REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS


The four tables here are \( \sqrt{n} \) and \( 1/\sqrt{n} \) rounded to 20D for \( n = 1(1)1000 \) together with the sums \( \sum_{m=1}^{n} \sqrt{m} \) and \( \sum_{m=1}^{n} 1/\sqrt{m} \). The tables were checked by squaring and by the Euler-Maclaurin summation formula. A physical problem which was the motivation for the tables is discussed in the appendix. In the introduction there is a discussion of previous tables of \( \sqrt{n} \) and of the two topics: “Benötigt man heute noch Tafeln mathematischen Funktionen?” and “Benötigt man Rechnungen mit großer Stellenzahl?”

D. S.


There are two tables here. The first is of \( C_n^p = n!/p! (n - p)! \) to 10S for \( n = 2(1)100 \) and \( p < n \). The second is of \( n! \) to 20S for \( n = 1(1)1775 \). Both tables are in floating decimal.

The introduction gives no bibliography, although many similar tables have appeared. The exact binomial coefficients to \( n = 100 \), for example, are found in the recent volume [1] of Davis and Fisher. In Davis-Fisher these exact values are printed on 25 pages, while the present generous format requires 69 pages for the 10S approximations. The second table may be compared with the recent Reid-Montpetit table [2]. The latter is less precise (10S) but covers a much larger range: \( n = 0(1)9999 \). There also is the older Reitwiesner table [3] which is equally precise (20S), but does not have quite the range: \( n = 1(1)1000 \).

The preface was written by J. Legras. In it he states that since \( n! \) is “difficult” to compute for large \( n \), an investigator might content himself with approximate formulas, such as Stirling’s formula, “thus introducing poorly known errors”. It may be remarked that if an investigator has indeed computed the leading term, \( (2\pi n)^{1/2} (n/e)^n \), as an approximation to \( n! \), the asymptotic series [4] that multiplies this, namely, \( 1 + (1/12n) + (1/288n^2) - \cdots \), is relatively easy to compute and would suffice, for the twenty-place accuracy given here, for all \( n > 10 \).

D. S.


3. G. W. Reitwiesner, A Table of the Factorial Numbers and their Reciprocals from 1! through 1000! to 20 Significant Digits, Ballistic Research Laboratories Technical Note No. 381, Aberdeen Proving Ground, Md., 1951. (MTAC, v. 6, 1952, p. 32, RMT 955.)

4. For the first 20 terms of this series see F. D. Murnaghan & J. W. Wrench, Jr., The Converging Factor for the Exponential Integral, Report 1535, January 1963, David Taylor Model Basin, p. 34, 35, and 49.
41[A, D].—HANS A. LARSEN, Natural Sines and Trigonometrical Quadratic Surds to 50 Decimal Places, MS of one folded sheet (4 pages) deposited in UMT File.

In an accompanying explanatory note the author describes this manuscript table as the result of a check and extension of the corresponding data given to 30D by Herrmann [1]. In the present tables we find carefully checked 50D values of sin x for x = 1°(1°)90° and of comparable approximations to the 15 quadratic surds appearing in the closed expressions for the sines of integral multiples of 3°, all computed on a Facit desk calculator.

More extended approximations (to 230D) to the sines of 10°, 50°, and 70° are also included; they were evaluated as the roots of the appropriate cubic equation.

As a result of these calculations the author detected six rounding errors in Herrmann’s values and three similar errors in Gray’s 24D approximations [2] to the trigonometric quadratic surds. These errata are described elsewhere in this issue.


These two short books, which are bound together, form a valuable guide for students to some of the literature and problems of modern number theory.

The first book contains: (a) short introductions to five fields of number theory; and (b) brief descriptions of twelve topics including pertinent references to the bibliography of 46 items that follows. The six chapters of the book have appeared previously in different issues of L’Enseignement Mathématique and are listed below:

“Introduction à la géométrie des nombres,” by C. Chabauty
“Introduction à l’analyse diophantienne,” by F. Châtelet
“Problèmes d’approximation diophantienne,” by R. Descombes
“Introduction à la théorie des nombres algébriques,” by Ch. Pisot
“Le théorème de Thue-Siegel-Roth,” by G. Poitou
“Bibliographie de l’arithmétique,” by A. Châtelet

In his article F. Châtelet suggests that Fermat’s Last Theorem may not be due to Pierre Fermat at all, but rather to his son Samuel. He states that our only access to Pierre Fermat’s notes is in the edition of them put out by the son Samuel, and that the latter may have misunderstood P. Fermat, who perhaps merely meant that the proposition has been proven for the exponents 3 and 4.

The book by Erdős (55 pages long) contains statements of 76 problems together with discussion and references. The problems are of a considerable variety both as regards their subject matter and their status. Most of them are related to papers
of Erdös, and the book may be regarded as an introduction to some of the work of this prolific mathematician. The problems are classified by subject matter: divisibility problems concerning finite and infinite sequences, additive problems, congruences, arithmetic progressions, primes, diophantine equations, etc. Here is problem no. 60 (for which no discussion or references are given):

\[ m = 2^k - 2 \text{ and } n = 2^k(2^k - 2) \text{ have the same prime divisors. Likewise } m + 1 \text{ and } n + 1 \text{ have the same prime divisors. Are there any other such examples?} \]

D. S.


By almost every standard this is a good book; the subject matter receives careful treatment, the presentation is on an elementary level, interesting and important material is covered, and many exercises are included together with answers. After informally introducing the basic ideas of Boolean Algebra, the book proceeds to an axiomatic treatment of the subject. There then follows a chapter on Boolean equations, a chapter on "sentence logic", and finally a chapter on lattices. The neophyte will gain much from this short text.

