AN APPROXIMATION TO THE FERMI INTEGRAL \( F_{1/2}(x) \)  

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The Fermi Integral as defined, for instance, in the *Handbuch der Physik*, Bd. XX, S. 58 [1], is given by

\[
F_p(x) = \int_0^\infty \frac{t^p}{e^{t+x} + 1} \, dt.
\]

The function \( F_{1/2}(x) \) has for negative values of \( x \) an expansion of the form

\[
F_{1/2}(x) \sim x^{3/2} \left[ \frac{2}{3} + \frac{\pi^2}{12} \cdot \frac{x}{2} + \left( \frac{1}{3} \right) \cdot \frac{7}{60} \cdot \frac{\pi^4}{x^4} + \cdots \right] + \left( \frac{1}{2n - 1} \right) \frac{2^{2n-1} - 1}{n} \cdot B_{2n} \cdot \frac{\pi^{2n}}{x^{2n}} + \cdots \];

and for large positive \( x \) the asymptotic expansion

\[
F_{1/2}(x) \sim x^{3/2} \left[ \frac{2}{3} + \frac{\pi^2}{12} \cdot \frac{x}{2} + \left( \frac{1}{3} \right) \cdot \frac{7}{60} \cdot \frac{\pi^4}{x^4} + \cdots \right] + \left( \frac{1}{2n - 1} \right) \frac{2^{2n-1} - 1}{n} \cdot B_{2n} \cdot \frac{\pi^{2n}}{x^{2n}} + \cdots \];

compare [2], formulas (10) and (12);

\( B_{2n} \) are the Bernoulli numbers, given for example in [3], page 298. We obtained Chebyshev approximations to \( F_{1/2}(x) \), based upon the table by McDougall and Stoner [4]. This table was subtabulated by interpolation with a fifth-degree polynomial. The approximations are

\[
F_{1/2}^*(x) = e^x \sum_{r=0}^5 \alpha_r e^{rx} \quad \text{for} \quad -\infty < x \leq +1,
\]

\[
F_{1/2}^*(x) = x^{3/2} \left[ \frac{2}{3} + \sum_{r=0}^5 \frac{b_r}{x^{3r+2}} \right] \quad \text{for} \quad +1 < x < +\infty.
\]
the coefficients

\[
\begin{array}{ccc}
\nu & a_\nu & b_\nu \\
0 & +0.8860 & 7596 & +0.8435 & 00 \\
1 & -0.3087 & 1705 & +0.7108 & 09 \\
2 & +0.1463 & 8520 & -3.7124 & 56 \\
3 & -0.0584 & 3877 & +6.7056 & 28 \\
4 & +0.0143 & 1771 & -5.5948 & 77 \\
5 & -0.0015 & 0176 & +1.7777 & 87 \\
\end{array}
\]

With these approximations, the relative error \(|F_{1/2}(x) - F^*_{1/2}(x)|/F_{1/2}(x)|< 2 \cdot 10^{-4} \text{ and } 5 \cdot 10^{-4},\) respectively.

Another intensive table of \(F_p(x)\) has been given by G. A. Chisnall [5] who also discusses in [6] a method for the interpolation of the existing tables of \(F_{1/2}(x)\). It is not difficult to obtain analogous Chebyshev approximations to \(F_p(x)\) for any fixed values of \(p\) to a prescribed degree of accuracy if one is able to generate the function with this (or slightly more) accuracy.

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On the Congruences \((p - 1)! \equiv -1 \text{ and } 2^{p-1} \equiv 1 \text{ (mod } p^2)\)

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The results of computations to determine primes \(p\) such that one of the relations

\[
\begin{align*}
(p - 1)! & \equiv \text{ -1 (mod } p^2), \\
2^{p-1} & \equiv \text{ 1 (mod } p^2)
\end{align*}
\]

holds have been published previously [1-5]. The known Wilson primes (those satisfying (1)) are 5, 13, and 563, the last having been determined by Goldberg [3] in testing \(p < 10^6\). Froberg [4] tested \(10^6 < p < 30,000\) without finding additional Wilson primes.

Froberg [4] determined \(p = 1093\) and \(p = 3511\) to be the only primes less than

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