REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS


This is a revised, enlarged, and reset edition of a well-known, popular mathematical handbook.

As in the earlier editions, the book is divided into two parts; namely, Part One, "Formulas, Definitions, and Theorems from Elementary Mathematics"; and Part Two, "Tables".

The current trend in mathematical education is reflected in this latest edition in the inclusion in Part One of sections devoted to sets, logic, algebraic structures (including Boolean algebra), number systems, matrices, and statistics.

The second part of the book consists of 39 tables, generally to 5D or 5S. The tables in the third edition have been retained and partially rearranged. These tables include natural and common logarithms, natural and logarithmic values of trigonometric functions, exponential and hyperbolic functions, squares, cubes, square roots, cube roots, reciprocals, circumferences and areas of circles, factorials and their reciprocals, binomial coefficients, probability functions, interest and actuarial tables, the complete elliptic integrals $K$ and $E$, common logarithms of the gamma function, factors and important constants. Two additional tables have been included in this new edition: one of these, which constitutes part of Table 32, contains 4D square roots of certain common fractions; the other gives 3D values of the $\chi^2$ distribution. One error exists in the latter table: the entry for the 1% point ($\epsilon = 0.01$) and three degrees of freedom ($m = 3$) should read 11.345 instead of 11.341. This has been tabulated correctly by Fisher and Yates [1].

Another improvement over the earlier editions is the inclusion of a glossary of symbols and an index of numerical tables, as well as an appropriately enlarged subject index.

A further feature is a "table locator," which enables the user to readily locate any one of the tables by merely flexing the pages. An abbreviated list of mathematical symbols, abbreviations and the Greek alphabet are now presented on the inside of the front cover and the facing page, respectively.

This improved and expanded edition should be even more useful than its predecessors.

J. W. W.


The five tables in this report are concerned with the distribution of the first six million primes, from $p_1 = 1$ (which is here counted as a prime) to $p_{600000} = 104395$.
289. This study followed the printing of these primes [1] from the same tape. For convenience of description, let us define the difference $\Delta_i$ by

$$\Delta_i = p_{i+1} - p_i.$$ 

Table 1 lists, for each $\Delta = (1)220$, (a) the first $p_i$ such that $\Delta_i = \Delta$ (if one such exists), and (b) the number of differences $\Delta_i = \Delta$ within this range.

Table 2 tabulates all 6433 of the pairs $\Delta_i$ and $p_i$ such that $\Delta_i > 100$.

Table 3 lists, for each interval $50000K < p < 50000(K + 1)$, $K = 0(1)2087$, the following five quantities.

(a) The number of $p_i$ therein.
(b) The largest $p_i$ therein.
(c) The number of $p_i$ therein such that $\Delta_i = 2$ and $\Delta_{i+1} = 4$.
(d) The number of $p_i$ therein such that $\Delta_i = 4$ and $\Delta_{i+1} = 2$.
(e) The number of $p_i$ therein such that $\Delta_i = 2$, $\Delta_{i+1} = 4$, and $\Delta_{i+2} = 2$.

The last interval, $K = 2087$, is incomplete, and has instead an upper bound of $p_{6000000}$. Through an oversight, this value of $p_{6000000} = 104395289$ is nowhere indicated in the entire report. Cumulative counts are unfortunately not given, except for the grand totals: 6000000 primes in (a), 57658 triples in (c), 57595 triples in (d), and 4917 quadruples in (e).

Table 4 tabulates all 4917 of the $p_i$ that initiate the quadruples just mentioned.

Finally, for the intervals $100000K < p < 100000(K + 1)$, $K = 0(1)1042$, Table 5 lists the number of $p_i$ therein such that $\Delta_i = 2$ (the twin primes). Again, cumulative counts are lacking, except for the grand total of 456998 twin pairs up to 104300000.

With some hack work, though (it would have taken the machine about 0.01 sec.) the reviewer finds that up to $10^8$ there are 440312 twins, 55600 triples of the type (c) above, 55556 triples of type (d), and 4768 quadruples of type (e).

No attempt is made in the report to compare these statistics with the famous Hardy-Littlewood conjectures, and it would be too tedious to do this thoroughly now by hand. Some checks are, however, easily made. By the conjectures, primes $p$ such that $p + 2$ is also prime should be equinumerous with primes $p$ such that $p + 4$ is also prime, but only one-half as numerous as primes $p$ such that $p + 6$ is prime. In Table 1 we find 457399 primes with $\Delta_i = 2$, 457478 primes with $\Delta_i = 4$, and 798900 primes with $\Delta_i = 6$. Adding to the last number the 57658 + 57595 triples of Table 3, we do find the pleasing ratios:

$$\frac{457399}{457478} = 0.99983, \quad \frac{457399}{914153} = 0.50035.$$ 

Further, one would expect about

$$1.32032 \cdot \frac{10^8}{(\log 10^8)^2} = 389.11$$

pairs of twins in each block of $10^8$ numbers near $10^8$. One finds in Table 5 that the ten such blocks closest to $10^8$ do contain 406, 385, 363, 405, 409, 377, 425, 378, 379, and 392 twins, respectively.

Much more detailed comparisons have been previously given by D. H. Lehmer [2] of these distributions in the smaller range of primes less than $37 \cdot 10^6$. 
The reviewer is pleased to announce that the belated appearance of this review is solely due to the belated appearance of a review copy in the editorial office of this journal.

D. S.


This is the fourth edition of a compact textbook on group theory, which first appeared in the year 1921. Though there is probably not a single sentence in common to the two editions, the book has retained the pedagogical skill of the exposition and of the many exercises (now 151) illustrating the concepts developed in the text in unbroken sequence.

The content of the present edition may be characterized as a substantial portion of the union of the textbooks on group theory by A. Kurosch and by the reviewer (first edition) emphasizing basic concepts, but not considering transfer theory, lattice theory, extension theory, theorems on not finitely generated abelian groups, etc. The attractive historical references and sections on geometric groups of the first edition have given way to a treatment of group theory governed entirely by the restrained abstract viewpoint of the thirties and forties. The group tables appended to the book are very useful for teaching and self-study purposes.

