Note on "A Method for Computing Certain Inverse Functions"

The method for computing certain inverse functions, one binary digit at a time, which was described by D. R. Morrison in a recent issue of *MTAC* [1] has been used in this laboratory. In particular, a subroutine for computing an inverse cosine, $x = \arccos y$, based on the method was given in [2] (part III, subroutine T4, p. 152–153). It was, however, pointed out by van Wijngaarden [3] that the method gives poor accuracy for certain values of the argument, namely, those for which one or more of the functions $\cos x$, $\cos 2x$, $\cos 4x \cdots \cos 2^k x$ are near unity. When x is near zero the error is, perhaps, of little importance since the equation $x = \arccos y$ does not then determine x with any great precision, but this is not the case when x is near $\pi/2$, $\pi/4$, etc. In general abnormally large errors may occur if, in Morrison's notation,

$$dy_n/dx = 0 \text{ for any } n \leq N,$$

since δ will then be of order $\sqrt{\epsilon}$ if $d^2y_n/dx^2 \neq 0$, and of larger order otherwise. The number of correct figures in the value obtained for an inverse cosine or similar function may, as a result, be only about half as many as Morrison suggests.

Although, in cases in which the above objection does not apply, digit by digit methods of computing functions may sometimes be useful in a digital computer on account of their simplicity, they are, in general, slow in operation, and unless storage capacity is very restricted other methods are, in our experience, generally to be preferred.

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1. D. R. Morrison, "A method for computing certain inverse functions," *MTAC*, v. 10, 1956, p. 202–208.
2. M. V. Wilkes, D. J. Wheeler, & S. Gill, *The Preparation of Programs for an Electronic Digital Computer*, Addison-Wesley Press, Cambridge, Mass., 1951.
3. A. van Wijngaarden, "Erreurs D'Arrondiment dans les Calculs Systématiques," Centre National de la Recherche Scientifique, *Colloques Internationaux*, v. 37, 1953, p. 285–293.

REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

75[A].—Walter Schmidt, *Der Rechner*, Technischer Verlag Herbert Cram, Berlin, 1955, xix + 200 p. DM 18.00.

This gives on page n, $n = 1(1)200$, the product mn, to one place of decimals, where $m = p + \theta$, $p = 0(1)100$, $\theta = 0.1(.1).9$; $\frac{1}{6}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{5}{6}$, $\frac{1}{4}$, $\frac{3}{4}$. The format is satisfactory and the printing tolerable. The accuracy has not been checked.

There is a rather lighthearted introduction, which gives various examples of the use of the table.

J. T.

This review was prepared by J. Todd for *Mathematical Reviews.*

76[A].—FRANZ TRIEBEL, *Rechen-Resultate.* Achte Auflage, Technischer Verlag Herbert Cram, Berlin, 1956, ii + 285 p. DM 26.00.

This gives, on page $n + 2$ for $n = 1(1)100$, the product mn to two decimal places, where $m = 1(1/4)100$. There follows the product mn for $m = 101(1)1000$, $n = 1(1)100$, each page covering five values of m; there is also a table of mn, $m = 1(1)300$, $n = \frac{1}{6}, \frac{1}{3}, \frac{2}{3}$.

There is a brief introduction showing the use of the tables. There are elaborate thumb indices. Printing is satisfactory; the accuracy has not been checked.

J. T.

This review was prepared by J. Todd for *Mathematical Reviews.*

77[B].—H. NAGLER, *Table of Square Roots of Integers*, on microfilm, 101 frames, deposited in UMT FILE.

This table gives $\sqrt{N}$ to 15D (17S) for $N = 1(1)10,000$. The table was computed on an Elliott 402 digital computer, and printed on an automatic printer from five-hole punched paper tape. The author states that the method of computation insures that the error is within one unit in the last decimal place. A random spot check by the reviewer with 18D hand computed values revealed an error of 1 in the last place in two cases out of ten, the other eight cases being correct.

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78[B, C, D, E].—V. BELEVITCH & F. STORRER, "Le calcul numérique des fonctions élémentaires dans la machine mathématique IRSIA–FNRS," Acad. r. de Belgique, *Cl. d. Sciences, Bull.*, s. 5, v. 52, 1956, p. 543–578.

This article treats in detail the problems of approximating elementary functions by polynomials, for use on a digital calculator using floating decimal arithmetic. The specific calculator the authors have reference to is the Belgian machine ISRIA–FNRS [1], which uses a floating decimal arithmetic with 15 significant digits in the mantissa, and an exponent between -50 and 50. The basic command structure allows the machine to add, subtract, and multiply; all other arithmetic operations must be programmed.

It is clear that in dealing with floating point operations, the desirable criterion for an approximating polynomial is that the *relative* error, rather than the *absolute* error, of the approximation be bounded. Two methods of deriving a polynomial with a given relative error from a polynomial with a given absolute error are discussed; the first method depends on the property that if the function $f(x)/x$ is approximated by a polynomial $p(x)$ on an interval I with an absolute error $= e$, then the approximation $xp(x)$ to the function $f(x)$ will have a *relative* error $= Me$, where $m = \sup|x/f(x)|$, for x in I. The second method utilizes the fact that if the derivative $f'(x)$ is approximated by a polynomial with a certain absolute error, then the integral of the polynomial will approximate $f(x)$ with a bounded relative error.

Several methods of obtaining the approximating polynomials are discussed, and illustrated by deriving the polynomials used in the calculation of $1/x$, $x^{-\frac{1}{2}}$, $\sin x$, $\arctan x$, 10^x, $10^x - 1$, $\log_{10} x$. Tables are given of the coefficients of the polynomials derived, to 15S.

An analysis of the rounding error in the calculation of polynomials is also made, and the question of which elementary functions should be chosen as basic is investigated. For example, for x near zero, the function $10^x - 1$ is chosen as the basic function, and 10^x is gotten from it by addition of unity; this has obvious advantages in the calculation, for example, of $\sinh x$ for x near zero.

The basic law of floating point coding is "avoid subtracting two nearly equal numbers," and the authors have shown ingenuity in complying with this law while calculating the elementary functions. The ideas contained in this article should prove valuable to everyone who is coding in floating decimal (or floating binary).

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1. OFFICE OF NAVAL RESEARCH, Department of the Navy, *A Survey of Automatic Digital Computers*, Washington, D. C., 1953, p. 53.

79[D].—L. S. KHRENOV, *Pyatiznačnye tablicy trigonometričeskikh funkcii s argumenton, vyrazennym v čsasovoi mere* (*Five-place tables of the trigonometric functions with argument expressed in hourly measure*). Second edition. Gosudarstv. Izdat. Tehn.-Teor. Lit., Moscow, 1954, 172 p. Price 7.70 rubles.

The main table gives values of the six trigonometrical functions at interval of 4 seconds of time (i.e., 1 minute of arc), usually to 5S, in the range 0 to 3 hours. (Whenever the leading figure is unity, five further digits are given.) First differences are usually given, with their proportional parts alongside the tables. An auxiliary table, new in this second edition, gives $\cot x$, $\operatorname{cosec} x$ to 5S for $x = 0(.1^s)8^m(1^s)40^m$. There is a table of $\sin^2 (\frac{1}{2}x)$ for $x = 0(4^s)12^h$. Entries with terminal 5 are marked with $\pm$ to indicate in what direction rounding should be made. There is a collection of conversion tables, constants and formulae from plane and spherical trigonometry and there are worked examples showing the direct and inverse use of the tables. The tables are clearly printed.

There are no references to sources, nor is there a description of the construction. It is stated that the entries are correct to half a unit in the last place. The tables should be convenient for those who require something between the two volumes issued by L. J. Comrie [1] which gives 4+ decimals at 10^s interval, and the British Nautical Almanac Office [2] which gives 7D at 1 interval.

