Some Additional Factorizations of $2^n \pm 1$

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Herein are set forth some details of three new factorizations of integers of the form $2^n \pm 1$.

The first of these is the complete factorization of $2^{119} - 1$, which possesses as algebraic factors the Mersenne primes $2^7 - 1 = 127$ and $2^{17} - 1 = 131071$. The quotient is known to be divisible by 239 and 20231. There then remains the factorization of the integer

$$N = 82 57410 95583 43357 90279,$$

which was proved composite by E. Gabard of Poitiers, France.

The method of factorization employed was that described by Kraitchik [1] and used by the writer in a previous factorization [2]. Briefly, the procedure consists of exhibiting the integer $N$ as the difference of two squares $a^2 - b^2$, where the integer $a$ is suitably restricted by a process of exclusion based on a knowledge of several quadratic residues of $N$.

In this manner the representation $a = 1019592x + 90874619060$ was obtained, and this was found to yield a square value for $a^2 - N$ when $x = 6051$; whence we obtain the factorization
\[ N = 62983048367 \cdot 131105292137, \]
which completes the factorization of $2^{119} - 1$.

The second factorization is that of $2^{129} + 1$, which has the algebraic factor $2^{43} + 1 = 3 \cdot 2932031007403$, and the quotient is divisible by 3 and 1033. The remaining factor
\[ N = 249 \cdot 66522 \cdot 25083 \cdot 17105 \cdot 80243 \]
was also proved composite by E. Gabard. In this case the same method of exclusion leads to the representation
\[ a = 133128z + 158008074298, \]
which corresponds to a square value of $a^2 - N$ when $x = 57734583$. Accordingly, we obtain the following decomposition into prime factors:
\[ N = 1591582393 \cdot 15686603697451, \]
and the factorization of $2^{129} + 1$ is thus complete.

The last factorization considered here is that of $2^{141} + 1$, which is divisible by $2^{47} + 1 = 3 \cdot 283 \cdot 165768537521$. The quotient, $2^{94} - 2^{47} + 1$, is divisible by $3 \cdot 1681003$, and the resulting integer is
\[ N = 39 \cdot 27623 \cdot 49394 \cdot 29899 \cdot 21473. \]
Here the representation $a = 636192x + 62673972097$ was found, in which the value $x = 16720$ was discovered to result in a square value for $a^2 - N$. Hence,
\[ N = 35273039401 \cdot 111349165273, \]
which completes the factorization of $2^{141} + 1$.

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