But I would like to take this opportunity to point out a serious omission in content that this book shares with many other mathematics books of this type. The revival of interest in Boolean Algebra is undoubtedly due to its use in switching-circuit theory. For the reader who is studying Boolean Algebra with this application in mind, the book does not meet the need. In no place is switching-circuit theory mentioned. And the methods are presented only in the abstract: computational methods and techniques for solving problems are studiously avoided.

For example Boolean equations are discussed and particular solutions are given to certain selected equations. The solutions are given first, and then it is demonstrated that these solutions do indeed satisfy the equations. How one obtains these solutions to begin with is left a mystery, even though methods for determining solutions to the simple equations considered are quite elementary. For instance, consider the equation \((A \cap X) \cup (B \cap X') = 0\), discussed on page 62 of the book. There are only four possible combinations of values that \(A\) and \(B\) can have together; consequently for each of these combinations we can see what value of \(X\) will satisfy the equation. The following table demonstrates these, where it is clear that the case \(A = 1, B = 1\), can not lead to a solution:

\[
\begin{array}{|c|c|c|c|c|}
\hline
(A \cap X) \cup (B \cap X') & 0 & 0 & 1 & 0 \\
\hline
0 & 0 & 1 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 \\
0 & 1 & 1 & 0 & 0 \\
1 & 1 & 1 & 1 & 1 \\
\hline
\end{array}
\]

Immediately one sees that the two possible solutions are \(X = \bar{A}\) and \(X = B\) where \(A \cap B = 0\).

Too often in mathematics texts, the applications are ignored. This I believe to be a serious defect, not just in this text but in a large majority of books in the English language. This is not to say that a mathematical text should lack rigor or
detail in its treatment of the subject matter, but rather that some degree of attention should be paid to various practical applications of the subject matter. Such applications will tend to stimulate the student and better orient him with respect to the role of mathematics in modern science. This remark holds particularly for Boolean Algebra, where the eventual application will more likely than not be the basis for the reader's interest in the subject.

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We have recently had a flurry of interest in the tabulation of values of the Clebsch-Gordan coefficients for ever higher numerical values of the angular-momentum parameters. These coefficients are the quantum-mechanical vector-coupling coefficients denoted by Condon and Shortley [1] as \( (j_1 j_2 m_2 | j_1 j_2 j m) \), with \( j_1, j_2, j \) restricted to nonnegative integers or half-integers satisfying the "triangle" conditions, with \( m_1 \) ranging from \( j_1 \) to \(-j_1\) in integral intervals, \( m_2 \) similarly from \( j_2 \) to \(-j_2\), and with \( m = m_1 + m_2 \).

Three tabulations, of different types, have recently been reviewed in this journal [2, 3, 4]. The present volume contains, in its introduction, a useful bibliography of all the tables that have been computed. These tables are of three types:

(a) Algebraic tables: if \( j_2, m_2, \) and \( j - j_1 \) are given fixed numerical values, the coefficient can be written as a relatively simple algebraic function of \( j_1 \) and \( m \).

(b) Numerical tables in which the coefficients are expressed as square roots of rational numbers.

(c) Numerical tables in which the coefficients are expressed as decimal numbers.

The present table is of the third type and extends the available decimal tables to all values of \( j_1 \) and \( j \) from 0 to 10 in steps of \( \frac{1}{2} \), but only for \( j_2 = 1 \) to 6 in steps of 1. The coefficients are given to 7 decimal places. No apology is given for the restriction of \( j_2 \) to integral values.

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From the editor's preface: "The six expository lectures appearing in this volume are the first in a series of eighteen lectures being given at George Washington Uni-
Two subsequent volumes will contain the remaining lectures. Our intention was to invite each of the eminent men represented here to delineate a substantial research area, to describe it broadly and comprehensively for an audience of mathematicians who are not specialists in that area, and to contribute to this description his individual evaluation of the esthetic and practical aspects of the field, its position in mathematical development as a whole, and its future, as that might be implied in the conjectural exposition of its unsolved problems.”

The six chapters are:

“A Glimpse into Hilbert Space,” by P. R. Halmos
“Some Applications of the Theory of Distributions,” by Laurent Schwartz
“Numerical Analysis,” by A. S. Householder
“Algebraic Topology,” by Samuel Eilenberg
“Lie Algebras,” by Irving Kaplansky

The lectures are at an advanced level and would not be of much benefit to undergraduates or the general scientific public, but for trained mathematicians and graduate students of mathematicians, the present volume, together with the two to appear, must be regarded as a valuable and stimulating collection of introductions.

Halmos and Brauer emphasize unsolved problems. Halmos, in his usual lively style, listed ten, mostly concerning algebraic aspects of a single bounded operator. Three of these ten problems have already been solved, and one is not sure whether congratulations or condolences are due the author. Brauer’s chapter is more extensive; he lists 43 problems, and he adds much supplementary comment plus an appendix of definitions.

Eilenberg, Kaplansky, and Schwartz emphasize known results. Eilenberg starts with a concrete problem (vector fields on a sphere) but spends most of his time developing the definitions of concepts. Kaplansky has more of a survey of results, including five connections between Lie Algebras and several types of groups, and a classification of simple algebras. Schwartz (a brilliant lecturer) has less ground to cover, and after a survey of fundamentals gives applications to the fundamental solutions of partial differential equations, and to the “problem of division,” due, primarily, to Hörmander and Łojasiewicz respectively. Schwartz makes the interesting remark: “It appears more and more that some of the greatest mathematical difficulties in theoretical physics, for instance, in quantum field theory, proceed precisely from this impossibility of multiplication [of distributions].”