J. T.

This review was prepared by J. Todd for *Mathematical Reviews*.

1. L. M. MILNE-THOMSON & L. J. COMRIE, *Standard Four-Figure Mathematical Tables*, Edition B, MacMillan & Co., Ltd., London, 1931; Edition A, MacMillan & Co., Ltd., London, 1944.
2. H. M. NAUTICAL ALMANAC OFFICE, *Seven-Figure Trigonometrical Tables for Every Second of Time*, H. M. Stationery Office, London, 1939.

80[E, H, P].—JØRGEN RYBNER, *Nomogrammer over komplekse hyperbolske funktioner* (*Nomograms of Complex Hyperbolic Functions*), Jul. Gjellerups Forlag, Copenhagen, 1955, 39 + 60 p. of illustrations, 30 cm. Price Dan. Kr. 44.00.

This is a second edition of this useful book. (The first edition was reviewed by K. G. Van Wynen, RMT **526**, *MTAC*, v. 3, 1948, p. 174–175.) The unhandy binding of the first edition has been improved, errors have been removed, notation has been changed (complex arguments are now $A + jB$ instead of $b + ja$), nomograms have been added extending the range of the argument for hyperbolic tangent, and nomograms for interaction loss and interaction phase shift have been added.

In addition to discussion and formulas, alignment charts are included for $\sinh (A + iB)$ and $\cosh (A + iB)$, $0 \leq A \leq 4$, for $\tanh (A + iB)$, $0 \leq A \leq 3$ and for various incidental functions as follows:

$$x + iy = r(\cos \theta + i \sin \theta);$$

$$R(\cos \alpha + i \sin \alpha) = 1 + r(\cos \theta + i \sin \theta), \quad r \leq 1;$$

$$A_r = \ln\left(\frac{1 + Z}{2\sqrt{Z}}\right), \quad 1 \leq |Z| \leq 10;$$

$$B_r = \arg\left(\frac{1 + Z}{2\sqrt{Z}}\right);$$

$$A_x = \tfrac{1}{2} \ln (1 - 2e^{-2A} \cos 2B + e^{-4A}), \quad 0 \leq A \leq 1, \quad 0 \leq B \leq 90°;$$

$$B_x = \arctan \frac{\sin 2B}{e^{2A} - \cos 2B}, \quad A \geq 0;$$

$$f = \frac{1}{2\pi\sqrt{LC}}, \quad 10^{-9} \leq L \leq 10^{5} \text{ henry}, 10^{-12} \leq C \leq 10 \text{ farad}$$

$$K = \sqrt{\frac{L}{C}}, \quad 10^{-9} \leq L \leq 10^{5}, \quad 10^{-12} \leq C \leq 10.$$

The tables and their units are chosen for convenience in electrical filter design and other similar problems. Accuracy is adequate for most such applications.

The author notes the following errata:

"On the four new charts for tgh $(A + jB) = r/\theta$ covering the ranges A = 2,00–2,25; 2,25–2,50; 2,50–2,75; 2,75–3,00 nepers, the $B_1 \cdots B_4$ scales are erroneously marked $a_1 \cdots a_4$.

In the list of contents, page 6, the formula 7 for the interaction loss should read:

$$A_x = \tfrac{1}{2} \ln (1 - 2e^{-2A} \cos 2B + e^{-4A}).$$

In the Danish preface, page 9, line 7 from the bottom, the word monogrammerne should read nomogrammerne."

C. B. T.

81[E].—HOMER S. POWLEY, *Table of log cosh x*. One typewritten sheet, 28 cm., deposited in the UMT FILE.

This table lists log cosh x, 3D, for $x = 10(.5)20(5)40$.

Because of the high value of x, the table is essentially $x \log e - \log 2$.

C. B. T.

82[F, Z].—CARL-ERIK FRÖBERG, *Hexadecimal Conversion Tables*, C. W. K. Gleerup, Lund, Sweden, 1957, 26 p., 22 cm. Price 3 kr.

A revised version of the conversion table so widely used around computers with hexadecimal input. (See RMT **1042**, *MTAC*, v. 7, 1953, p. 21.) In this edition the binary point follows the first binary digit of fractions; the first digit is used as a sign. This is clearly important in use of the table; for some machines the numbers must be doubled and the sign inserted properly.

Using A, B, C, D, E, and F for the hexadecimal digits ten through fifteen respectively the table lists:

1. Decimal and hexadecimal integers over the decimal ranges $1(1)1024(16)4096$ and $10^k(10^k)10^{k+1}$, $k = 2(1)12$;
2. Hexadecimal equivalent of decimal fractions x, $x = 10^{-k}(10^{-k})10^{-k+2}$, $k = 2(2)16$;
3. Conversion of $n \cdot 10^k$, to normalized hexadecimal numbers, $n = 1(2)9$, $k = -12(1)12$ and conversion of 10^k and 10^{-k} for $k = 13(1)25$;
4. Hexadecimal form of constants frequently met;
5. Decimal form of hexadecimal fractions $x = 16^{-k}(16^{-k})16^{-k+1}$ (subject to the sign convention mentioned earlier) $k = 1(1)10$.

For similar octal-decimal tables, see van Wijngaarden [1], Karst [2], and Causey [3]. None seems to have been widely distributed.

The following erratum was communicated to the author by S. Arkéus:

On page 23, *for* $^2\log 10$ = 6A4D3 C25E8 1209D 82 *read* 6A4D3 C25E6 8DC58 82.

C. B. T.

1. A. VAN WIJNGAARDEN, "Decimal-binary conversion," Report R-130 of the Computation Department, Mathematical Center at Amsterdam (see following review **83**).

2. EDGAR KARST, *Tables for converting 4 digit decimal fractions to periodic octal fractions*. [See Review 6, *MTAC*, v. 10, 1956, p. 37.]

3. ROBERT L. CAUSEY, *Decimal to octal and octal to decimal conversion tables*. [See Review 65, *MTAC*, v. 10, 1956, p. 227.]

83[F, Z].—A. VAN WIJNGAARDEN, *Decimal-Binary Conversion and Deconversion*, Report R-130 of the Computation Department of the Mathematical Centre, Amsterdam, 1951, 41 p., mimeographed, 33 cm.

This useful table, hard to read, gives decimal equivalents of octal numbers 0(10)303230 (all digits OCTAL!). In addition it gives decimal values of 2^n, $n = 1(1)50$, exact, decimal values of 2^{-n}, $n = 1(1)50$, 20D, octal values of 10^n, $n = 1(1)18$, exact, and octal values of 10^{-n}, $n = 1(1)18$, to twenty octal digits.

References to similar tables and their usage may be found in Review **82**, above.

C. B. T.

84[F, X].—P. DAVIS & P. RABINOWITZ, "Abscissas and weights for Gaussian Quadratures of High Order," NBS *Jn. of Research*, v. 56, 1956.

This paper contains 20D values of weights and abscissas for Gaussian quadrature rules with $n = 2, 4, 8, 16, 20, 24, 32, 40$, and 48 points. Corresponding values are available from the authors for the cases $n = 64, 80$, and 96.

The weights, a_{kn}, and abscissas, x_{kn}, enter the approximate formula

$$\int_{-1}^{1} f(x)dx \approx \sum_{k=1}^{n} a_{kn} f(x_{kn}).$$

These numbers, which have been adequately checked by the National Bureau of Standards, should prove useful in cases where: (a) the integrand $f(x)$ can feasibly be computed for arbitrary x and (b) a high degree of accuracy (result is exact if $f(x)$ is a polynomial of degree $\leq 2n - 1$) is needed.

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85[I, X].—K. A. KARPOV, *Tablit͡sy Koėffit͡sientov interpoliat͡sionnoĭ Formuly Lagranzha* (*Tables of Lagrangian Interpolation Coefficients*), Akad. Nauk SSSR, Moscow, 1954, 79 p., 26 cm. Price (including [3]) 61 rubles.