HANS ZASSENHAUS

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75[G].—C. A. CHUNIKHIN, Podgruppy Konečnych Grupp (Subgroups of Finite Groups), Nauka i Technika (Science and Technology), Minsk, 1964, 158 pp., 21 cm. Price 57 kopecks.

In honor of the ninetieth anniversary of Sylow's theorems (1872) the author devotes a four-chapter monograph to the exposition of the known theorems of finite group theory about the existence of subgroups of given order of a finite group $G$, starting with Sylow's theorem on the existence of $p$-subgroups for every $p$-power divisor of the order of $G$ and the conjugacy of the Sylow $p$-groups under $G$, continuing with P. Hall's theorems on II-subgroups of solvable groups (II a given set of prime numbers), and concluding with a detailed exposition of the author's results contained in more than 30 research papers.

In Chapter I the known generalizations of Sylow's theorems on $p$-groups to the corresponding theorems on II-groups are studied. In Chapter II the factorization of the finite groups utilizing the indices of the principal or composition series is treated. In Chapter III the construction of the subgroups of a finite group, with the help of the "indexials", is discussed. Given a principal chain $G = G_0 \supseteq G_1 \supseteq \cdots \supseteq G_\mu = 1$ ($\mu \geq 1$) of $G$ and a chain of subgroups $F_i \mid G_i$ of $G_{i-1} \mid G_i$ for $i = 1, 2, \cdots, \mu$ such that any conjugate of $F_i \mid G_i$ under $G \mid G_i$ already is a conjugate
under \( G_{i-1} \mid G_i \), then the factorization \( h = \prod_{i=1}^{r} (F_i ; G_i) \) is called an indexial. Chapter IV deals with complects of non-nilpotent subgroups of \( G \). A \( \Pi \)-complect is defined as a mapping \( \sigma \) of \( \Pi \) into the subgroup set of \( G \) such that the order of the image of \( \sigma p \) is divisible by \( p \) and the images of different members of \( \Pi \) are non-isomorphic.

HANS ZASSENHAUS


The term “Fedorov group” is used in this book to denote what is more commonly called a space group, i.e., an infinite discrete group of Euclidean motions and reflexions of 3-dimensional Euclidean space which leave no point and no line or plane invariant. Space groups are fundamental in crystallography, and there are, in all, 230 of them. The subgroup, \( H \), of any space group, \( G \), which consists of the Euclidean translations contained in \( G \), is an Abelian normal subgroup of \( G \), and the factor group \( G/H \) is one of 18 different finite groups. The integral unimodular 3-dimensional representations of these finite groups define what are commonly known as crystal classes, of which there are in all a total of 73. The elements of \( H \) define a crystal lattice, which may also be denoted by \( H \), and the lattice reciprocal to \( H \) is denoted by \( H^* \). The lattice \( H^* \), combined with the factor group \( G/H \), furnishes a space group \( G^* \), and certain space groups \( G \) have the property that the transform of a given vector, \( u \), from the fundamental region of \( G^* \), by any element of \( G \) differs from \( u \) by an element of \( H^* \). Every vector \( u \) from the fundamental region of \( G \) determines an irreducible unitary representation of \( G \), and when \( G \) has the property mentioned, this representation is termed basic. It is these basic representations which are tabulated, for all 73 crystal classes, in the present book. A short indication of how to determine nonbasic representations from the basic representations is furnished.

The book is carefully printed and should be very useful to anyone working in the field of crystallography.

F. D. MURNAGHAN

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This book, the proceedings of an advanced seminar on matrix theory held at the Mathematics Research Center, University of Wisconsin, on October 14–16, 1963, is a collection of the following six papers:


Briefly, each author makes a penetrating study of a particular facet of matrix theory, and each author unifies and summarizes the important results in his area. This book is, without question, a very valuable collection of results and references in modern matrix theory, and the editor, Hans Schneider, is to be congratulated for his successful efforts in bringing together such distinguished researchers and for editing the final results.

R. S. V.


As the title implies, this report deals with the convergence of Jacobi methods for the determination of the eigenvalues (and eigenvectors) of real symmetric matrices. Specific methods considered are the classical Jacobi method, the cyclic-Jacobi method, and the threshold-cyclic-Jacobi method.

For a number of cyclic methods a new proof of convergence is given which indicates quadratic convergence for a matrix with distinct eigenvalues. In the case of multiple and close eigenvalues, the classical Jacobi method and the cyclic-threshold-Jacobi method are examined. It is shown, for these methods, that convergence is improved for matrices with multiple eigenvalues. Close eigenvalues also improve the convergence.

Some numerical examples are discussed. For matrices of low order, high-accuracy computations were performed and the results obtained confirm the theoretical results about the rates of convergence of the methods employed.

For matrices of higher order, computations were performed with ordinary accuracy. Results obtained permit a comparison of the methods, with regard to speed and accuracy, and thus permit an evaluation of the methods for the practical determination of all eigenvalues and eigenvectors of a real symmetric matrix.

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Professor Fano’s valuable textbook on modern information theory (for, certainly, it is not a research monograph) is the considered outgrowth of nearly ten
years' effort in teaching the subject at M.I.T. and of associating with the men who founded the physico-mathematical theory—notably, C. E. Shannon, N. Wiener, A. Feinstein, P. Elias, and J. M. Wozencraft. He follows Shannon's School [1], [2], [3], [4], [5], with its emphasis on reliable communication in the presence of noise, rather than Wiener's School [6], [7], [8], with its emphasis on the theory of extrapolation and prediction. Since the book is directed to graduate-level engineers, there is no pretension to the full rigor available in other, more mathematical, treatises, such as those by Khinchin [2], Feinstein [3], and Wolfowitz [4]. Frequently, the author uses refreshing physical insights to motivate the careful proofs of theorems.

The nine chapters of the work, which is reproduced by photo-offset, include such basic topics as: "a measure of information," where the functional form of the entropy function is obtained by using a geometrically oriented proof, in distinction to the arithmetical argument given in Feinstein [3], which uses the unique factorization into prime numbers; "the optimum encoding procedure of D. Huffman"; "the weak and strong laws of large numbers"; "the Sampling Theorem"; "Shannon's Coding Theorem and its weak converse" (Wolfowitz' results [4] are not given); and "various estimates for multinomial distributions" (previously unpublished results from Shannon's 1956 Seminar on Information Theory). Most results are stated for finite probability spaces, and Lebesgue integration is completely ignored even in the continuous cases.