Householder states that numerical analysis, as a recognized discipline, originated in three events during 1947. He indicates that the two problems which concern the numerical analyst, namely, truncation error and roundoff error, require quite different techniques. The latter problem is due to the differences between real-number arithmetic and the much more difficult pseudo-arithmetic using finite precision. These latter, “dirty,” problems are discussed first, starting with the von Neumann-Goldstine paper on matrix inversion (one of the three events), and continuing with work of Turing, Gastinel, Bauer, Givens, and especially Wilkinson. Turning to the “clean” problems—the development of techniques—he indicates that many of these may be discussed on the basis of several organizing principles:
the concept of a norm, an effective notation for particular types of matrices, and
the so-called König ratio and its generalizations. The emphasis throughout is on
matrix problems, and presumably much of this material will be developed in
greater detail in his forthcoming book on matrices.

D. S.

46[K].—A. E. Sarhan & B. G. Greenberg, editors, Contributions to Order Sta-
$11.25.

If the random observations $X_1, X_2, \cdots, x_n$ of a sample drawn from a continuous
population are arranged in ascending order of magnitude, $X_{(1)} \leq X_{(2)} \leq \cdots \leq X_{(n)}$,
then we have the order statistics of the sample and $X_{(i)}$ is called the $i$th order
Statistic. Order statistics are inherently much more informative than the ordinary
random sample alone, and therefore have considerable practical value. It is prob-
ably for this reason that within the last fifteen years there has occurred a rather
large-scale attack on the theory of order statistics.

Interest in order statistics runs high. For example, are the least or greatest
values, or both, “outliers” which perhaps should be discarded? What are the dis-
tribution properties of the order statistics and how efficient are the order statistics
(in particular, various linear combinations of them) in estimating population
parameters? The great, practical point regarding order statistics is that computa-
tions involved in their use are rather minimal compared to that for the “most
efficient” statistics, while the loss in efficiency is not very significant.

The present volume brings together the more pertinent theoretical background,
applications, and tables required to use order statistics. Indeed, it provides a very
worthwhile manual, which is sorely needed at the present state of progress in this
area of Mathematical Statistics. As examples of topics covered, we mention in
particular the exact and approximate distributions and moments of order statistics
from normal, exponential and gamma populations, the range $X_{(n)} - X_{(1)}$, best
linear estimates of population parameters, theory and applications of extreme values,
tests for suspected outlying observations, theory and applications of extreme values,
tests for suspected outlying observations, theory and applications of extreme values,
test for independent samples, multiple-decision and multiple-comparison techniques for
ranking treatment means, optimum grouping and spacing of observations, short-cut
tests, and tolerance regions. From this list alone, we get a general idea concerning the
over-all value of the book as a welcome addition to the statistical library. The
editors of the book are to be congratulated for a job well done.

FRANK E. GRUBBS

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47[K, X].—Edward O. Thorp, Beat the Dealer: A Winning Strategy for the
Price $4.95.

Although volumes have been written about blackjack, the first mathematical
attempt to obtain an optimal strategy was made in 1956 by Baldwin, Cantey,
Maisch, and McDermott. To simplify the computations, they assumed that all
hands were dealt from a complete shuffled deck. As the game is actually played, however, the later hands come from a decreasing deck. Thus, the probability of winning and the optimal strategy should fluctuate. Further, the player should have the advantage frequently. Using an IBM 704, the author computed, as a function of the cards in the depleted deck, the situations when the player has the advantage.

The book begins with a discussion of the rules of the game and then proceeds to describe the optimal strategy as a function of the amount of information (the cards depleted from deck) the player is able to remember. If no information is remembered, the optimal strategy yields 0.21 percent advantage to the casino. However, keeping track of the fives, the player obtains an advantage of 3 percent. If a player is able to keep track of more than four cards, tens and aces, he can obtain an advantage which ranges from 4 to 15 percent.

The book contains an account of the author's successful test in Nevada. The chapter on how to spot cheating is unique. The book also contains an appendix giving the probabilities for hands dealt from a complete deck.

Melvin Dresher

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Following a one-page introduction, which gives the general definition of the Struve functions, their expansions in both power series and asymptotic series, and an outline of the contents of the tables, the author presents decimal approximations to $L_0(x)$ and $L_1(x)$ to 5 and 6S for $x = 0.02(0.005) 4(0.05) 10(0.1) 19.2$, calculated by power series, and approximations to 2S, in floating-point form, for $x = 6(0.25) 59.50(0.5) 100$, calculated by asymptotic series. All calculations were performed on an IBM 650 at North Carolina State College, where the author is a member of the Department of Mechanical Engineering.

No bibliography is presented, and apparently no comparison of these data was made with existing tables such as those of the National Bureau of Standards [1]. A single comparison with the latter tables revealed numerous last-place errors (ranging up to 5 units) in the tables under review.

Apart from these discrepancies, the manuscript tables appear to be reliable, and they supply tabular information corresponding to a range of the argument extending considerably beyond that of previous tables of these functions.

J. W. W.


The main subject of this book by Avner Friedman is a somewhat specialized topic in the theory of partial differential equations; namely, the Cauchy problem
for partial differential equations of elliptic and hyperbolic type with constant coefficients, in various classes of functions and distributions. However, the book is of value to the general reader, since it contains good accounts of such topics of general interest as a theory of linear topological spaces, generalized functions, distributions, convolutions, and Fourier transformations of generalized functions and distributions. It also contains a good account of such important topics in the theory of partial differential equations as fundamental solutions of equations with constant coefficients and the differentiability of solutions of hyperelliptic equations. It contains a statement and proof of the Sobolev lemma, and it is the only book which contains a proof of the important Seidenberg-Tarski theorem. One typographical error was noted: in the statement of the theorem on page 218, change sigma to $a$ on the third line from the bottom and 16 spaces from the left.

P. D. L.


The second edition of Lefschetz's now classical book has a considerable amount of important new material. After the preliminary chapters containing standard information on existence theorems and linear systems, including Floquet theory and stability, the author proceeds to a detailed study of Liapunov stability. Considerable emphasis is put on the direct method. An important feature is the treatment of the converses of the Liapunov theorems in case the system is suitably stable at a critical point.

