Computation of Fresnel Integrals

By J. Boersma

Two approximations, one valid for \( x \) less than 4 and the other valid for \( x \) larger than 4, have been established by means of the \( \tau \)-method of Lanczos [1] for the Fresnel integrals defined in the form

\[
f(x) = \int_{0}^{x} \frac{e^{-it}}{\sqrt{2\pi t}} \, dt = C(x) - iS(x).
\]

These approximations are the following:

1. For \( 0 \leq x \leq 4 \)
   \[
f(x) = e^{-ix} \sqrt{\frac{x}{4}} \sum_{n=0}^{11} (a_n + ib_n) \left( \frac{x}{4} \right)^n
\]

2. For \( x \geq 4 \)
   \[
f(x) = \frac{1}{2} - i + e^{-ix} \sqrt{\frac{4}{x}} \sum_{n=0}^{11} (c_n + id_n) \left( \frac{4}{x} \right)^n.
\]

The numerical values of the coefficients \( a_n, b_n, c_n \) and \( d_n \) are given by

<table>
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<tr>
<th>( a_0 )</th>
<th>( a_1 )</th>
<th>( a_2 )</th>
<th>( a_3 )</th>
<th>( a_4 )</th>
<th>( a_5 )</th>
<th>( a_6 )</th>
<th>( a_7 )</th>
<th>( a_8 )</th>
<th>( a_9 )</th>
<th>( a_{10} )</th>
<th>( a_{11} )</th>
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</thead>
<tbody>
<tr>
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<td>-0.00000033</td>
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<td>+0.850665781</td>
<td>-0.025639041</td>
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</table>

<table>
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<th>( b_0 )</th>
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<th>( b_2 )</th>
<th>( b_3 )</th>
<th>( b_4 )</th>
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<tbody>
<tr>
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<td>+0.000039386</td>
<td>-7.780030400</td>
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<td>-0.000609892</td>
<td>+4.255387524</td>
</tr>
</tbody>
</table>

The derivation of these approximations is given in [2].

The maximum error is \( 1.6 \times 10^{-8} \) for the first approximation and \( 0.5 \times 10^{-9} \) for the second approximation.

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