(b) With a slightly different algorithm i.e.
\[ x = p^2 - q^2 - r^2, \]
\[ y = 2pq, \]
\[ z = 2pr. \]
We find for \( x = 495, y = 840, z = 448, \)
\[ x^2 + y^2 + z^2 = 1073^2, \quad x^2 + y^2 = 975^2, \quad z^2 + y^2 = 952^2, \]
\( x^2 + z^2 \) not a square.

(c) The sets \((1008, 1100, 1155)\) and \((1008, 1100, 12075)\) have two numbers in common.

(d) There are several sets of \((x, y, z)\) which have one value in common e.g. \((2964, 9152, 9405)\), \((2964, 6160, 38475)\) and \((5643, 43680, 76076)\), \((5643, 14160, 21476)\).

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Some Designs for Maximal \((+1, -1)\)-Determinant of Order \(n = 2 \pmod{4}\)

By C. H. Yang

When \(n = 2 \pmod{4}\), Ehlich [1] has shown that

(i) the maximal absolute value \(\alpha_n\) of \(n\)th order determinant with entries \(\pm 1\) satisfies

\[ \alpha_n^2 \leq 4(n - 2)^{n-2}(n - 1)^2 = \mu_n, \]

(ii) matrices \(M_n\) of the maximal \(n\)th order \((+1, -1)\)-determinant whose absolute value equals \(\mu_{n^{1/2}}\) exist for \(n \leq 38\), provided that \(\left((n - 1, -1)_p = 1\right)\) (Hilbert's symbol) for any prime \(p\), which is also equivalent to \(\text{any prime factor of squarefree part of } n - 1 \text{ is not congruent to 3 } \pmod{4}\)."

It is found that \(M_{42}, M_{46}\) also exist by Ehlich's method and such maximal matrices \(M_n\) are likely to exist for all \(n = 2 \pmod{4}\) if \((n - 1, -1)_p = 1\) for any prime \(p\). This means that for \(n < 200\), all such matrices are likely to be found except for \(n = 22, 34, 58, 70, 78, 94, 106, 130, 134, 142, 162, 166, 178,\) and 190.

The maximal matrix \(M_n\) such that

\[ M_n M_n^T = \begin{pmatrix} 2 & n \\ \vdots & \vdots \\ n & 2 \end{pmatrix} \]

where \(P = \begin{pmatrix} \vdots & \vdots \\ 2 & n \end{pmatrix} \)

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and $M_n^T = \text{the transpose of } M_n$, can be constructed from the following (cf. Ehlich [1]):

$$M_n = \begin{pmatrix} A_1 & A_2 \\ -A_2^T & A_1^T \end{pmatrix},$$

where $A_1, A_2$ are circulant matrices of order $n/2$.

For $n = 42, 46$, the designs for the maximal matrices $M_n$ are:

- $n = 42; \quad A_1 : \quad - - - - + + - - + - - - - + - + - ;$
  \hspace{1cm} A_2 : \quad - - + + + - - - - + - - - - - - - - ;$
- $n = 46; \quad A_1 : \quad - - - - + + - - + - - - - + - - - ;$
  \hspace{1cm} A_2 : \quad - - + + - - - - + - - - - - - - - ;$

where $-$ stands for $-1$, $+$ for $+1$.

Another design for $n = 38$ is found as follows:

- $A_1 : \quad - - - + - - - - - - - - - - - - - - ;$
- $A_2 : \quad - - - - + - - - - - - - - - - - - - - ;$

For $n = 50$, the maximal matrix $M_n$ can be constructed by taking $A_1 = A_2 = \text{the matrix of Raghavarao [3]},$ without circulancy.

As noted in the design of above maximal matrices, the numbers $n_1$ and $n_2$ of $-1$'s respectively in each row of $A_1$ and $A_2$ can not be arbitrary. For example, when $n = 38; n_1, n_2$ must be either 6 or 7, provided $n_1, n_2 < n/4$. Similarly, when $n = 42; n_1, n_2$ must be either 6 or 10; when $n = 46; \text{either 7 or 10}$. For $54 \leq n < 200$, the following table of $n_1$ and $n_2$ is helpful to construct the maximal matrices. ($n_1, n_2 < n/4$)

| $n$ | 54 | 62 | 66 | 74 | 82 | 86 | 90 | 98 | 102 | 110 | 114 | 118 | 122 | 126 |
|-----|----|----|----|----|----|----|----|----|-----|-----|-----|-----|-----|
| $n_1$ or $n_2$ | 9 | 10 | 12 (11) | 13 | 16 | 16 (15) | 16 | 18 | 20 | 21 | 21 | 22 | 25 | 25 (24) |

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<td>42 (45)</td>
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