After a study of the $n$-dimensional case, where many of the results are still fragmentary, there is a detailed study of two-dimensional systems, including the critical cases and structural stability.

The remainder of the book is concerned with equations of the second order, including the Cartwright-Littlewood theory and the Hill-Mathieu equations.

The methods used are both analytic and topological. The reader untrained in geometry may have difficulty with the close geometric reasoning of the latter chapters. On the other hand, the material is not readily available in any other single source.

H. Pollard

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This monograph is a translation from Russian of a text written in 1950 by one of the leading pioneers in the theory of anisotropic elasticity. His earlier book Anisotropic Plates, written in 1947, is already classical. The present monograph represents the results of the author's investigations (and related works) on another class of important problems in anisotropic elasticity.

The author’s stated purpose in writing this book was to bring together some scattered results on anisotropic problems which have appeared in the literature
and to present these results in a systematic and orderly manner, so that the basic material would then be readily available to the scientific public. The book makes no attempt to investigate all questions of the theory of elasticity for anisotropic bodies, but restricts its attention to certain parts of the theory which have been rather thoroughly studied but not previously organized. For instance, the author does not treat the questions of stability and deflections of elastic plates, since these problems were covered in his earlier book. He also omits all problems of equilibrium and stability of anisotropic shells as well as questions connected with plasticity or large deformations of anisotropic bodies.

Chapter I deals with the general equations of elasticity of an anisotropic body. It contains numerous examples and the details of the derivation for various types of anisotropy. Chapter II investigates the simplest cases of elastic equilibrium—stretching and bending of rods and plates under various conditions and with various types of anisotropy. Chapters III and IV treat problems connected with an anisotropic body bounded by a cylindrical surface for which the stress does not vary along the generator. Here the author first derives the general governing equations and then treats in detail generalized plane stress problems, torsion problems, bending problems, etc., giving particular consideration to the case of cylindrical anisotropy. In these sections he extends Muskhelishvili's work in the plane theory of isotropic elasticity to the anisotropic case. Chapter V deals with the state of stress of an anisotropic cantilever of constant cross section deformed by a transverse force. The final chapter covers symmetric deformation and torsion of bodies of revolution. Here a number of examples are treated in detail.

The author has consistently kept his exposition brief but lucid. As a result the book is well within reach of the senior graduate student. The English translation will certainly be welcomed by research scientists in the physical and engineering sciences and by design engineers.

L. E. Payne

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The advent of the Sputniks has brought a rash of what purport to be Space Age textbooks; and this is another. Some appear to be motivated by an uncontrollable desire to rush into print; and, on the other extreme, some appear to make a serious effort to make the material comprehensible to beginners and the uninitiated. The present volume under review has a peculiar place in this spectrum: it is not elementary, and it attempts to cover all aspects implied by its adopted title. Within the bounds of 410 pages, this is patently impossible, and therein lies the principal criticism. Many books come into being naturally from the notes of a course which the author has taught several times. They become "tried and true". It may well be that the present material came from a course which was taught once. But there is a wide disparity between treatments, e.g., Cowell's method is presented on one page of prose, whereas Musen's modification of Hansen's lunar theory as applied to artificial satellites is copied in great detail from the published papers. One also
detects the influence of the Yale Summer Institutes in Dynamical Astronomy, which the author undoubtedly attended.

The chapters cover two-body motion, orbit determination, analytical dynamics, general perturbations, special considerations for artificial satellites, nongravitational forces, special perturbations (for which the author seems to have a "blind spot"), since most of this chapter consists of Musen's numerical general theory), reduction of radio observations, orbit improvement, transfer orbits, and a final chapter on the problem of three bodies. The organization of the references leaves much to be desired. One must only deplore the dissipation of the author's obvious competence on such a "broad-brush", transparent treatment as this is.

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A new consistent set of values for the fundamental physical constants has been recommended by the above committee. It is anticipated that these will be adopted by the International Unions of Pure and Applied Chemistry and Physics. A full report of the background entering into this set of values was discussed by J. W. M. DuMond and E. R. Cohen at the Second International Conference on Nuclidic Masses, Vienna, July 1963.

The meter is defined as 1650763.73 wavelengths in vacuo of the unperturbed transition 2p\(_{10}\) — 5s\(_{5}\) in \(^{85}\)Kr. The second is 1/31556925.9747 of the tropical year at 12\(^{th}\) ET, 0 January 1900. (The latter definition does not appear to be very neat operationally, since it is not clear how a direct comparison could be made. As a colleague remarks: "Times have changed since then.")

Of the famous atomic constants we mention here only the proposed values:

\[
\begin{align*}
    c &= 2.997925 \pm 0.000003 \cdot 10^{10} \text{ cm/sec}, \\
    e &= 4.80298 \pm 0.00020 \cdot 10^{-10} \text{ esu}, \\
    h &= 6.6256 \pm 0.0005 \cdot 10^{-27} \text{ erg sec.}
\end{align*}
\]

All errors listed are 3 standard deviations, and it is stated: "It is therefore unlikely that the true value of any of the constants differs from the value given in the table by as much as the stated uncertainty." Consistent with the above values is

\[
\frac{hc}{2\pi e^2} = 137.0388 \pm 0.0019,
\]

which, at least on the face of it, contradicts Eddington's notion that this ratio equals 137 exactly.

A plastic wallet-sized card listing some of these constants is available from the National Bureau of Standards for 5 cents.

D. S.
REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS


The term "hypersonic" distinguishes aerodynamic problems at speeds far greater than sound from associated problems at moderately supersonic speeds. The new features of hypersonic flows can be divided into hydrodynamic characteristics (associated with high Mach numbers) and physical or chemical characteristics (associated with high temperatures arising because of high velocities). The subject matter of the book is devoted primarily to the hydrodynamic features.