The book terminates with a number of well-chosen problems, which will challenge most first-year graduate students in engineering. Further, it contains a storehouse of inequalities to be generalized.

Albert A. Mullin
Lawrence Radiation Laboratory
University of California
Livermore, California


The first eight nonzero coefficients in the power-series expansion of

\[ \phi(z) = \sec \pi z \sin \pi \{(1 - 4z^2)/8\} \]

are given (multiplied by a factorial) in the form of polynomials in \( \pi \) with integer coefficients, multiplying the numbers \( \cos \pi/8 \) and \( \sin \pi/8 \). Numerical values of these coefficients to 10D and 20D, respectively, have been given by Lehmer [1] and
Haselgrove and Miller [2]. The integer coefficients are related to the Euler numbers and were calculated from recurrence relations.

Author's summary


This report contains 10D tables of both complete and incomplete elliptic integrals of all three kinds in Legendre's form. Table I consists of such decimal approximations to $F(\phi, k)$ and $E(\phi, k)$ for $\phi = 5^\circ(5^\circ)90^\circ$ and $k = 0(0.01)1$, while Table II gives similar information for $k^2 = 0(0.01)1$. Table III gives 10D values of $\Pi(\phi, \alpha^2, k) = \int_0^\phi (1 - \alpha^2 \sin^2 \theta)^{-1/2} (1 - k^2 \sin^2 \theta)^{-1/2} d\theta$ for $\phi = 5^\circ(5^\circ)80^\circ(2.5^\circ)90^\circ$, $k^2 = 0(0.05)0.9(0.02)1$, $\alpha^2 = -1(0.1) -0.1, 0.1(0.1)1$ (except that when $\alpha^2 = 1$, $\phi$ extends only to $87.5^\circ$). The authors' description of the tables contains some minor errors with reference to the ranges of the parameters.

To insure reliability in the final rounded values, the underlying calculations were performed to 16S on an IBM 1620, using a subroutine based on Gauss's transformation [1], which is given for all three integrals in the accompanying explanatory text. A discussion of the several checking procedures applied to the tabular entries is included; however, the problem of interpolating in the tables is not considered.

The authors refer to tables of elliptic integrals of the third kind by Selfridge & Maxfield [2] and by Paxton & Rollin [3], but appear to be unaware of the extensive 7S tables of Beliakov, Kravtsova & Rappaport [4].

The present tables contain the most accurate decimal approximations to the elliptic integral of the third kind that have thus far been published, and constitute a significant contribution to the tabular literature.

J. W. W.


82[L, Z].—Jurgen Richard Mankoff, Über die periodischen Lösungen der Van der Polschen Differentialgleichung $x + \mu(x^2 - 1)x + x = 0$, Forschungsberichte des Landes Nordrhein-Westfalen, Nr. 1307, Westdeutscher Verlag, Opladen, 1964, 55 pp., 24 cm. Price DM 41.

This expository monograph analyzes the Van der Pol equation. The work is concerned primarily with the free-vibration equation. The case corresponding to large values of $\mu$ is discussed in a very brief section.
One section of the investigation applies general results from nonlinear vibration theory to the Van der Pol equation. Another section develops a perturbational technique to obtain approximations to periodic solutions.

The final section discusses analogue computer techniques to generate solutions of the equation.

Paul Brock
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This report contains numerical tables of the real and imaginary parts of the diffraction integral

\[ F = \int_{-\infty}^{\infty} f(z) \exp(-i\pi z^2/2) \, dz \]

in the following three cases:

I. Linear tapered aperture,

\[ f(z) = \left( \frac{1}{2} + \frac{\beta}{2} (z - z_0) \right), \quad z_0 - \beta^{-1} \leq z \leq z_0 + \beta^{-1}, \]

\[ = 0, \quad -\infty \leq z \leq z_0 - \beta^{-1}, \]

\[ = 1, \quad z_0 + \beta^{-1} \leq z. \]

II. Cubic tapered aperture,

\[ f(z) = \left( \frac{1}{2} + \frac{\beta}{2} (z - z_0) - \frac{2\beta^3}{27} (z - z_0)^3 \right), \quad z_0 - \frac{3}{2\beta} \leq z \leq z_0 + \frac{3}{2\beta}, \]

\[ = 0, \quad -\infty \leq z \leq z_0 - \frac{3}{2\beta}, \]

\[ = 1, \quad z_0 + \frac{3}{2\beta} \leq z. \]

III. Exponential tapered aperture,

\[ f(z) = \{1 + \exp[-2\beta(z - z_0)]\}^{-1}, \quad -\infty \leq z \leq \infty. \]

In Tables 1 and 2, associated with Cases I and II, respectively, the integral \( F \) is tabulated to 5D for \( \beta = 0.2, 0.5, 1, 2, 5, 10 \), and \( z_0 = 0(0.1)5 \). Table 3, associated with Case III, consists of 5D approximations to \( F \) for \( \beta = 0.5, 1, 2(0.5)3(1) \) \( 5, 10, \infty \), and \( z_0 = 0(0.1)5 \).

The authors note that, in all three cases, as \( \beta \) tends to infinity, \( F \) approaches \( \frac{1}{2}(1 - i) - F_0(z_0) \), where \( F_0(z) = \int_0^z \exp(-i\pi x^2/2) \, dx \) is the Fresnel integral. This fact permitted a check on the entries in Table 3 corresponding to \( \beta = \infty \), through a comparison with the corresponding data in the tables of Wijngaarden and Scheen [1]. Complete agreement was found.
The authors state that these tables have been designed for use in the numerical solution of diffraction or scattering problems involving objects with tapered distributions of density.

J. W. W.


This informal monograph presents a comparative discussion of four related approaches to finding self-similar solutions to boundary-value problems in engineering. These approaches are called the “free parameter” method (Riabouchinsky’s method), “separation of variables” (reference is made to D. E. Abbott and S. J. Kline), “group-theory methods” (developed by A. J. A. Morgan, following ideas of the reviewer), and (lastly!) “dimensional analysis” (following L. I. Sedov).