These tables contain for four-point Lagrangian interpolation the coefficients $A_i(t)$, $i = -1(1)2$, $t = -1(.001)2$ and for five-point interpolation the coefficients $A_i(t)$, $i = -2(1)2$, $t = -2(.001)2$, 6D. It was issued as a supplement to [3].

Several entries were checked against the Mathematical Tables Project's more extensive tables [1], and no discrepancies were found. The printing of the present volume is clear and easy to read, and the small size of the volume makes the tables much handier than [1] for the many times when four- or five-point interpolation with these increments suffices.

The numbers listed seem to have been rounded individually rather than as a group, so that $\Sigma_i A_i(t)$ may differ from 1 in the last digit. This happens for $t = 0.001$ in the four-point coefficients, for example, where the entries are $A_{-1} = -0.00033\ 3$, $A_0 = 0.99949\ 9$, $A_1 = 0.00100\ 0$, and $A_2 = -0.00016\ 7$. For a slowly varying function tabulated to many places subtraction of a constant (usually common leading digits) may be required to reduce the absolute value of the tabulated entries in order not to introduce a rounding error; restitution of the subtracted portion must follow the interpolation. This process is not always convenient, and the reviewer would prefer the forced rounding used in [1] to assure that $\Sigma_i A_i(t) = 1$. It would seem to be reasonable to mark each digit which should be rounded up in a further truncation of the coefficients so that this unit sum could be maintained; thus if only 3D values are needed the user would round up an overscored third digit and leave others as printed in the table. The additional cost of printing might well be justified in ease of using the tables.

An estimate of the continuing need for Lagrangian interpolation is made in the *MTAC* review of [1]. More extensive tables are available in [1] (and this is noted in the present work), and tables for sexagesimal arguments are available in NBS AMS No. 35, [2].

C. B. T.

1. NYMTP, A. N. LOWAN, technical director, *Tables of Lagrangian Interpolation Coefficients*, Columbia University Press, New York, 1944. [RMT 162, *MTAC*, v. 1, p. 314–315.]
2. NBS Applied Mathematics Series, No. 35, *Tables of Lagrangian Coefficients for Sexagesimal Interpolation*, U. S. Govt. Printing Office, Washington, D. C., 1954. [Rev. **54**, *MTAC*, v. 11, p. 108.]
3. K. A. KARPOV, *Tablitsy Funktsii* $w(s) = e^{-s^2} \int_0^s e^{x^2} dx$ *v Kompleksnoi oblasti*, Akad. Nauk SSSR, Moscow, 1954.

86[K].—BELL AIRCRAFT CORPORATION, *Table of Circular Normal Probabilities*, Operations Analysis Group, Dynamics Section, Report No. 02-949-106, 1956, iv + 305 p., 22 × 27 cm. (oblong). A limited number of copies are available by writing to the Research Division, Bell Aircraft Corp., Buffalo 5, New York. One copy deposited in the UMT FILE.

Circular normal probability integrals

$$P(T, \sigma) = \frac{1}{2\pi\sigma^2} \int\int_{x^2+y^2 \leq R^2} e^{-\frac{(x-a)^2+(y-b)^2}{2\sigma^2}} dydx$$

for $R/\sigma = 0(.01)4.59$ and $\sqrt{a^2 + b^2}/\sigma = D/\sigma = 0(.01)3$, 5D.

These tables were calculated on an IBM model 650 computer and checked against other available tables with perfect agreement over regions of overlap. Two thousand values chosen randomly were checked and first and second differences were examined.

The calculation is described and a most elementary illustrative example given.

Printing by a photographic offset process is adequate.

No differences or aids to interpolation are given.

C. B. T.

87[L].—T. PEARCEY, *Table of the Fresnel Integral to Six Decimal Places*, Cambridge, England, at the University Press, 1957, 63 p., 24 cm. Price $2.50.

This table was compiled for, and printed by, the Commonwealth Scientific and Industrial Research Organization, which published an Australian edition at Melbourne in 1956. The functions tabulated are

$$C = \frac{1}{2}\int_0^x J_{-\frac{1}{2}}(t)dt = \frac{1}{\sqrt{(2\pi)}}\int_0^x \frac{\cos t}{\sqrt{t}} dt$$

$$S = \frac{1}{2}\int_0^x J_{\frac{1}{2}}(t)dt = \frac{1}{\sqrt{(2\pi)}}\int_0^x \frac{\sin t}{\sqrt{t}} dt.$$

Tables with argument u, where $x = \frac{1}{2}\pi u^2$, are also common, but the argument of the present table is x, though a few expansions in terms of u are displayed in the explanatory text. The values of C and S are listed to 7D for $x = 0(.01)1$ and to 6D for $x = 1(.01)50$, with δ^2 throughout.

The values were mostly computed by subtabulation of a 7D table for

$x = 0(.02)1$ given by Watson [1], and a 6D table for $x = 1(.5)50$ due to Lommel, and reproduced by Watson; Lommel's table is unreliable in the sixth decimal, but was differenced and corrected by recomputation where necessary. The compiler states that "errors in rare cases may amount to 1 unit in the last figure."

The reviewer compared the values up to $x = 2$ with 8D values (with possible error up to 2 final units) computed by Corrington [2]. Apart from a number of cases (almost all in the 7D portion) in which the two tables differ by between 0.5 and 0.9 units of the last place printed in the Australian tables, the comparison revealed the following errors, all in S:

x	*For*	*Read*
0.05	.00298 12	.00297 30
0.07	.00492 43	.00492 40
1.97	.55507 4	.55507 3

These three corrections were verified by the reviewer by independent computation; the error in the third case is about 1.3 final units. The compiler states that some values for small arguments were specially computed; it appears that this part of the work was not well done. On the other hand, it is surprising that a 6D table produced by subtabulating a 6D table should contain so *few* rounding errors in the portion tested; perhaps the compiler knows more about the seventh decimal than he has claimed. The second differences appear to be modified when necessary, though this fact is not stated. The table does not pretend to be definitive, but will nevertheless be found very useful.

A. F.

1. G. N. WATSON, *A Treatise on the Theory of Bessel Functions*, second edition, Cambridge University Press, England, 1944.
2. M. S. CORRINGTON, *Tables of Fresnel Integrals, Modified Fresnel Integrals, the Probability Integral, and Dawson's Integral*, Radio Corporation of America, R.C.A. Victor Division. [*MTAC*, v. 7, UMT 166, 1953, p. 189.]

88[L].—M. FERENTZ & C. HARRISON, "A tabulation of the function $\frac{1}{x}\int_0^x J_0(y)dy$, $x = 0(.01)31$; 4D," Argonne National Laboratory, Lemont, Illinois, 8 ozalid sheets, 28 cm. Three copies deposited in the UMT FILE.

This is a table of the function $f(x)$ given in the title for $x = 0(.01)31$, to 4D. The table was computed from Taylor's expansion. Values of $\frac{x}{2} f(x)$ are given for $x = 0(.02)1$ to 7D by G. N. Watson [1], p. 752. For $x = 0(.02)16$, 7D values of $f(x)$ can be computed from tables given by Watson, l.c., p. 666 et seq., using the formula

$$f(x) = J_0(x) + \frac{\pi}{2}(J_1(x)\mathbf{H}_0(x) - J_0(x)\mathbf{H}_1(x)).$$

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1. G. N. WATSON, *A Treatise on the Theory of Bessel Functions*, 2nd ed., Cambridge University Press, Cambridge, 1944.

89[P].—R. E. D. Bishop & D. C. Johnson, *Vibration Analysis Tables*, Cambridge University Press, New York, 1956, viii + 59 p., 28 cm. Price $2.00.