The chapter headings are:

I. General Considerations
II. Small-Disturbance Theory
III. Newtonian Theory
IV. Constant-Density Solutions
V. The Theory of Thin Shock Layers
VI. Other Methods for Blunt-Body Flows
VII. Other Methods for Locally Supersonic Flows
VIII. Viscous Flows
IX. Viscous Interactions
X. Free Molecule and Rarefied Gas Flows

Chapter I gives an excellent qualitative description of the general features of hypersonic flow fields for both blunt and slender bodies. The shockwave patterns, the characteristics of the flow in the vicinity of the body (Mach number, temperature, pressure, etc.), and the characteristics of the flow in the wake are described. This qualitative description leads naturally to a categorization of the assumptions upon which various hypersonic theories for inviscid flow are based. These assumptions depend on shape of the body and the freestream flow as follows:

A. \( M_\infty \gg 1 \) "Basic Hypersonic"
B. \( \sin \theta_b \ll 1 \) "Slender Body"
C. \( M_\infty \sin \theta_b \gg 1 \) "Strong Shock"
D. \( \epsilon \ll 1 \) "Small Density Ratio"
E. \( M_\infty \sin \theta_b \ll 1 \) "Linearization"

where \( M_\infty \) = freestream Mach number
\( \theta_b \) = inclination of body surface to the freestream flow
\( \epsilon \) = density ratio across the shock wave

The authors indicate the assumptions which apply to the several inviscid theories developed in a very orderly way in Chapters II through VII.

The theories considered in Chapters II through V are indicated by the headings. Additional methods for blunt body flows given in Chapter VI include stream-tube continuity methods, method of integral relations, and relaxation techniques. Chapter VII covers shock-expansion theory for locally supersonic flows and tangent-wedge and tangent-cone methods.

In addition to the thorough treatment of inviscid flow theories, the book is an
excellent reference for hypersonic viscous effects which can be treated with boundary-layer theory (Chapters VIII and IX).

Chapter X gives a very complete qualitative discussion of the general features of rarefied gas flows, covering the gamut from low Reynolds number continuum flow to free molecule flow. For the free molecule flow regime, results for forces and heat transfer are given.

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55[V, X].—Charles J. Thorne, George E. Blackshaw & Ralph K. Claassen,
Steady-State Motion of Cables in Fluids, Part I., Tables of Neutrally Buoyant
Cable Functions, NAVWPS Report 7015, Part 1 NOTS TP 2378, China Lake,
California, 1962, xxxii + 400 (approx.) unnumbered pages, 22 cm.

An approximate solution for the shape and tension of a neutrally buoyant
flexible cable in a stream is expressible in terms of the functions

\[ \tau = \exp \left( \frac{F}{R} \cot \phi \right), \quad \xi = \int_{\phi}^{\pi/2} \tau \cot \phi \csc \phi \, d\phi, \quad \eta = \int_{\phi}^{\pi/2} \tau \csc \phi \, d\phi \]

where \( R/F = 45 \). A brief table of these functions was given by Landweber and Protter (Jour. Appl. Mech., 1947). In the present work these functions are tabulated for much smaller increments of the variable. Various combinations of these functions that are useful in solving certain types of cable problems are also tabulated.

Since the assumed laws of the forces on a cable are empirical and approximate, it is interesting to observe that by a slight alteration in the physical assumptions, due to R. K. Reber of the Navy Department, Bureau of Ships, the differential equations can be made integrable. Assuming that, instead of a constant tangential component, there is a constant force \( F \) per unit length in the downstream direction, the differential equations (5) and (6) in the book would be replaced by

\[ \frac{dT}{ds} = F \cos \phi \]

\[ T \frac{d\phi}{ds} = -R \sin^2 \phi - F \sin \phi \]

It is readily verified that the functions corresponding to \( \xi \) and \( \eta \) obtained from these differential equations are exactly integrable. This has the obvious advantage of enabling neutrally buoyant cable problems to be solved with the aid of trigonometric tables.

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56[X].—Charles Andersen, "The Ruler Method, An Examination of a Method
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REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS


Let $f(x)$ be given graphically on the closed interval $(0, 2\pi)$. Required are the Fourier coefficients; for example, $a_n$, where $f(x) = \sum_{n=0}^{\infty} a_n \cos nx$, and $a_n = \left(\frac{1}{n}\right) \sum_{x \in [0,2\pi]} f(x) \cos (n\pi/n)$. The idea is to have a set of rulers so graduated as to facilitate location of $x_k$.

There is a discussion of the relation between $a_n$ and $A_n = (1/n\pi) \int_0^{2\pi} f(x) \cos nx \, dx$.

Y. L. L.


Three tables concerning graphs are given in the appendices of the monograph above.

For $p = 4(1)7$ and $k = 0(1)\frac{1}{2}p(p - 1)$, Appendix 2 lists the number of graphs with $p$ points and $k$ lines of the following six types: $N_{p,k} =$ labeled graphs; $C_{p,k} =$ labeled connected graphs; $S_{p,k} =$ labeled stars; $\pi_{p,k} =$ free graphs; $\gamma_{p,k} =$ free connected graphs; and $\sigma_{p,k} =$ free stars.

The interesting Appendix 3 shows diagrams of each topologically distinct connected graph for $p = 2(1)6$ and $k = p - 1 (1)\frac{1}{2}p(p - 1)$. For each of these there is given $n$, the number of such graphs if they were labeled; $d$, the so-called “complexity” (an invariant of the graph matrix); and finally a symbolic designation of the corresponding graph group. For $p$ and $k$ fixed the number of topologically distinct graphs is the quantity $\gamma_{p,k}$ above, while the sum of the corresponding values of $n$ is the quantity $C_{p,k}$ above. (There is an error in the first graph for $p, k = 6, 7$; the leftmost vertical line should be deleted).