A modest mathematical background is assumed; for instance, the notion of a group of transformations is explained carefully. Physical, philosophical, and experimental questions are avoided; the emphasis is on working out in detail solutions to specific boundary-value problems, some of which have been published only recently in the periodical literature on fluid flow and heat transfer.

Though in no sense profound, the book should be helpful in introducing various recent extensions of Rayleigh’s “method of similitude” to wider circles of mathematically-minded scientists.

Garrett Birkhoff
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This is one of the few books which presents a balanced approach to both analog and digital computing as used in engineering at the elementary level; the more imaginative and exciting uses are simply ignored. Thus, it is a good, solid, pedestrian text for a beginning computing course in engineering.

The first chapter, entitled Basic Analog-Computer Theory, covers the pertinent electronics sufficiently to make the operation clear and avoids getting mired in details. It goes on to show, by many clear examples, how to use the computer to solve practical problems.

The second chapter, Simulation of Discontinuous and Nonlinear Physical Systems, supplies all necessary details.

The third chapter, The Role of Analog Computers in Engineering Analysis, is a very good one, as it makes its points very well.

Chapter 4, The Digital Computer and the FORTRAN System, is simply an introduction to FORTRAN.

Chapter 5, Numerical Methods for Use with the Digital Computer, covers finding zeros, solution of simultaneous linear equations, eigenvalues, integration, differ-
entiation, and the solution of ordinary differential equations. The part on partial differential equations is a bit brief but does convey much information that is needed by the engineer.

All in all, it is a fine text for engineers.

R. W. Hamming

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The inclusion of the articles by Letov and Feldbaum make the volume of particular importance. Not only are these authors outstanding in their domains, but, in addition, they are able to give the American reader an overall view of both American and Russian work in these new areas.

The book is highly recommended for students and teachers, and, in general, for all those who want to understand what some of the problems and achievements of modern control theory are.

Richard Bellman

The RAND Corporation
Santa Monica, California


In this monograph the author proves 21 coding theorems, 16 strong converses, and eight weak converses for different kinds of channels.

There are 10 chapters: one on the discrete memory-less channel, with particular treatment of the binary symmetric channel and the finite-state channel with state calculable by both sender and receiver or only by the sender; another chapter on compound channels (classes of channels) including channels with feedback; two chapters on finite- and infinite-memory channels; one on the semicontinuous memory-less channel; and one on continuous channels with additive Gaussian noise.

Since its publication the monograph has had a considerable and positive influence on mathematical work on coding theory. Many of its results are due in part to, or have been refined by, the author. The proofs are clear and elegant.
Though the book is perhaps somewhat too dogmatic in judging, for instance, upon the relative merit of weak and strong converses, and though it is hardly an “Ergebnisbericht”—lacking an index and having only a one-page bibliography—it does well achieve the purpose stated in its preface, “to provide, for mathematicians of some maturity, an easy introduction to the ideas and principal known theorems of a certain body of coding theory.”

Volker Strassen

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Editorial note: This book has also been published by Springer-Verlag, Berlin in 1962 as v. 31 of the new series of Ergebnisse der Mathematik und ihrer Grenzgebiete.


There are altogether 28 papers in this collection, of lengths varying from four pages to 59 pages, and covering a wide range of topics. The longest paper is the first, by Bakhvalov, on Monte Carlo methods. Other topics include probabilistic error estimates in the solution of differential equations, methods of quadrature for the solution of singular integral equations, difference methods in regions of instability of systems of linear differential equations, several papers on differential-difference equations, asymptotic solution of integro-differential equations, nonlinear boundary-value problems, and a group of papers on special applications in the study of waves, diffraction, and other topics. An overall evaluation would be difficult, but it should be a useful collection for specialists.

A. S. H.


This book is an outgrowth of a Statewide Lecture Series on Applied Combinatorial Mathematics offered by the University of California in the spring of 1962. In it are collected eighteen expository articles which are applied, combinatorial, and mathematical in varying degrees and proportions and which together cover a wide range of subjects. Thoughtfully written and handsomely presented, accompanied with diagrams and extensive up-to-date bibliographies, the articles form a valuable addition to the literature. For many readers they will serve as enjoyable introductions to certain fields of lively current interest; for others they will call attention to problems not yet solved. Since most readers will be especially interested in particular articles, we list here the titles and authors:

2. Techniques for Simplifying Logical Networks, by Montgomery Phister, Jr.
5. Pólya’s Theory of Counting, by N. G. de Bruijn.
8. Graph Theory and Automatic Control, by Robert Kalaba.
10. Stopping-rule Problems, by Leo Breiman.
11. Combinatorial Algebra of Matrix Games and Linear Programs, by Albert W. Tucker.
17. Combinatorial Principles in Genetics, by George Gamow.
18. Appendices, by Hermann Weyl.

G. N. Raney

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Storrs, Conn.


In this collection of papers we are offered a banquet whose menu includes a number of main dishes plus many side dishes and tidbits. That some of these offerings do, and some do not, appeal to the reviewer (the taster) is only to be expected.

In the opinion of the reviewer, the best major dish is Hurwitz’ paper, “On the Conditions Under Which an Equation Has Only Roots with Negative Real Parts”, which is a model of lucidity by a major mathematician and mathematical artist. The topic is also of practical importance for control technique. On the other hand, the worst tidbit seems to be the paper by the two banquet organizers on the work of Liapunov and Poincaré (four pages, one consisting of a list of references).

A few remarks will now be made concerning some of the main dishes.

With considerable propriety, the first paper in this “control” collection is Clerk Maxwell’s “On Governors” (1868), the first in which control problems were dealt with scientifically and with appropriate mathematics. However, for lucidity we would have preferred the treatment by Pontryagin in his book on differential equations, or Vishnegradski’s very early and independent treatment (1876) of Watt’s steam-engine governor.

H. Bateman’s “The Control of an Elastic Fluid” (1945) is the longest of the collection and is, in fact, too long for its content.

H. Nyquist’s “Regeneration Theory” is not too lucid, but is interesting as the history of a noted application of mathematics to technology.