In writing their book, *Mechanics of Vibration*, Cambridge University Press, the authors collected works from various previous engineering publications and managed, in part, the computation of formulae and numerical tables intended to help the analysis (and synthesis) of conservative mechanical systems. These aids, isolated from the main volume, have an independent value for a professional analyst; accordingly they have been extracted in this booklet form.

The structural elements, of which the more complicated systems are assumed to be composed, and their motions are: lateral vibration of a taut, uniform string; torsional vibration of a uniform circular shaft; longitudinal vibration of a uniform bar; flexural vibration of a uniform beam. For the three first cases, where the motion is governed by the same second order differential equation, the receptances, natural frequencies, and modes of vibration are given in terms of structural dimensions for three combinations of the simplest boundary conditions, namely, either the distortion or its longitudinal derivative vanishing at the ends. In giving the receptance formulae for the flexural vibration, the boundary conditions considered are those of a beam with clamped, pinned, sliding, or free ends in all proper combinations. The related functions, composed of products and product sums of trigonometric and hyperbolic functions are given to 5S for $x = 0(.05)11$. For the boundary condition combinations, pinned-pinned, clamped-clamped, free-free, clamped-free, clamped-pinned, and free-pinned the characteristic functions and their derivatives up to the fifth mode are given to 5S for $x/l = 0(0.02)1$, and likewise the five first roots of the associated characteristic equations.

These lucid tables form a valuable contribution to the similar and previous German and Russian collections.

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90[U].—Einar Anderson (Director), Torben Krarup, & Bjarner Svejgaard, *Geodetic Tables, International Ellipsoid*, Geodaet. Inst. Skr. (Mémoires de l'Institut Géodésique de Danemark), Ser. 3, v. 24, Copenhagen, 1956, 8 p. introduction + 184 p. tables, 28.5 cm.

Geodesists employ tables based on a standard assumed ellipsoid of revolution to extrapolate latitudes and longitudes from an astronomically determined point to new points. This procedure was originally worked out in the eighteenth century as a method of determining the figure of the earth by comparing a series of astronomical determinations with the results to be expected from some assumed shape of the earth. It was extended to the general mapping of the European countries as the original arcs of the meridian were expanded into the modern national triangulation nets.

The present tables are for the most part carried to a precision of 12 decimal places or 12 significant figures. Since the question of the requirements for such precision have in the past been discussed in *Mathematical Tables and Other Aids to Computation*, it may not be out of place to discuss the problem here.

The results of precise triangulation have a precision of the order of $0''.3$ in the angles, or six-figure accuracy. Since this accuracy may be demanded between points only a few kilometers apart, it may, under some circumstances, imply accuracies of a few millimeters. Since 1 millimeter is $0''.00004$ of latitude, the use of five decimals of seconds of arc may, under some circumstances, be required.

At the same time, the triangulations of rather large areas, such as Europe or North America, have been brought to consistency by the simultaneous adjustment of the measured angles in very large systems of equations. Hence the latitudes and longitudes within the scheme must be treated formally as though they represented measurements with a precision of 10 or 11 significant figures.

The problem comes to a head when it is necessary to transform the latitudes and longitudes into a coordinate system of the type which is used for military or cadastral surveys. There are only two ways to avoid the use of a large number of significant figures: first, to permit the precision of the computations to suffer, by rounding off. This loses the power of the coordinates to define the direction and distance between nearby points. Second, to make use of small systems, thereby risking trouble at the junctions. Neither is desirable; and hence the requirement for precise tables.

It might be thought that the tables, although formally precise, need not be theoretically correct. An outstanding example of this scheme of thought is the tables for the Lambert Nord de Guerre, prepared in 1916 for France. Of the five constants at the head of this table, no three could be made to agree; and no two agreed with the table itself. This was not serious until, in 1943, it became necessary to extend the tables. One Allied agency extended them by differencing the tables and extrapolating; another solved from the tables for the constants of the spheroid which would produce the given tables; while a third managed to guess the approximation which the originators of the table had made. Any one of these tables would have done the job, but the discrepancies between them meant that they could not be used together. Ultimately one was adopted.

The present, very precise tables are based on the International Ellipsoid, adopted at Madrid in 1924, with the dimensions:

a (equatorial radius)	6,378,388 meters
f (flattening)	1/297.

They replace logarithmic tables prepared at that time. They include:

$W = \sqrt{1 - e^2 \sin^2 \phi}$, $\phi = 0°(1')90°$, 12D,
$10^7/N = 10^7 W/a$, in meters^{-1}; $\phi = 0°(1')90°$, 12S,
$10^7/M = 10^7 w^3/a(1 - e^2)$ in meters^{-1}; $\phi = 0°(1')90°$, 12S,
$\gamma = g_e\left[W + \dfrac{\omega^2 aC \sin^2 \phi}{g_e W}\right]$ in milligals, $\phi = 0°(1')90°$, 8S,
$\phi - \phi^*$, defined by $\tan(\phi^*/2 + \pi/4) = \tan(\phi/2 + \pi/4)\left(\dfrac{1 - e \sin \phi}{1 + e \sin \phi}\right)^{e/2}$
and expressed in seconds of arc; $\phi = 0°(1')90°$, 6D.
$\beta - \phi$, defined by $\tan \beta = b/a \tan \phi$; $b = a\sqrt{1 - e^2}$
and expressed in seconds of arc; $\phi = 0°(1')90°$, 6D.
$\rho''/2MN$, where $\rho'' = 648{,}000/\pi$, $\phi = 0°(1')90°$, 6S.

The quantities M and N are the radii of curvature of the ellipsoid along the meridian and parallel to it. They are formed from the auxiliary quantity W. The terrestrial gravity γ, is here defined by the International gravity formula, slightly modified to make it rigorously consistent with the International ellipsoid. Note that the geodesists include centrifugal force in gravity, which they distinguish from gravitation, which is taken as not including centrifugal force.

The Gaussian latitude, ϕ^*, is identical with the quantity called the isometric latitude in the tables of the U. S. Coast and Geodetic Survey and the U. S. Lake Survey. The term isometric latitude is used by the Europeans for $gd^{-1}\phi^*$, the antigudermannian of the Gaussian latitude. The antigudermannian of ϕ^* is proportional to the north distance corresponding to the given latitude on a Mercator map. For a sphere, it reduces to the log tangent of the semi-colatitude. The Gaussian latitude is fundamental for the calculation of conformal projections (especially for coordinate computations).

The reduced latitude, β, is also called the parametric latitude. The meridional arc can be expressed as an elliptic integral of the second kind in terms of the parametric latitude.

The tables have been computed from Fourier series by an IBM 602A, and printed directly from an IBM 416 tabulation. The figures are clear and readable.

A cursory check of the table of $\phi - \phi^*$ against the tables of latitude functions for the International Ellipsoid from the Lake Survey [1] indicated agreement to the 4th place of decimals, the limit of the Lake Survey tables.

J. A. O'Keefe

Army Map Service
Washington, D. C.

1. War Department, Corps of Engineers, U. S. Lake Survey, *Latitude Transformation, Hayford Spheroid, Geodetic Latitude to Isometric Latitude and Isometric Latitude to Geodetic Latitude*, New York Office, Military Grid Unit, 1944.

91[V].—Speer Products Company, The SPEER BALLISTICS CALCULATOR, Speer Products Company, Lewiston, Idaho, 23 × 10 cm. Price $1.00.

This little instrument is a slide rule for the computation of drop and remaining velocity of small arms bullets in terms of muzzle velocity, range, and ballistic coefficient. The scales were prepared on the basis of Ingall's tables, which were in turn based upon the Siacci approximation to flat trajectories and the Gâvre drag function. As an example, the range scale (versus drop) goes from 50 to 1000 yards, and may be read within about ten percent. Ballistic coefficients are supplied for each of the manufacturer's bullets.