Appendix 4 lists $n(p, k, d)$ for $p = 2(1)7$, $k = p - 1 (1)\frac{1}{2}p(p - 1)$ and all pertinent values of $d$. This is obtained by adding the values of $n$ for all graphs with the same values of $p$, $k$, and $d$.

Besides the physical application indicated in the title, the monograph contains a certain amount of graph theory, defining the above concepts and quantities and giving formulas. On page 197 is an unproved conjecture concerning the asymptotic behavior of $n(p, k, d)$.

D.S.


This is the first volume in a series on spacecraft structures, and is intended to present the mathematical methods that are most useful to structural engineers. The emphasis is on numerical methods, and the contents run over a small gamut of topics from interpolation to partial differential equations. The exposition is simple, and, since the author avoids involvement with knotty questions, the
material should be comprehensible and useful to anyone with a few years of college mathematics.

P. J. D.


Of the papers presented, that which offers the main interest for computation is "Monotonic operating in numerical mathematics," by L. Collatz, a survey of part of the recent work done by the author and his school, and published mainly in the Archive for Rational Mechanics and Analysis and in Numerische Mathematik.

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According to the introduction, this second edition represents a completely changed and substantially expanded version of the work; and, without question, this is indeed a rather modern introduction to the programming field.

A striking feature of this book is the complete parallelism of presenting the entire material in a symbolic machine-oriented language and at the same time in an algorithmic language. Throughout its entire design and terminology the presentation in algorithmic notation has been directed completely and consistently toward ALGOL, and accordingly the book is in part an elementary introduction to ALGOL-60. The symbolic programming is based on the Zuse Z-22. The Z-22, a widely used computer in German universities, has a 2\textsuperscript{nd}, 38-bit word drum memory with 5 ms mean access time and a 16-word core memory, which is in part used as accumulator and for various registers. The symbolic notation employed is based on the Freiburger Code, an assembly language developed at the University of Freiburg/Br.

Chapter 1 provides an introduction to flow charts and programming in general, after which it presents a brief description of the Z-22. The second chapter gives a concise definition of the Z-22 order code and a convention for flow chart notations. Chapter 3, the longest chapter in the book, enters into a detailed discussion of simple programs, including the concept of a loop; at the same time the fundamentals of ALGOL are introduced (although the name ALGOL is not yet mentioned). This is followed by a short Chapter 4 on symbolic addressing, while Chapter 5 deals with programs with multiply nested loops. Chapter 6 then discusses the different ways of supplying a subroutine with parameters and includes a very good, brief discussion of recursive subroutines. Chapter 7 contains a systematic presentation of address changes, relocation of programs, indexing and related topics. Chapter 8 introduces the succeeding chapters by discussing briefly the differences between interpretive routines and assembly programs, etc. Chapter 9 then furnishes a presentation of the actual machine language of the Z-22 and of some of the principles of assembly programs as, for example, address assignment and subroutine calls. Chapter 10 concerns itself with compiler languages and with some aspects of
formula translation; in particular, Rutishauser's and Bauer and Samelson's work is briefly discussed, with very little reference to relevant developments in the U. S. A. This is followed by a very general section on language unification. As mentioned earlier, starting with Chapter 3 the author has built up and used an algorithmic language based on ALGOL. Chapter 11 now gives a precise definition and summary of ALGOL-60, using Backus' notation. The final Chapter 12 is entitled "parallel programming" and gives a survey of various aspects of this field, including such topics as input/output buffering and program interrupts, and including a fairly representative citation of basic references.

Many programming examples are used in every chapter to elucidate the discussion. Each example stresses the parallelism of the language presentation and consists of a general problem statement, a detailed flow chart, a full Z-22 symbolic program as well as an ALGOL program. The examples are taken in part from numerical analysis, but there are also a large number of problems from various areas of business data processing.

The book is written in a lucid style and makes very good reading. The overall presentation shows the commitment of the German computing community to ALGOL. At the same time, a significant feature of the book is its complete lack of material on programming and monitoring systems, the use of tapes or disks, and similar problems of importance to users of large-scale computing systems.

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Editorial note: The first edition, entitled Einführung in die Programmierung digitaler Rechenautomaten, was reviewed in Math. Comp., v. 15, 1961, p. 316.


This is a most welcome volume to everybody in the computer field who cares to know what happens outside of his own particular area of computer applications. In the words of the editor, "it is addressed primarily to those readers already familiar with computers, and the computing specialists, who, for example, wish to learn about their neighboring areas or about new trends which have not yet become common knowledge".

Each of the 25 contributors had complete freedom as far as the presentation of their subject and the expression of their opinions was concerned. Eight contributions are in German, seven are in English, and each is preceded by English, French, and German summaries.

The first article by Heinz Zemanek on automata and thought processes is a critical study on the nature of automation. He analyzes automata and thought processes from the point of view of one skilled in the automation engineering art. Then he summarizes and critically reviews published work. He goes on to single out the basic processes of reproduction, reduction, and expansion of information and describes the famous "artificial animals". Specific problems such as learning, composing, game-playing, and problem solving by machine are discussed. The paper ends on a philosophical note: "Is there an end to the natural sciences?"
The second article by Ambros P. Speiser on new technical developments is an account of the current work which will pave the way for the second computer generation. While the first generation is conveniently defined as the sum of all computers up to the present time, the present efforts are classified in two areas: (a) perfection of the techniques for the construction of our present computers, (b) increase of the speed and memory capacity of large and small computers. The second stage of development involves many aspects of the physical sciences and is the main topic of this paper. Most of it is devoted to magnetic-core storage, trapped-flux-super-conducting memory kryotrons, disk storage, read-only memories, flying-spot memories, and different output devices.