In H. W. Bode’s paper “Feedback—The History of an Idea”, the all important role of feedback in the development of long-distance telephony is interestingly described in full detail.
B. van der Pol's "Forced Oscillations in a Circuit with Nonlinear Resistance (Reception with Reactive Triode)" discusses a highly interesting application by the author of the famous van der Pol equation.

The paper by N. Minorsky, entitled "Self-Excitation in Dynamical Systems Possessing Retarded Action", represents pioneer work on retarded action.

Here an appropriate main dish would have been the omitted paper by D. Bushaw entitled "Optimal Discontinuous Forcing Terms", which appeared in Contributions to the Theory of Nonlinear Oscillations, vol. IV, Annals of Mathematics Studies, no. 41, pp. 29–52. This is one of the earliest papers on optimization.


Attractive side dishes are: "Time Optimal Control Systems", by J. P. LaSalle, which is a high-grade contribution on optimization in the United States, originally published in 1959; and "On the Theory of Optimal Processes", by Boltyanskii, Gamkrelidze, and Pontryagin.

This reviewer hopes that his culinary description has not masked the fact that the editors have presented (à la earlier Bellman) quite a noteworthy collection of control papers—not an easy choice in such a popular field and from such varied directions. A suggestion to the editors for a future edition: Do not hesitate to present extracts from long, but classical, memoirs such as those of Poincaré and Liapunov.

Detailed references to many other articles are included in this collection.

SOLOMON LEFSCHETZ

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91[X].—Harry M. Colbert, Asymptotic Expansion for the Characteristic Values of Mathieu's Equation, ms. of 5 pp. + 1 table of 3 pp., deposited in UMT File.

The author repeats the well-known BWK procedure for obtaining the asymptotic expansion of

$$ y'' + [A - kq^5(x)]y = 0. $$

He then specializes this equation to that of Mathieu, and uses the same procedure as Ince did to obtain the characteristic values. The equation is taken actually in the form

$$ y'' + (a + 2q - 4q \sin^2 x)y = 0, $$

which becomes identical with the usual form of Mathieu's equation if $q$ is replaced by $-q$.

For this specialized equation, the author tabulated $G_n$ and $c_m$, occurring in the series

$$ a_r \sim b_{r+1} = -q \sum_{n=1}^{\infty} \frac{\epsilon_n(\nu)}{2^{n-1} q^{(n-1)/2}}, $$

$$ \epsilon_n(\nu) = \sum_{m=1}^{(n+\nu)/2} c_m \nu^{2n-1-\nu}, $$
where \( \nu = 2r + 1, \sigma = 1 \) if \( n \) is odd, and \( \sigma = 0 \) if \( n \) is even. The integer coefficients \( c_j \) are given to a maximum of 28 digits, corresponding to \( n = 1(1)28 \).

This reviewer doubts that the table will be very useful, since the asymptotic expansion gives little accuracy unless the order is very low or \( q \) is extremely large. It should be noted that for the first 15 orders, the characteristic values of Mathieu’s equation have been tabulated from \( q = 0 \) to \( q = \infty \). (See references 20.53 and 20.58 on p. 746 of the NBS Handbook [1].)

Nevertheless, no one can predict with certainty that a table will find no application.

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The author’s aim, as stated in the Preface, is to produce a textbook that will appeal to mathematicians. In order to do so, he has tried to suppress the art and emphasize the mathematical discipline by stressing unifying principles. To achieve a balance between the theoretical and practical, he has made a clear-cut distinction between algorithms and theorems.

The table of contents shows the range of material covered:
Introduction.
Chapter 1. What is numerical analysis?
  2. Complex numbers and polynomials.
  3. Difference Equations.
Part One. Solution of Equations.
  4. Iteration.
  5. Iteration for Systems of Equations.
  8. The Quotient-Difference Algorithm.
Part Two. Interpolation and Approximation.
  9. The Interpolating Polynomial.
  15. Number Systems.

Bibliography.
Index.
It will be observed that linear algebraic equations and matrix theory have been omitted "... because I feel that this topic is best dealt with in a separate course". It should also be observed that Part Three, on Computation, covers less than 10% of the entire book.

The book is written in the author's usual clear, elegant style and achieves what he set out to do.

R. W. Hamming


This book is a well-organized introduction to approximation theory. The presentation begins with fundamentals and progresses to the frontiers of present knowledge in many areas. The book is rather short, but is organized so as to present a surprisingly large number of results. This is accomplished by the use of many "small type" sections where references to proofs, rather than proofs, are given. The level is about the same as the book of Achieser (i.e., roughly second-year graduate level). The computational problem for Tchebycheff approximation is considered in some detail and depth.


There is a well-selected bibliography of about 160 items. It is up to date, and includes both Russian and western literature.

In conclusion, this book is highly recommended as an introduction to modern approximation theory.

John R. Rice

Purdue University
Lafayette, Indiana


This is a short book devoted to various applications of the elementary inequalities, which is to say, those associated with the names of Cauchy-Schwarz, Hölder, Minkowski, Jensen, et al., and to the derivation of numerous special inequalities for the elementary functions of analysis. There is also a brief chapter on geometric inequalities.

It reads well, is attractively printed, and is highly recommended. It will be
particularly useful to analysts of all kinds who need an unconventional inequality at a critical juncture in a proof, to high school and college teachers who want interesting problems for their classes, and to students who wish to practice their analytic skills.

Richard Bellman


Most of the volume is concerned with the problem of best approximation of a given real function $f$ by a linear combination of given real functions $\phi_1, \phi_2, \ldots, \phi_n$ over a closed interval or over a finite (real) point set. Special cases emphasized are the classical ones of best approximation by polynomials and by trigonometric polynomials, but the more general setting is stressed to a much larger degree than that common in other texts.

The first chapter (entitled Fundamentals) is an introduction to the subject, in which the author seeks to give the reader a feeling for approximation theory and its methods. Also some theorems relating to the foundation of the theory are proved in this chapter.

The second chapter is a brief introduction to the subject of orthogonal systems of functions. The author succeeds in clearly showing the advantages of least-squares approximation from the points of view of ease of computation and simplicity and elegance of theory.