J. W. Green

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92[V].—Homer S. Powley, *Extension of Ingalls' Table IIA*. One typewritten sheet, 28 cm., deposited in the UMT File.

For $Z = 20000(1000)45000$ and 50000, the functions $A\,V^2/700^2$, $\log V/u$ and N are listed to 2D, 4D, and 2D respectively. For values of Z divisible by 5000 $A\,V^2/700^2$ is listed to 4D and N to 3D.

Ingall's Tables are described by Bliss [1]. According to the author the present tables are intended to extend the tables for solution of problems relating to some low power weapons.

C. B. T.

1. G. A. Bliss, *Mathematics for Exterior Ballistics*, John Wiley and Sons, New York, 1944, p. 33–35.

93[W, X].—S. Vajda, *The Theory of Games and Linear Programming*, John Wiley & Sons, Inc., New York, 1956, 106 p., 17 cm. Price $1.75.

This handy pocket-sized book contains an exposition of some of the most useful aspects of the theory of two-person zero-sum games and of linear programming.

Any work of this size must necessarily be incomplete. Thus the present volume contains no reports of experience with extensive computations involved with games and linear programming problems. There is no indication of the handling of non-linear situations, and no real full use of the continuous space in which the problems are invented. A final chapter devoted to Beale's "Method of Leading Variables," and a few introductory and incomplete historical remarks might profitably have been omitted in favor of other topics (the genesis of problems, for example), but on the whole there can be no real objections to the choice of material expounded.

A major effort is devoted to a simplex method, which is certainly the most widely used method resolving games and linear programming problems. The description of the theory covered is elementary and clear. However there are no exhibited statements, such as theorems, to summarize the arguments that come to mind.

Several examples are given and solved. The number of drawings included to clarify the presentation is impressive, particularly in the early chapters which are devoted to graphical representations of the theory. This all contributes to the exposition.

In all the book is a valuable contribution to the literature in this popular field, and it contains a valuable selection of material presented clearly.

C. B. T.

94[P, W, X, Z].—*Proceedings of the Second Annual Computer Applications Symposium*, held October 24–25, 1955, sponsored by the Armour Research Foundation of the Illinois Institute of Technology, Chicago, Illinois, 1956, 108 p., 23 cm. Price $3.00.

This is a collection of reports of talks delivered at the symposium, together with the discussion which followed each paper. For all practical purposes, the word "digital" could be in the title of the symposium.

The program was devoted to seven papers having to do with "Computers for Business and Management," and seven concerned with "Computers for Engineering and Research." Twelve papers and one abstract appear in the *Proceedings*, as follows:

Computers for Business and Management: "The use of digital computers in industry," by R. F. Clippinger, "A dollar and cents approach to electronics," by

John L. Marley, "An application of computers to general bookkeeping," by W. F. Otterstrom, "User experiences and applications of the ERA 1103," by George E. Clark, "Automobile selective underwriting and automatic rating on the IBM 650," by C. A. Marquardt, "Cutting costs with linear programming," by Jacob E. Bearman, "Probability forecasts in management decisions (abstract)," by Stanley Reiter.

Computers for Engineering and Research: "Use of the IBM 650 in scientific computations," by A. W. Wymore, "Engineering applications of large scale computers," by C. B. Ludwig, "High speed computation of engine performance," by J. T. Horner, "Pyrolysis reactor design computations," by H. C. Schutt and R. H. Snow, "Aircraft flight test data processing," by T. M. Bellan, "Programming a Monte Carlo problem," by J. F. Hall and J. M. Cook.

The *Proceedings* contain some information which is very useful for those interested in the two broad fields covered.

T. H. SOUTHARD

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Los Angeles, California

95[G, H, X].—RICHARD B. SMITH, *Table of Inverses of Two Ill-Conditioned Matrices*, Westinghouse Electric Corporation, Bettis Atomic Power Division, Pittsburgh, Pennsylvania, 1957, ii + 68 p., 29 cm. Deposited in the UMT FILE.

The listings are the inverses ($n = 2, 3, \cdots, 15$) for $A_n = (a_{ij})$, $a_{1j} = 1$ for $j = 1, 2, \cdots, n$, $a_{ij} = 1/(i + j - 1)$ for $i = 2, 3, \cdots, n$ and $j = 1, 2, \cdots, n$; and $B_n = (b_{ij})$, $b_{ij} = 1/(p + i + j - 1)$ for $p = 0$ and $i, j = 1, 2, \cdots, n$. The matrix A_n is described by M. Lotkin [1] and B_n is described by Savage and Lukacs [2] and Collar [3, 4].

The calculations were carried out on an IBM 650 using a complete double precision rational number interpretive system. The listings are believed to be correct.

RICHARD B. SMITH

Westinghouse Electric Corporation
Bettis Atomic Power Division
Pittsburgh, Pennsylvania

1. MARK LOTKIN, "A set of test matrices," *MTAC*, v. 9, 1955, p. 153–161.
2. I. R. SAVAGE & E. LUKACS, "Tables of inverses of finite segments of the Hilbert matrix," NBS Applied Mathematics Series, No. 39, *Contributions to the Solution of Systems of Linear Equations and the Determination of Eigenvalues*, 1954, p. 105–108.
3. A. R. COLLAR, "On the reciprocation of certain matrices," Royal Soc. Edinburgh, *Proc.*, v. 59, 1939, p. 195–206.
4. A. R. COLLAR, "On the reciprocal of a segment of a generalized Hilbert matrix," Cambridge Phil. Soc., *Proc.*, v. 47, 1951, p. 11–17.

96[H, X].—ZYGMUNT DOWGIRD, *Krakowiany i ich zastosowanie w mechanice budowli.* (*Cracovians and their application in structural mechanics.*) Państwowe Wydawnictwo Naukowe, Warszawa, 1956, 168 p. zł. 18.

Cracovians are rectangular arrays of numbers which are added like matrices, but which are multiplied column-by-column. They were developed by T. Banachiewicz, apparently because column-by-column multiplication is easier in desk computing than row-by-column multiplication. The present work expounds the definitions, notations, and properties of cracovian theory, and their use in problems of linear algebra. The second chapter is devoted to the solution of systems of linear equations by various methods of triangular decomposition. In the third chapter are discussed simple iterative methods for solving linear systems and for computing eigenvalues. The exposition is elementary. There are problems from structural mechanics, and many numerical examples of orders up to 5 or 6. The examples are oriented towards desk computation.

The fact that cracovian multiplication is non-associative causes various strained notations, and appears to the reviewer as an overwhelming impediment to fundamental progress. Nevertheless, cracovians have a minority of enthusiastic supporters, largely but not exclusively in Poland.

G. E. FORSYTHE

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(After September, 1957, will be at Stanford University, Stanford, California.)
This review was prepared by G. E. Forsythe for *Mathematical Reviews*.

97[S, X].—SIR HAROLD JEFFREYS & BERTHA SWIRLES (LADY JEFFREYS), *Methods of Mathematical Physics*, Cambridge University Press, Great Britain, 1956, iv + 714 p., 25 cm. Price $15.00.

This is the third improved edition of this impressive textbook of mathematics as it should be applied to physics. The authors are firm in their feeling that the physicist needs a rigorous proof of all theorems used, and they present such proofs under conditions which are suitable for the physics applications they have in mind.

At the beginning of the chapter on numerical methods the authors quote Lord Kelvin, "I have no satisfaction in formulas unless I feel their numerical magnitude." For purposes of this book the authors have no interest in mathematical theorems unless they are applicable to physics (their stated standard is applicability in at least two branches), and they have no satisfaction unless the theorem is proved under adequate conditions for their application and an application illustrated.

Numerous problems are included.

