TABLE 2
The Hardy-Littlewood Constants

$h_{-10} = 0.67111392$	$h_{-3} = 1.38342429$	$h_4 = 1.37281346$
$h_{-9} = 0$	$h_{-2} = 1.85005441$	$h_5 = 0.52824557$
$h_{-8} = 1.85005441$	$h_{-1} = 0$	$h_6 = 0.71304162$
$h_{-7} = 0.75737123$	$h_0 = 0$	$h_7 = 1.97304317$
$h_{-6} = 1.03575587$	$h_1 = 1.37281346$	$h_8 = 0.71306310$
$h_{-5} = 1.77330507$	$h_2 = 0.71306310$	$h_9 = 0.91520897$
$h_{-4} = 0$	$h_3 = 1.12073275$	$h_{10} = 1.08240211$

From Table 1, in turn, we may compute [3], [4] the Hardy-Littlewood constants h_a for $a = \pm 5$ and ± 10. Together with previously computed values, we may thus complete an 8D table of h_a for $a = -10(1)10$ except for $a = \pm 7$. The $L_{\pm 7}(s)$, needed to fill this gap, may also be expressed in terms of $I_s(\alpha)$ and $R_s(\alpha)$, but this time the arguments α are not given explicitly in [2], and elaborate interpolation would be required to obtain comparable precision.

Alternatively, as is known, generalized harmonic series, including $L_a(s)$ for integer s, may be expressed in terms of the *polygamma* functions [5], [6]. However, the same difficulty arises for $L_{\pm 7}(s)$, and again elaborate and laborious interpolation is necessary. At the author's request John W. Wrench, Jr. has kindly computed $L_7(2)$, $L_7(4)$, $L_{-7}(3)$ and $L_{-7}(5)$ in this way, and these numbers, together with the closed-form $L_{\pm 7}(s)$, suffice to complete our tabulation of h_a. This is given in Table 2.

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New Factors of Fermat Numbers

By Claude P. Wrathall

Eleven new factors of Fermat numbers $F_m = 2^{2^m} + 1$ are listed below. A summary of the present status of the sequence F_m is presented in Table 2.

The method used was suggested by Dr. J. L. Selfridge. Simply stated, the method consisted of forming a sieve array to eliminate possible factors divisible by a prime $\leqq 499$. The remaining possible factors were tested to determine if any of the congruence relationships

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TABLE 1

Factors $k \cdot 2^n + 1$ of Fermat Numbers F_m

k	n	m	k	n	m
308385	21	19	149041	32	30
534689	23	21	127589	33	30
48413	29	25	1479	34	32
143165	29	26	2653	40	38
141015	30	27	43485	45	42
			4119	54	52

TABLE 2

m	Character of F_m
0, 1, 2, 3, 4	Prime
5, 6	Composite and completely factored
10, 11, 12*, 19, 30, 38	Two or three* prime factors known
9, 15, 16, 18, 21, 23, 25, 26, 27, 32, 36, 39, 42, 52, 55, 58, 63, 73, 77, 81, 117, 125, 144, 150, 207, 226, 228, 250, 267, 268, 284, 316, 452, 1945	Only one prime factor known
7, 8, 13, 14	Composite but no factor known
17, 20, 22, 24, 28, 29, 31, etc.	Character unknown

$$2^{2^m} \equiv -1 \mod(k \cdot 2^n + 1) \qquad m = 7, 8, \cdots, n-2$$

were satisfied.

The search program was executed on the IBM 709 at the University of Washington and on the IBM 7090 at the UCLA Computing Facility. All factors have been checked using the SWAC programs of Robinson [1].

For references relevant to Table 2 see Robinson [1], the references cited there, and the more recent papers [2] through [5].

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