the coefficients

ν	a_ν	b_ν
0	+0.8860 7596	+0.8435 00
1	−0.3087 1705	+0.7108 09
2	+0.1463 8520	−3.7124 56
3	−0.0584 3877	+6.7056 28
4	+0.0143 1771	−5.5948 77
5	−0.0015 0176	+1.7777 87

With these approximations, the relative error $|F_{1/2}(x) - F^*_{1/2}(x)|/F_{1/2}(x)$ is less than $2\cdot10^{-4}$ and $5\cdot10^{-4}$, respectively.

Another intensive table of $F_p(x)$ has been given by G. A. Chisnall [5] who also discusses in [6] a method for the interpolation of the existing tables of $F_{1/2}(x)$. It is not difficult to obtain analogous Chebyshev approximations to $F_p(x)$ for any fixed values of p to a prescribed degree of accuracy if one is able to generate the function with this (or slighty more) accuracy.

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On the Congruences $(p-1)! \equiv -1$ and $2^{p-1} \equiv 1 \pmod{p^2}$

By Erna H. Pearson

The results of computations to determine primes p such that one of the relations

$$(p - 1)! \equiv -1 \pmod{p^2}, \tag{1}$$

$$2^{p-1} \equiv 1 \pmod{p^2} \tag{2}$$

holds have been published previously [1–5]. The known Wilson primes (those satisfying (1)) are 5, 13, and 563, the last having been determined by Goldberg [3] in testing $p < 10^4$. Froberg [4] tested $10^4 < p < 30{,}000$ without finding additional Wilson primes.

Froberg [4] determined $p = 1093$ and $p = 3511$ to be the only primes less than

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50,000 satisfying (2). Kravitz [5] extended the range of primes tested in (2) to $p < 10^5$ and found no additional primes of this type.

The author recently tested primes $30{,}000 < p \leqq 200{,}183$ in (1) and $10^5 < p \leqq 200{,}183$ in (2). No primes satisfying either relation were found in these ranges.

The computations were carried out on the Control Data 1604 Computer at The University of Texas. The formula used as a basis for programming the computations was

$$(3) \qquad (p - 1)! \equiv (-1)^{(p-1)/2}\, 2^{2p-2}\, ([(p - 1)/2]!)^2 \pmod{p^2}$$

for an odd prime p. This formula is given as Theorem 133, Hardy and Wright [6].

The primes not exceeding 200,183 were generated and stored on tape, from which they were called in blocks to be tested individually. The 1604 computer is a binary computer with a 48-bit word length. The residue of 2^{p-1} was determined by successions of left shifts and reductions modulo p^2, where the left shifts were long enough to multiply each intermediate residue by a reasonably large power of 2, yet short enough to avoid end-around carry. This residue was tested in (2), then squared, and reduced for use as a factor in (3). The residue of $[(p - 1)/2]!$ was built up by successive multiplication and reduction modulo p^2, finally being squared and reduced for use in (3). The computation time per prime was roughly proportional to the size of the prime, about 8.5 seconds for a prime of the order of 10^5.

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Editorial Notes: (1) The 18,000th prime is 200,183; (2) the residues were not saved and are not available for comparison.

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