Much of the work concerns numerical analysis; this varies from the classical studies of finite differences through various uses of analysis and studies of special functions. Thus the book is a valuable but not a complete textbook on many aspects of numerical analysis.

The authors have continued to improve the book through these three editions evidently seeking advice wherever it is available. Thus they have put together a sound book containing much material not easily available elsewhere.

The chapter headings follow:

Chapter 1. The Real Variable, 2. Scalars and Vectors, 3. Tensors, 4. Matrices, 5. Multiple Integrals, 6. Potential Theory, 7. Operational Methods, 8. Physical Applications of the Operational Method, 9. Numerical Methods, 10. Calculus of Variations, 11. Functions of a Complex Variable, 12. Contour Integration and Bromwich's Integral, 13. Conformal Representation, 14. Fourier's Theorem, 15. The Factorial and Related Functions, 16. Solution of Linear Differential Equations of the Second Order, 17. Asymptotic Expansions, 18. The Equations of Potential, Waves, and Heat Conduction, 19. Waves in One Dimension and Waves with Spherical Symmetry, 20. Conduction of Heat in One and Three Dimensions, 21. Bessel Functions, 22. Applications of Bessel Functions, 23. The Confluent Hypergeometric Function, 24. Legendre Functions and Associated Functions, 25. Elliptic Functions.

There is a reasonably detailed index.

C. B. T.

98[S, X].—F. B. HILDEBRAND, *Advanced Calculus for Engineers*, Prentice-Hall, Inc., New Jersey, 1956, xiii + 594 p., 22 cm. Price $7.75.

The reviewer belongs to a school of thought which holds that the simplest numerical methods (Euler's method of forward differences in ordinary differential equations, for example) may be the soundest introduction to many mathematical subjects. He would have liked to find more numerical material in this text by this author of one of our better numerical analysis texts.

The volume does contain the following tables: A table of 41 Laplace transforms, a table of $\sqrt{\frac{\pi}{2}}\, x^m J_m(x)$ and $\sqrt{\frac{\pi}{2}}\, x^m I_m(x)$ for $m = -\frac{1}{2}(1)9/2$, $\Gamma(x)$, $x = 1(.01)1.99$, 4D, and the first five zeros of $J_p(x)$ $p = 0(1)5$.

A chapter on numerical methods for solving ordinary differential equations includes Taylor's series, the Adams method, the Runge-Kutta method, and the Picard method. A chapter devoted to series solutions of differential equations introduces Bessel functions, Legendre functions, and the hypergeometric function (and the gamma is introduced in connection with Laplace transforms).

Various iterative methods for computing eigenvalues or solutions of equations are discussed, and the author does pay attention to the requirement for numerical answers which engineers frequently face. However, when treating partial differential equations the author does not take up difference equation methods of approximating solutions, which were not popular at the time of writing.

Several topics usually included in advanced calculus texts are omitted; multiple integrals and surface integrals are treated only as they arise in vector analysis.

Problems are carefully chosen.

The book more or less parallels material in the much more ambitious and difficult work on *Methods of Mathematical Physics*, by Jeffreys and Jeffreys [1]. Chapter headings are listed below.

Solutions of Linear Ordinary Differential Equations
The Laplace Transformation
Numerical Methods for Solving Ordinary Differential Equations
Series Solutions of Differential Equations
Boundary-Value Problems and Orthogonal Functions
Vector Analysis
Partial Differential Equations
Solutions of Partial Differential Equations of Mathematical Physics
Functions of a Complex Variable.

C. B. T.

1. HAROLD JEFFREYS & BERTHA SWIRLES JEFFREYS, *Methods of Mathematical Physics*, Cambridge, England, at the University Press, 1946.

99[X, P].—S. H. CRANDALL, *Engineering Analysis, A Survey of Numerical Procedures*, McGraw-Hill Book Co., Inc., New York, 1956, x + 417 p., 24 cm. Price $9.50.

This textbook has grown out of the author's continuing efforts to acquaint engineering students with numerical methods of attacking problems in engineering analysis. The author describes engineering analysis as the performance of two steps: "1. Construction of a mathematical model for a physical situation. 2. Reduction of the mathematical problem to a numerical procedure."

The present book is largely devoted to the second of these steps, but the arrangement of material conforms more nearly with the first step than with the arrangement of mathematics courses in most schools. Actually, to a mathematician, the book is more nearly a most valuable catalogue of methods (with careful references to existing literature) than a textbook, for proofs are presented only in summary form.

Three classes of problems are considered: Equilibrium problems, Eigenvalue problems, and Propagation problems. Each of these classes is considered first as a lumped-parameter problem (or a problem with a finite number of variables) and then (three chapters later) as a continuous problem.

The reviewer has long felt that use of at least the most elementary numerical methods is the soundest introduction to many courses in mathematics. Euler's method of forward differences for numerical solution of an ordinary differential equation, for example, offers the student a feeling for the properties described in the Picard existence and uniqueness theorem; calculation of difference quotients (particularly when they are stated in terms of displacement and time) seems to be a sound introduction to the derivative, and so on. Thus one important purpose which this book can serve is presenting this feeling concerning the nature of solutions of the problems treated. Just as a student who knows Euler's method can do something with any ordinary differential equation initial value problem he is likely to meet, so the student of the present text may make efficient progress on any problem he is likely to encounter from the fields studied. The study (or at least the perusal) of a book of this type would seem to provide an excellent introduction to many aspects of abstract analysis through the concrete examples furnished. Here the motivation for the non-numerical studies would be the promise that some of the problems can be solved more generally and with less effort—a situation to which many students seem highly attracted.

In any event, since recent engineering problems have more and more demanded numerical solution, the book (or its equivalent, which seems not to exist) seems necessary for any complete training in several fields of engineering, and the reviewer suggests that teachers of more abstract courses might well extract much material from the book for introductory use.

The author has chosen methods which he feels are most likely to be useful in engineering analysis, and he has expounded them carefully and in a scholarly way consistent with the level of maturity at which he aims. In approximate methods he mentions stability considerations carefully, but he does not swamp the student with all technicalities which can crop up in stability studies. In connection with relaxation he mentions overrelaxation, but stops short of the detailed studies available on the subject.

The material to be covered requires a study of methods of solution of systems of linear algebraic equations, and ordinary and partial differential equations with various kinds of initial and boundary conditions. Numerical methods (of scope indicated below) are presented in detail along with an outline of the basic mathematics involved.

The material in the book is illustrated with many worked examples and with a spectacular number of figures and much tabulated material. Problems for the student in each section vary from routine application to problems demanding some knowledge of the theory—determination of degree of convergence of discrete methods, for example.

There is an impressive number of references to basic material used in the book and to additional material which is available for more extensive study.

The book is the second which we have seen recently stressing the importance of numerical calculation in engineering analysis; Purday's book [1] covers much of the same material but with considerably less detail.

A reasonable idea of the contents is gained from the section headings, which follow.

1. *Equilibrium problems in systems with a finite number of degrees of freedom:* Particular examples—Formulation of the general problem—Mathematical properties—Extremum problems—Elimination method for linear systems—Iteration—Relaxation—Iteration combined with elimination—Procedures applicable directly to physical systems.

2. *Eigenvalue problems for systems with a finite number of degrees of freedom:* Particular examples—Matrix notation—Formulation of the general problem—Mathematical properties—An extremum principle for eigenvalues—Direct methods of solution—Iteration—Intermediate eigenvalues—n-step iteration—Relaxation methods—Upper and lower bounds for eigenvalues—Diagonalization of matrices by successive rotations.

3. *Propagation problems in systems with a finite number of degrees of freedom:* Particular examples—Formulation of the general problem—Mathematical properties—Iteration—Series methods—Trial solutions with undetermined parameters—Finite-increment techniques—Introduction to step-by-step integration procedures—Recurrence formulas with higher-order truncation error—Step-by-step integration methods for systems with several degrees of freedom.