The third chapter deals quite extensively with the theory of best Tchebycheff approximation.

Chapter 4 discusses the problem of best approximation in the $L_1$ norm.

Chapter 5 (The Weierstrass Theorem and Degree of Convergence) deals mainly with classical results of Weierstrass, Fejér and Jackson, namely, those theorems which (together with Tchebycheff’s work) form the classical backbone of approximation theory in the real domain.

The sixth chapter (Computational Methods) gives a survey of methods for the actual construction of best (or merely good) approximations. Two of the methods discussed are the method of descent and linear programming.

The book is rich in problems (of which some serve as exercises and others as an integral part of the text) and in illustrations. It is suitable both as a reference and as a classroom text.

As to the material presented, the book combines classical results with recent ones, including contributions of the author to approximation theory.

It is a highly valuable work that should attract many to study the theory of approximation and to contribute to it.

Oved Shisha

Applied Mathematics Research Laboratory
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This introductory textbook in numerical analysis presupposes mathematical preparation equivalent to the completion of a first course in the calculus, supplemented by some knowledge of advanced calculus and differential equations.

An unusual feature, which the author recognizes in the Preface, is the inclusion of a chapter on graphical and nomographic methods.

The book begins with a good introduction to approximate numbers and computational errors, followed by a well-motivated chapter on computation with power series and asymptotic series. Typical of the manipulation of power series therein is the derivation of recursion formulas for the Bernoulli numbers and for the logarithmic numbers (which are later identified with the coefficients in Gregory's integration formula).

Also included in this book are conventional treatments of interpolation (divided differences; the Aitken-Neville procedure; formulas of Lagrange, Newton, Gauss, Stirling, Bessel, and Everett), the numerical solution of algebraic and transcendental equations (regula falsi, the method of chords, iterative methods such as that of Newton-Raphson), numerical differentiation and integration in terms of differences and of ordinates (formulas of Gregory, Newton-Cotes, and Gauss), the numerical solution of ordinary differential equations (methods of Picard, Runge-Kutta, Adams, and Milne; use of power series), and curve fitting (method of averages, least squares).

In the opinion of the reviewer, the discussion of the solution of simultaneous linear equations is regrettably superficial. The procedure recommended by the author is Gauss elimination, although he does not identify it as such. The problem of the evaluation of errors in the solution arising from errors in the coefficients of the system of equations (which is considered in detail by several authors, such as Milne and Hildebrand) is here merely alluded to. Furthermore, iterative techniques, such as that of Gauss-Seidel, are omitted, and the existence of such procedures merely noted.

The text is supplemented by approximately 40 diagrams, 13 tables (all relating to numerical differentiation and integration), and a bibliography of 17 selected references.

Numerous illustrative examples appear throughout, and exercises for the student are appended to each of the principal sections in all nine chapters.

The reviewer has noted a total of 22 typographical errors, which are relatively minor and are obvious, except on page 38, where the numerator of the logarithmic number \( \log \) should read 8183 instead of 8193. (This number is listed correctly on page 260 and in the table on page 262.)

Although the computational procedures therein are intended to be carried out on desk calculators, this book should serve as an introduction to numerical analysis for those students interested in high-speed computers and their applications.

J. W. W.

Academic mathematics is principally concerned with the exact solution of ideally posed problems: practical mathematics requires mainly the approximate solution of problems deriving from models of acknowledged imperfection. The insistence on rigor necessary for the former is more often than not a positive hindrance when approaching the latter. In this book, which is written at the high school or freshman level, an attempt is made to acquaint the mathematician who wishes to be useful, with the facts of his life, not by brutal confrontation after he has taken his degree, but by preparation for them at an earlier age.

The book deals mainly with the solution of equations. By means of simple examples (a stone falling down a well, Achilles and the tortoise), the way in which equations arise in Physics and Engineering is illustrated. Iterative methods (the method of chords, Newton's method, etc.) are then discussed; their motivation is explained with the help of numerous diagrams, and some conditions for convergence are derived.

The standard of exposition is extremely high, and the book is attractively produced.

In view of the level at which the material is presented this book is hardly of interest to the research numerical analyst, nor will it command the direct attention of those teaching numerical analysis, but for the enterprising student and the inquisitive layman it is certainly a welcome addition to the literature.

P. Wynn

University of Wisconsin
Madison, Wisconsin


This book consists of the transactions of the symposium dedicated to Professor Langer and held at Madison, Wisconsin, May 4–6, 1964. Survey articles, as well as detailed presentation of recent results, are included. A careful reading of the book yields a very good idea of the methods of obtaining asymptotic solutions of differential equations, as well as their tremendous importance in the applications. Each article also contains a very good bibliography. The authors of the articles are: Clark, Erdélyi, Kazarinoff, Lewis, Lin, McKelvey, Olver, Sibuya, Turrittin, and Wasow.

Jack K. Hale

Brown University
Providence, Rhode Island


This book is the Proceedings of a conference on Computing Methods in Optimization Problems held at UCLA in January, 1964. The papers appearing in this volume will be reviewed individually, and, by necessity, these reviews must be brief.

In the first paper, entitled "Variational theory and optimal control theory," by Magnus R. Hestenes (pp. 1–22), a general problem in optimal control is formu-
lated and is shown to be equivalent to the problem of Bolza. This paper is an outline of the approach used by Pontryagin to establish first-order conditions. The author was one of the first to see the importance of optimal control problems in variational theory and was one of the first to formulate a general control problem.

The second paper, entitled "On the computation of the optimal temperature profile in a tubular reaction," by C. Storey and H. H. Rosenbrock (pp. 25–64), treats the problem of selecting the temperature profile and the final time so as to maximize the yield of a reaction for which the kinetic equations are linear, with coefficients depending on the time and the temperature profile used. The results of various computational methods tried and reported on herein lead the authors to believe that effective methods would consist of direct hill-climbing followed by the gradient method in function space or, perhaps even better, by an analog in function space of the Newton-Raphson method. Neither dynamic programming nor Pontryagin's maximum principle were found to be as good as other methods.

