2 x, \quad 2^{-1} \leq x \leq 2^4,
\]

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