4. *Equilibrium problems in continuous systems:* Particular examples—Formulation of the general problem—Mathematical properties—Extremum problems—Trial solutions with undetermined parameters—Finite-difference methods.

5. *Eigenvalue problems in continuous systems:* Particular examples—Formulation of the general problem—Mathematical properties—Extremum principles for eigenvalues—Iteration—Trial solutions with undetermined parameters—Finite-difference methods.

6. *Propagation problems in continuous systems:* Particular examples—Formulation of the general problem—Mathematical properties—Trial solutions with undetermined parameters—Finite-difference methods for parabolic systems—Finite-difference methods for hyperbolic systems.

One note of criticism concerns the first sentence of the preface: "The advent of high-speed automatic computing machines is making possible the solution of engineering problems of great complexity." However, references in the book to such machines or experience with machines are sparse. This valuable publication certainly did not need this implicit and unfulfilled promise of an introduction to the bright new electronic world.

C. B. T.

1. H. F. P. PURDAY, *Linear Equations in Applied Mechanics*, Interscience Publishers, Inc., New York, 1954. [Rev. 66, *MTAC*, v. 9, p. 131.]

100[X, Z].—GEORGE R. STIBITZ & JULES A. LARRIVEE, *Mathematics and Computers*, McGraw-Hill Book Co., Inc., New York, 1957, vi + 228 p., 23 cm. Price $5.00.

The eleven chapters of this book cover nearly all phases of digital computer design and use. The first three chapters cover the basic elements of computers and mathematics, such as the basic differences between the analog and digital computers and the mathematical definition of a function. These early chapters also include some of the history of mathematics and computing, and outline the types of problems arising in applied mathematics which can be and are solved by computing techniques. The fourth chapter is a discussion of the history of computers which is both interesting and informative but, unfortunately, does not go beyond the ENIAC computer to a description and discussion of the more recent history of stored program computers. The fifth chapter contains discussions of some numerical techniques including those for finding roots of polynomials, solving linear equations, and solving ordinary and partial differential equations. This chapter gives an excellent insight to the beginning student on some of the techniques, but many of the techniques presented are not practical for the modern electronic computer. Bernoulli's method for finding roots of a polynomial equation, for example, is of little importance in the modern computing world. The next three chapters discuss digital computer components, number systems, computer memories, and analog-digital converters. These chapters also outline techniques for performing arithmetic processes in the digital computer, and contain an elementary description of how the computer carries out its operations on the basis of a stored program. The analog computer is likewise treated in these chapters, and the techniques for the mechanical differential analyzer in performing integration are explained. Chapter 9 is a discussion of Monte Carlo techniques and includes an explanation of the solution of linear equations by these techniques. The discussion on sampling techniques includes brief discussions on the solution of certain partial differential equations and techniques for generating "random" sequences. Chapter 10 is a discussion on the various errors which can occur in both analog and digital computers. Chapter 11 summarizes various special-purpose applications of computers such as those seen in the automatic factory, language translation, and, in a lighter vein, computers which play games.

The central difficulty of this reviewer in evaluating this book is in deciding to whom the information is directed. The book seems to be too elementary to serve as a reference book for any computer professional, either user or designer.

On the other hand, the book does not go into sufficient detail in any area to be a successful textbook for most university courses. It could, however, serve as a textbook on an introduction to computers given to freshman or sophomore students or more advanced non-science students. It could also serve as a reference for the beginner in the field or for a non-professional.

The most serious shortcoming of the book is that the material is not sufficiently modern. Many of the numerical techniques presented are old-fashioned and fundamentals are often described in terms of old-fashioned equipment rather than modern equipment. (An analog-digital converter could be easily described in terms of halving and comparing the voltage rather than in terms of the mechanical device which transmits a shaft position to a mechanical digit position.) The authors leave the false impression that tables, especially those involving the elementary functions, are frequently stored in computer memory. Often modern words are not used; for example, the words "subroutine," "programmer," and the term "parity check bit," are not used in describing and discussing these items.

The style of the book is light and enjoyable and makes interesting reading. The bibliography is good and extensive.

WALTER F. BAUER

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Los Angeles, California

101[X, Z].—W. J. ECKERT & REBECCA JONES, *Faster, Faster*, McGraw-Hill Book Co. Inc., New York, 1955, vii + 160 p., 23 cm. Price $3.75.

Quoting from the preface, "This monograph is an attempt to explain in nontechnical language how a calculator operates, the nature of the problems it solves, and how the problems are presented to the calculator." Actually, it consists almost entirely of a description of "NORC," the Naval Ordnance Research Calculator designed and built by the International Business Machines Corporation. Probably but few experts would agree to the claim that NORC ". . . is also easiest to understand" (preface) yet the explanatory attempt must be judged very successful; it is completely devoid of engineering details (such as component types, circuit diagrams, etc.), relying instead upon simple block diagrams and schematics accompanied by descriptions of the essential properties involved. The language is indeed nontechnical, even such common descriptive contractions as "and/or gate" being avoided, although various colloquialisms native to business machinery, such as "echo-pulse," "print cycle," are explained and used. Here and there the viewpoint tends to be a bit insular; for instance the basic electronic building block turns out to be a binary pulse shaper producing 1 microsecond time delay; this is called a Dynamic Pulse circuit, written always with capitals, like a Thing. Again, no allusion is made to any except IBM equipment, nor to technical contributions by any outsiders except F. C. Williams, the Red Queen, Leibniz and Newton.

Chapter I is introductory, and begins with a general outline of the need for, and requirements of automatic *en masse* arithmetic, and sketches the typical mental, organizational, and instrumental hurdles that must be overcome. Lucid examples of basic arithmetical and scaling operations are given, as well as various notions about electronic components, timing relationships and the rôle of main functional units such as the arithmetical, memory, input-output, etc. Halfway through the chapter all such generality is jettisoned without rites and supplanted by a clear outline of NORC'S code-language, organization, structure, and philosophy; these and other features of NORC are more fully developed in all chapters but the last of the ten that follow, and in five summarizing appendices. The level of exposition is, on the whole, excellent; one may take exception—as does this reviewer—to the practice of attaching grammatical commas and periods to arithmetical examples, and to the use of the word "accuracy" instead of the word "reliability" in contexts where freedom from accidental errors is meant, but aside from these trivialities of taste, the exposition leaves little to be desired.

NORC is revealed to be quite a fast machine (70 microseconds to multiply; 50 to add or subtract; 250 to divide; 8 for memory access), with ample memory capacity (3600 words, each 66 bits) composed of 4 banks of Williams CRT storage, supplemented by 8 magnetic tape units of impressive capacity (400,000 words each) and speed (4000 words/second), plus printing (19 words/sec) and card punch/read facilities (400 words/min). Each order is an instruction plus three addresses, the latter automatically modifiable en route. The instruction list is almost lavishly flexible; all standard arithmetic operations are available with options of floating point (± 30), automatic sizing shifts, extractions for order modification, transplantation of addressees, etc. There are no less than 21 control transfer instructions, plus 5 for print control and 9 for tape control; the grand total is 98. Data entries are normally preserved to 13 decimal places, and there appears to be an efficient provision for double-precision arithmetic. Beyond doubt, NORC should be classed as an outstandingly flexible and effective computing system.

Electronically, NORC belongs in the category of "pulse recirculating" machines typified by SEAC of the National Bureau of Standards. These machines avoid the use of static memory cells of Eccles-Jordan "flip-flop" type in their arithmetical and logical units by routing binary pulses through various circulating paths consisting of time delay and pulse re-shaping components. Since identification of pulse position on this moving coordinate system is vital, a central clock is used to quantize the time coordinate and great stress is laid upon matters of timing, pulse shaping, and synchronism.