A paper by R. Tarján deals with logical machines, which are classified according to their problem-solving ability. The main topic is the abstract theory of automata consisting of two-valued decision elements.

T. Erisman presents an article on digital differential analyzers. He describes the demand for such devices and the circuitry required. The historical development is outlined and the main features of “Integromat”, “Maddida”, “Bendix DDA”, and “Trice” are described. The author discusses the advantages and disadvantages as compared with analog and digital computers. In general, in a given time the DDA will produce more accurate results than analog computers. But using the same number of arithmetic units, the analog computer is cheaper in price and analog information has to be converted into digital form before input in the DDA. But it is added that both advantages shrink somewhat since every error in a DDA is of a systematic nature, and that coupling of many integrators reduces speed considerably.

In an article “Interrelations between Computers and Applied Mathematics”, Herman Goldstine analyzes the effects that large-scale, electronic computers have had upon applied mathematics. One kind of effect relates to the impact of the new machines on numerical analysis, another shows the opportunity for gaining new insights into areas as yet inaccessible by conventional means. In the first category three kinds of errors are discussed: input errors in the data (empirical or experimental), the truncation error of the discrete step methods, and digital noise, due to finite word length. He concludes with an excursus on Bliss’ techniques applied to a system of ordinary differential equations.

F. L. Bauer’s and K. Samelson’s paper on processing of programming languages by computers describes the way from early “automatic programming” to the modern problem oriented languages: FORTRAN, PACT, AP3, Mercury autocode, ALGOL 58, ALGOL 60, COBOL, and a language proposed by Lyapunov, which is widely used in Russia. A universal metalanguage is predicted to be the next step in this development, as already indicated by Zuse’s “Plankalkül” and the General Problem Solver (GPS) of Newell, Shaw and Simon and J. McCarthy’s work with LISP.

W. L. van der Poel’s article on micro-programming and trickology, shows, by example of the ZEBRA computer, ways to overcome the shortcomings of present day computers in searching lists, block transfer, sorting, etc. Ways are shown to build up macro-instructions from a coding system, where the programmer has immediate access to the micro-programming of the machine. Repeating of an instruction, or the generation of programs in fast registers, not written out beforehand is illustrated (“under-water programming”).
R. W. Bemer’s paper “The Present Status, Achievement and Trends of Programming for Commercial Data Processing” describes some of the basic elements of programming techniques developed during the last eight years in commercial data-processing problems. Automatic operating systems, tabular languages, input-output control systems, automatic production of automatic programming processors, remote operation of computers through communication links, standardization of techniques, and communication between different computers by common language are trends which are noted in modern programming. Generalizing of programs and program sharing among many users are other important features of programming today.

H. K. Schuff in an article on “problems in commercial data processing” discusses the basic philosophy of automatic data handling. He attempts a systematic classification of these problems by using control circuits, the elements of which represent the places where high densities of data occur.

Y. Bar-Hillel discusses theoretical aspects of the mechanization of literature searching. He distinguishes between data-providing and reference-providing systems. References can be provided in (at least) four stages: accession numbers, citations, abstracts and copies of the selected documents. The idea of shortcircuiting reference-providing by scanning the document collection directly with a high-speed computer is rejected. Automatic indexing and extracting are criticized as either leading to inferior products or as being uneconomical. He shows where an electronic computer might be employed usefully in the library problem and discusses the attempts to establish a general mathematical theory of literature search, which he feels to be failures. He delegates the computer to performing merely the routine operations and wishes to see the human having constant control over the operations.

A rather extensive paper by E. Reifler on machine language translation is divided in two parts. In the historical outline, the worldwide efforts on MT are described. Part two is a review of the fundamental bilingual lexicographic and linguistic problems of multiple grammatical and non-grammatical meaning, and the problems of the automatic identification and translation of potential future, and, therefore, still unrecorded, compound words. It finally demonstrates a sample of MT output from the University of Washington research material.

K. Zuse’s paper on the evolution of computer development from mechanics to electronics gives an account of the author’s efforts in this field, which date back to the year 1934. The prototypes Z1, Z2, Z3, Z4 which were built by the author during World War II, in Germany, are briefly discussed. The construction was resumed after an involuntary pause of four years, and led to the computers Z22, Z23 and Z31, the two latter ones being fully transistorized. The Z22 computer is described in some detail. An outlook on digital field computers follows, to which the author was led by the study of meteorological problems.

Jan Oblonský’s report on computer progress in Czechoslovakia is a description of the self-correcting features of the Czech computer SAPO, which stands for Samočinný Počítač, i.e., automatic computer. SAPO is a five-address machine with error detection through parity checks and re-reading after writing in memory, checking of transfers through parity check, checking of the operations by performing
them in triplicate, checking of the control, checking of input and output units through multiple read and punch features. The correction of errors is affected by entering a micro-program which checks address and instruction parities separately and induces necessary repeat instructions. Failure after one repeat cycle causes the machine to stop and print a diagnostic statement. After four years of successful operation, the author finds that microprogrammed error correction is more economical than simultaneous execution of the same instruction several times.

The second article of the same title by Antonín Svoboda, the chief designer of SAPO, is on the number system of residual classes (SRC). It is an interesting paper on the usefulness of SRC-encoding of numbers in computers and its effects on the arithmetic unit and logical design of a computer. The concept is relatively new, but it can be expected that interesting features, algorithms, and logical designs will emerge from it in the future.