The next paper, "Several trajectory optimization techniques," appears in two parts. Part I, by R. E. Kopp and R. McGill (pp. 65–89), consists of a discussion of numerical methods for optimizing trajectories, wherein computational algorithms and the advantages and disadvantages of various procedures are reviewed. Part II, by H. Moyer and G. Pinkham (pp. 91–105), is a discussion of, and a report on, the application of these methods to problems of minimum time, low thrust, and circle-to-circle transfer, with details relating to techniques and experience.

"A steepest-ascent trajectory optimization method which reduces memory requirements," by R. H. Hillsley and H. M. Robbins (pp. 107–133), presents a method resulting in reduced memory requirements which obviate the need for tape operations on an IBM 7000 and leads to the conclusion that in-flight use by a guidance computer of limited memory capacity appears feasible.

"Dynamic programming, invariant imbedding and quasilinearizations: comparisons and connections," by R. Bellman and R. Kalaba (pp. 135–145), treats a simple class of variational problems from the three points of view stated in the title and discusses the possibilities of combining these methods.

In "A comparison between some methods for computing optimum paths in the problem of Bolza," by F. D. Faulkner (pp. 147–157), the method of steepest descent of Bryson and Denham is compared with the author's direct method. A modification of the Bryson-Denham method is presented. All three methods are said to work satisfactorily, subject to minor computational difficulties relative, for example, to convergence. The relative merits of these methods are discussed.

"Minimizing functionals in Hilbert space," by A. A. Goldstein (pp. 159–165), contains a generalization of constructive techniques used by Kantorovich and Altman for the minimization of quadratic functionals. Examples from control theory and from approximation theory in $L_1$ are discussed.

"Computational aspects of the time-optimal control problem," by E. J. Fadden and E. G. Gilbert (pp. 167–192), examines convergence difficulties inherent in methods proposed for the solution of this problem and suggests remedies. An example, including computational results, is given.

“Method of convex ascent,” by Hubert Halkin (pp. 211–239), discusses a computational procedure for the solution of a class of nonlinear optimal control problems. The method is based upon properties of the reachable set. An elementary description is followed by a detailed one giving theorems and refinements. An application of the method to the Goddard problem indicates its usefulness and power.

“Study of an algorithm for dynamic optimization,” by R. Perret and R. Rouxel (pp. 241–259), is a pragmatic approach, as the authors state, to the organization of a particular class of dynamical systems. An experimental computer has been constructed, and the experimental results are reported elsewhere.

The last three papers are concerned with the use of hybrid analog-digital computation, in an attempt to use the best features of each type of computer. These papers are entitled, respectively: “The application of hybrid computers to the iterative solution of optimal control problems,” by E. G. Gilbert (pp. 261–284); “Synthesis of optimal controllers using hybrid analog-digital computers,” by B. Paiewonsky, P. Woodrow, W. Brunner, and P. Halbert (pp. 285–303); and “Gradient methods for the optimization of dynamic systems parameters by hybrid computation,” by G. A. Bekey and R. B. McGhee (pp. 305–327). The first two of these papers use algorithms based on the theory of time-optimal control, particularly the computational methods proposed by Neustadt and Eaton. Included are descriptions of computer programs and reports of computer studies.

As a whole, this book makes a valuable contribution in presenting under one cover our, as yet, primitive knowledge on the subject. In the opinion of this reviewer, it is the best book of its kind to date.

J. P. LaSalle

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Providence, Rhode Island


Since \((a + jb)/(c + jd) = [(a/d + b/c) + j(b/d - a/c)]/N\), where \(N = c/d + d/c\), the real and imaginary terms of the right-hand member can be computed on the same nomogram composed of three logarithmic scales.

The absolute value \(\alpha\) of the ratio is determined by the use of three logarithmic scales, of which two are

\[ L(x) = \log x - 1 \quad \text{and} \quad R(x) = 1 - \log x, \]

and the relation \(2m(y) = L(\sqrt{(1 - y)}) + R(\sqrt{(1 - 1/y)})\), where \(\log \alpha = 4m(\alpha)\).

The reviewer recalls a much simpler method which was described by Jesse W. M. DuMond in “(A complex quantity slide rule,” in the Journal of the American Institute of Electrical Engineers, v. 44, 1925, pp. 133-139.) This used a chart on a drafting table. This method has been mechanized recently in a commercial cylindrical slide rule.

Michael Goldberg

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Washington 16, D.C.

This is an elementary numerical analysis book designed for undergraduate use. According to the dust jacket, “The only mathematical prerequisites are a knowledge of elementary calculus and an awareness of the existence and importance of differential equations.” A unique feature is that most of the numerical methods are illustrated by complete programs written in FORTRAN II. The dust jacket also states, “Explanatory notes, flow charts, and beginners hints provide a complete handbook in FORTRAN.”

The FORTRAN programs have been placed in a separate section, occupying somewhat more than half of the book. There is ample cross-referencing to the numerical methods which are presented in the first section. The topics covered are indicated by the chapter heading: (1) Computers and Programming, (2) Approximate Computations, (3) Differentiation, Integration, Interpolation and Extrapolation, (4) Solution of Algebraic and Transcendental Equations, (5) Simultaneous Linear Algebraic Equations, (6) Ordinary Initial-Value Problems, (7) Two-Dimensional Problems. The numerical analysis section covers 146 pages. The FORTRAN programs cover 171 pages.

A textbook which tries to cover two complementary subjects often fails to cover either adequately. This book suffers from this difficulty. The chapters on numerical analysis are very brief. Since they contain a good number of problems and worked examples, it is clear that theory gets the short end of things. The last chapter, on two-dimensional methods, includes interpolation, integration, and partial differential equations, all in 21 pages. This is pretty thin stuff. The other chapters are more satisfactory and present standard numerical methods in a terse cookbook style. The last part of the book gives a set of complete FORTRAN programs together with flow charts, hints, and printouts of results. Many students and teachers would find this section useful. It does not, however, constitute a complete course in FORTRAN programming, and needs to be supplemented with additional material.

There have been a number of books published which purport to teach numerical analysis and computer programming. Even with their faults, it is often convenient to have one textbook instead of two, particularly if the price of the book is reasonable. At the quoted price of this book, the reviewer feels that one would get more for his money by buying two books which treat each subject separately.