The internal language of NORC consists of words $16\frac{1}{2}$ decimal digits long, each digit being represented by a tetrad of binary digits. It is actually true that arithmetic within the machine is carried out in this mixed base, elements of an essentially binary nature (aggregates of dynamic pulse units plus gangswitches) being combined so as to preserve the local identity (i.e., within tetrads of wires, elements, etc.) of decimal characters. Words are taken from or put into the memory as 66-bit parallel transfers, whereas within the arithmetic unit the basic process involves 4-abreast shifting of successive tetrads to effect, for instance, addition that is decimally serial. Multiplication is done by a similar shift of the multiplicand stepwise through a parallel circuit called a "product generator" equivalent to a multiplication table, so that as each multiplicand digit steps up to the altar, all ten multiples of it become simultaneously available. From this array all multiples specified by the multiplier digits can thus be accumulated at each multiplicand step, and the entire product assembled during one "pass." Consequently multiplication is almost as brief as addition, though division does not fare comparably well, requiring four or five times as long.

A little bookkeeping may suggest that NORC is somewhat extravagant with regard to information capacity, one binary tetrad capable of representing 16 alternatives being used to represent only 10, etc. This glaring redundancy is the decimal man's burden and seems unavoidable; in NORC some slight byproduct utility is recovered by using "12" and "13" for magnetic tape word-end and block-end signals. Much emphasis is placed upon automatic checking throughout the discussion and NORC uses two systems, (a) modulo 4 summing of the bits in each word, affixing half a tetrad of redundancy to tag this, and (b) "casting-out-nines" checking of the arithmetic operations. Together, (a) and (b) seem to make good sense, whereas (a) alone would be quite weak since repeated doublings play a key rôle in the product generator. In the Williams memory a parity check is also provided for the sum of the bits in each of the 66 parallel positions. Altogether, these features inspire confidence, yet in a machine having some $\frac{1}{3}$ of its capacity redundant, one wonders whether it might not have been feasible to make the checking density far more severe.

The reader concerned with machine design will appreciate that NORC has been moulded to fit rigorously within a framework of precepts: decimal, speedy, ample capacity, big vocabulary, pulsed circuits, checking. Probably not everybody would choose exactly this prescription, but few would deny that it is interesting and bold, and seems to have been carried out with systematic skill and generous material resources, and with the sort of opportunistic ingenuity that is the hallmark of elegant design. The field of computer design is advancing so rapidly both in technology and in concepts that no design group can claim ultimate insight, and all may well benefit from a study of the various species evolved. NORC deserves careful study, and it is to be hoped that a careful comparative examination of its operational performance will eventually be published.

The final chapter in *Faster, Faster* is entitled, "What is there to Calculate?", and consists of a very elementary but clear discussion of calculations applied to linear systems, ballistic trajectores, planetary motion, etc. Perhaps a slight improvement could be made by distinguishing more carefully the idea of solving a mathematical problem in general from the idea of calculating a numerical solution; on page 131 following a discussion of ballistics, the statement that "A slightly more complicated problem is the three-body problem" may illustrate this point. Aside from this, the discussion of numerical procedures is well suited to convey the flavor of this field to the nonspecialist.

J. H. B.

102[F, Z].—D. H. Lehmer, "Sorting cards with respect to a modulus," *J. Assn. for Comp. Machinery*, v. 4, 1957, p. 41–46.

The author gives a method of using an ordinary punched card sorter to separate a deck of punched cards into no more than *m* sub-decks in each of which all numbers are congruent mod *m*.

C. B. T.

103[Z].—L. PEASE, *An Improved Control Board for Card-Programming the IBM 602A Calculating Punch*, Atomic Energy of Canada, Limited, Chalk River, Ontario, A.E.C.L. Report No. 317, 1955, reprinted 1956, vi + 24 + 8 p. of diagrams and figures, 27 cm. Price $1.00.

The paper gives detailed wiring diagrams and instructions for use of a general purpose 602A board. Although the 602A is an electromechanical and, by electronic standards, a slow machine, it has a high degree of flexibility which is utilized in the board described. The board will perform operations on 8-digit numbers of the form: $(A + a)*(B + b) + c = d$ where * indicates $+$, $-$, X or /; the small letters indicate any of seven internal storage registers, and the capital letters fields on the input cards.

OWEN R. MOCK

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104[Z].—W. G. BROMBACHER, JULIAN F. SMITH, & LYMAN M. VAN DER PYL, *Guide to Instrumentation Literature*, NBS Circular 567, U. S. Gov. Printing Office, Washington, D. C., 1955, iv + 156 p., 26 cm. Price $1.00.

A bibliography listing pertinent works in instrumentation. It includes sections on automatic control, analogue and digital computers, and many other applications more or less related to computation. Books and reference works which appeared no more than about twenty years prior to the compilation of the bibliography were listed, and all periodical articles found were listed. The closing date is not stated, but it was presumably some time during 1954. Sources of material are generally described and listed as specifically as is reasonable.

C. B. T.

105[W, Z].—NATIONAL PHYSICAL LABORATORY, *Wage Accounting by Electronic Computer*, Report No. 1 of the Inter-Departmental Study Group on the Application of Computer Techniques to Clerical Work. Her Majesty's Stationery Office, London, 1956, 25 cm. Price 2s. 6d. net.

This 57-page booklet reports on one of the earliest applications of computers to commercial work in England; the application is a government payroll calculation, and the machine used is the DEUCE. The report is quite complete, including block diagrams, time and cost data, and even considerable discussion of computers per se, reliability, and a section on data sorting. Following are some of the report's conclusions:

(1) A "scientific computer" with magnetic tape input-output forms an adequate machine for payroll work.

(2) The reliability of this operation is satisfactory if attention is given to program checks, proper machine maintenance, and safety margins in the time schedule; automatic "built-in" checking is not essential.

(3) The computer permits reduction in clerical staff, but further economies could be obtained if input data were originally recorded in appropriate form.

(4) The report contains cautious statements indicating that payroll work may justify or at least help justify a computer installation in many organizations.

The DEUCE (successor to ACE) has a 250,000 bit magnetic drum, 32 bit word size, mercury delay line high speed store, and two milliseconds multiply time.

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TABLE ERRATA

The following errata are mentioned in this issue:

CARL-ERIK FRÖBERG, *Hexadecimal Conversion Tables*, Review **82**, p. 208.
H. NAGLER, *Table of Square Roots of Integers*, Review **77**, p. 205.
T. PEARCEY, *Table of the Fresnel Integral to Six Decimal Places*, Review **87**, p. 210–211.
J. RYBNER, *Nomogrammer over komplekse hyperbolski funktioner*, Review **80**, p. 207.
G. N. WATSON, *A Treatise on the Theory of Bessel Functions* [I. M. Longman paper, p. 179].

256.—GEORGE WELLINGTON SPENCELEY, RHEBA MURRAY SPENCELEY, & EUGENE RHODES EPPERSON, *Smithsonian Logarithmic Tables to Base e and Base 10*, The Smithsonian Institution, Washington, D. C., 1952. [Review **992**, *MTAC*, v. 6, 1952, p. 150–151.]

On p. 241 *for* log 1902 = 3,27921 0512*9* 01395 12706
read log 1902 = 3,27921 0512*6* 01395 12706.

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NOTES

Handbook of Mathematical Tables

National Bureau of Standards

The National Science Foundation has commissioned the National Bureau of Standards Applied Mathematics Division to prepare a Handbook of Mathematical Tables containing formulas and graphs. This project is an outgrowth of a conference on Mathematical Tables held at Massachusetts Institute of Technology on September 15 and 16, 1954. One of the principal recommendations made at this conference was that "an outstanding need is for a 'Computer's Handbook,' with usually encountered functions, together with a discussion of their analytic properties and a set of formulas and tables for interpolation and other techniques useful to the occasional computer."