A seven-part paper by Hideo Yamashita and others report on the digital computer development in Japan since 1939. Each part is written by a different author, tracing the development essentially through 1961. M. Goto and Y. Komamya describe the relay computer ETL Mark II (pilot model 1952), H. Takahasi and E. Goto report on the parametron, its principles and its application as a logical element in a computer. The same two authors also discuss memory systems for parametron computers in another article. S. Takahashi and H. Nishino describe the transitorized computer ETL Mark IV which is completed in 1957. Still a relatively slow and small computer (180 kc/sec cycle time, 1 K magnetic drum memory with 1.67 ms average access time), it is the second fully transisorized Japanese computer built. T. Motooka has a paper on magnetic core switching circuits and E. Goto reports on the Esaki diode, or the tunnel diode, as we know it. The seventh paper in this series is by N. Kuroyanagi on a specific high-speed arithmetic system built with an adder, a shifting register, detector, and a selective transmission unit. The purpose is to perform a chain of complicated logical operations of great depth in one step.

The last article is written by the editor of the volume, W. Hoffman. It is a capsule history of automatic computing from the Chinese abacus (1100 B.C.) through the mid-fifties in the U.S., the development in England, Germany, Belgium, Denmark, France, Italy, Yugoslavia, the Netherlands, Norway, Austria, Poland, Rumania, Sweden, Switzerland, USSR, Czechoslovakia, and Hungary. Other countries are mentioned and references to pertinent literature given, only to illustrate this worldwide effort and to show the impossibility for achieving completeness in this enumeration. A selection of digital computer literature is annotated and most journals which deal primarily with applications of digital computers are listed. Review journals are also given. The bibliography lists 697 titles, almost 30 percent of all titles combined in the entire 22 articles.

While few will read the entire book, it should be interesting to many who wish to inform themselves about those disciplines of computer science and developments outside their own activities.

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This book is concerned with the systems analysis and design aspects of computer utilization in business data processing systems. The reader is assumed to have a general understanding of programming business problems on conventional high-speed computers. The major emphasis has been placed on batched processing techniques for magnetic tape computers with fixed word length and word-addressed memory.

The book is divided into three parts. Part one deals with the conventional data processing procedures; namely, file-maintenance operations, conversion runs, sort-merge runs, editing runs, and computation runs. Part two discusses the principles in the design of input and output documents; subjects covered include form design of source documents, encoding of information, card design, paper tape design, and optimal utilization of printer. Part three delineates the major steps entailed in the design and installation of a computer data-processing system. A glossary and a list of references are given at the end of the book.

The authors have provided an excellent synthesis and elucidation of practical guidelines, techniques, and examples. Notwithstanding the profusion of literature on the subject of business systems, this book will be of inestimable use to anyone interested in applications programming.

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*Analogue Computing at Ultra-High Speed* is a treatment of the development of a high-speed, iterative, analog computer. The use of analog computing machinery at very high repetitive rates provides fast solutions for partial differential equations and integral equations which, with more conventional computers, are laborious to obtain and require a great deal of time.

Part I of this book is a report on investigations which were pursued in an effort to show that the art of analog computation was only in its infancy rather than having its possibilities already exhausted. The design of these explorations was unique in that information-theory concepts were applied to a continuum, whereas the usual application of these theories is to a discrete system. Early in the report, the information content of existing analog computers was evaluated and compared with that of contemporary digital computers. The analog computers were found to be sadly lacking in information content; however, it was shown that theoretically it was possible for an analog computer to compare favorably with digital computers in information content. Requirements for such a computer were then advanced.

Part II of this report is devoted to descriptions of the characteristics and design of the computing elements, displays, and measurement techniques used in the experimental computer. Since communication across the man-machine interface is of
prime importance in making use of information, Chapter 9, which deals with multidimensional displays, is a particularly important and interesting one. The laboratory breadboard model of this computer has been constructed and its use has provided much knowledge which made it possible to organize and write Part III.

The third portion of the report is concerned primarily with the mathematical problems associated with using computers to solve equations. Nine chapters are devoted to mathematical problems of computation, and solutions to some of these problems are suggested. The treatment is thorough and quite practical—being restricted to the area of obtaining solutions rather than an analytical treatment of characteristics and behavior. The solution of partial differential equations is given the greatest attention and is extremely well done. Other areas of interest include ordinary linear and non-linear differential equations, integral and integro-differential equations.

This book might well be considered good study material for graduate students in mathematics, engineering, and the physical sciences, for it contains a great wealth of information concerning the equations which describe physical phenomena. However, it should not be considered a textbook in any sense of the word. It will be of some value to practicing computer design engineers and of great value to anyone who programs and operates analog computers. The treatment of partial differential equations may be of aid to digital programmers also, since truncation error and high order difference techniques are treated. The real value of the book to an individual can be determined only by reading it. This presents no chore, however, as it is written in a fluid style which is easily read and yet in which the concepts are precisely stated. The authors have indeed performed a great service to analog computation, even though the equipment which they designed and built is now obsolescent in many respects.

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This book does quite well what it set out to do, i.e., to provide a self-instructional programming manual for the IBM 1401 data processing system. It is intended to be used by readers with no programming experience, and the material is explained in such detail that it should be effective in its purpose.

The material is organized in ten units, each subdivided into lessons. Student problems and answers are provided with each lesson, and a general quiz with answers appears at the end of each unit except the tenth, where a quiz with answers for the whole course is presented. One learns to program by doing, and the author implements this philosophy by the well graduated set of student problems throughout the book.

This manual starts off with basic machine language programming for a 1401 system limited to punched card input. The notion of symbolic programming is introduced in Unit V, and throughout the remainder of the book programming is discussed in these terms. Programming for a 1401 system utilizing magnetic-tape
input is introduced toward the end. This book does not purport to cover all the fine details of programming for a 1401 system; for this the reader is referred to the 1401 reference manual (A24-1403) published by IBM.

This manual fills a real need in the training of beginning programmers and is well worth the publisher's price. 

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