Richard C. Roberts
U.S. Naval Ordnance Laboratory
White Oak, Silver Spring, Maryland


The volume under review consists of seven articles by specialists. The first, “The role of computers in election night broadcasting,” by Jack Moshman, is very interesting and carries the weight of one of the leading experts in the field.
The second, "Some results of research on automatic programming in eastern Europe," by Wladyslaw Turski, is also very interesting, though at times it is a bit cryptic.

The third and longest (127 pp.), "A discussion of artificial intelligence and self-organization," by Gordon Pask, reveals that the field has developed its own literature (204 references), jargon, and conventions which the outsider (this reviewer) finds incomprehensible. Thus, "Since we are considering the real world, a computing machine is not a typical localized automaton. It has an aura of permanence which belies the fact that any localized automaton, open to the structural perturbations of the real world, has a finite life span. A better exemplar, perhaps, is an ape in a cage." That one should draw such distinctions, and continue to discuss them for some pages, makes the outsider wonder at the level of science being done.

The fourth article, "Automatic optical design," by Orestes N. Stravroudis, gives a good deal of information about one of the earliest applications of digital computers and presents both the history and the current state in a manner that is quite readable to anyone who knows some physical optics.

The fifth article, "Computing problems and methods in X-ray crystallography," by Charles L. Coulter, also treats a classic application in a manner that is reasonably clear.

The sixth, "Digital computers in nuclear reactor design," by Elizabeth Cuthill, being a topic of vast dimensions, tends at times to degenerate into listings of these and those codes, but is useful to the beginner (and perhaps the expert) in the field (386 references).

The last, "An introduction to procedure-oriented languages," by Harry D. Huskey, is a short article (28 pp.). It is interesting, and at least this reader wished that the author had taken more time and space on this important topic.

R. W. Hamming


This book is intended to be a primer of basic programming concepts and is directed toward readers with no previous knowledge of computers and with a mathematical background of high school algebra. The presentation is based on a simple hypothetical binary computer called the EX-1, and the material is very standard and by nature similar to that of various other texts of this type. The first two-thirds of the work cover the rudiments of number representation, data representation in machines, flow diagramming, and programming the EX-1 in a simple assembly language, using both fixed- and floating-point arithmetic. Topics discussed include indexing, symbol tables and table-lookup, elements of sorting, and the subroutine concept. Also included are chapters on input-output and on scaling techniques for fixed-point arithmetic. In its final third section, the book enters into a somewhat vague descriptive presentation of "modern" topics. This begins with a cursory discussion of a "typical modern giant computer" and continues with the introduction of a compiler language called TRIVIAL. In both cases the presentation is extremely hurried and would undoubtedly be quite unsatisfactory for a novice reader.
Although the presentation in general reads easily and is aided by many illustrations, the author has a very unfortunate habit of mentioning briefly important concepts in the form of a single sentence or a paragraph, without ever trying to explain these remarks further—a habit which leaves the reader with many questions unanswered. Altogether, the text is certainly only a very elementary primer, which could serve at best as a first introduction to the field of programming. However, it may well find some appreciative readers among high-school students and others interested in learning something about computers and the problems of programming for them.

Werner C. Rheinboldt
University of Maryland
College Park, Maryland


This is a short and elegant programming manual for the IBM 1620 Model I with the automatic-division feature, indirect addressing, and either paper tape or card input-output. The last of the seven chapters in this book outlines the features of the discpack. The first six chapters cover the 1620 central processor, principles of programming, input/output, and the Symbolic Programming System (SPS). The author is to be commended for a clear and particularly well-organized presentation and for having managed to include so many basic programming concepts in a manual devoted to a particular machine. Her explanations of symbolic addressing, macros, monitor systems, subroutines, iterative procedures and recursive techniques, though elementary, are remarkably lucid and to the point. They make this soft-covered little book much more than its title suggests.

E. K. Blum
Wesleyan University
Middletown, Conn.


This book is intended as an introductory text to the field of digital computers. It is composed of ten chapters, whose contents range from a discussion of the binary number system to a discussion of various features of advanced computers such as “instruction look-ahead.” The greater part of the book is devoted to engineering details of large digital system. Specific computers, the IBM 7090-94 and 7080 systems, are described and used as models of scientific and business computers, respectively.

Because the authors believe that “a complete comprehension of computers cannot be obtained without a basic understanding of programming,” they have included two chapters on “Fundamentals of Programming” and the “Fortran System.” The reviewer cannot agree that “these chapters are substantial and will enable the reader to write working programs.” The latter chapter is abstracted from the IBM 7090 Fortran manual and does not add very much to the contents of that manual.
These chapters are preceded by chapters entitled “Binary Arithmetic Operations” and “Floating Point Arithmetic Operations,” in which the discussion of round-off is entirely omitted. In fact, this topic is not listed in the index, and, although the instructions “Round” and “Multiply and Round” are listed (cf. p. 94), they are never discussed. The reviewer agrees with the authors when they say, “The competent programmer knows how his computer system operates and the best computer designers know programming.” He would like to add that both programmers and designers must know of the necessity of introducing round-off procedures in computers and the implications of a particular procedure on the accuracy of various computations. These topics are not even mentioned in the book. The reviewer feels that the book suffers greatly as a result of this omission.

A. H. Taub

University of California
Berkeley, California


This is the fourth volume in a series published for the Automatic Programming Information Centre, Brighton College of Technology, England. It contains the papers presented at a one-week symposium on programming held at the London School of Economics in July 1962.

The papers are generally well written and mark some sort of departure in style from the usual jargon which mars so many papers on programming. There are three expository papers on Fortran and Algol, which serve to introduce two papers on Fortran-like compilers and four papers on Algol-like compilers. The other papers cover commercial languages and their compilers (Cobol and Fact), aspects of programming systems (time-sharing, Atlas Supervisor) and advanced programming techniques. Subtopics in the last category include “syntactic analysis in compilers,” “addressing,” “list programming,” “stacks” and “continuous evaluation.” As in all previous publications of this sort, there is no attempt to formulate a theory or to define the problem. As the editor states in his preface, this volume is “experimental, seeking to develop a nucleus of material which might in the future become the basis of a science of programming.”

E. K. Blum