

## On Amicable and Sociable Numbers\*

By Henri Cohen

**Abstract.** An exhaustive search has yielded 236 amicable pairs of which the lesser number is smaller than  $10^8$ , 57 pairs being new.

It has also yielded 9 new sociable groups of order 10 or less, of which the lesser number is smaller than  $6.10^7$ ; the 9 sociable groups are all of order 4.

The sequence of iterates of the function  $s(n) = \sigma(n) - n$  starting with 276 has also been extended to 119 terms.

**Introduction.** Let  $n \geq 2$  be an integer, and

$$\sigma(n) = \sum_{d|n} d, \quad s(n) = \sigma(n) - n = \sum_{d|n, d \neq n} d.$$

We wish to study the behavior of the sequence:

$$\begin{aligned} a_0(n) &= n \\ a_{k+1}(n) &= s(a_k(n)) \end{aligned}$$

which will be called the aliquot series of  $n$ .

It is clear that if this sequence is bounded as  $k \rightarrow \infty$  it is periodic, since  $a_k(n)$  can take only a finite number of values.

Consequently the sequence can have essentially three distinct behaviors:

- (a) The sequence converges, i.e. there exists a  $k$  for which  $a_k = 1$  (or equivalently  $a_{k-1}$  prime).
- (b) The sequence is periodic of period  $t$ : there exists  $k_0$  such that

$$a_{k+t} = a_k \quad \text{for all } k \geq k_0.$$

If one can take  $k_0 = 0$ , the sequence is purely periodic; in this case: if  $t = 1$ ,  $n$  is a perfect number, if  $t = 2$ ,  $(n, s(n))$  is a pair of amicable numbers, and in general the  $t$ -uplet  $(a_0, \dots, a_{t-1})$  is a sociable group of order  $t$ .

- (c) The sequence is unbounded.

*Results on Amicable Numbers.* Two recent papers [6], [7], listed all pairs of amicable numbers up to  $10^6$  and  $10^7$  respectively.

Table 1 extends these lists and contains all amicable pairs with the lesser number between  $10^7$  and  $10^8$ . The 57 pairs marked with an asterisk are not found in the lists given by Escott [1], Poulet [2], Garcia [3], Lee [4], Lee [5], and seem to be new.

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TABLE 1  
AMICABLE NUMBERS

10254970 = 2 · 5 · 11 · 53 · 1759	10273670 = 2 · 5 · 11 · 59 · 1583
10533296 = 2 <sup>4</sup> · 19 · 34649	10949704 = 2 <sup>3</sup> · 29 · 109 · 433
10572550 = 2 · 5 <sup>2</sup> · 19 · 31 · 359	10854650 = 2 · 5 <sup>2</sup> · 31 · 47 · 149
10596368 = 2 <sup>4</sup> · 29 · 41 · 557	11199112 = 2 <sup>3</sup> · 53 · 61 · 433
* 10634085 = 3 <sup>4</sup> · 5 · 7 · 11 <sup>2</sup> · 31	14084763 = 3 · 7 · 11 <sup>2</sup> · 23 · 241
* 10992735 = 3 <sup>2</sup> · 5 · 13 · 19 · 23 · 43	12070305 = 3 <sup>2</sup> · 5 · 13 · 47 · 439
* 11173460 = 2 <sup>2</sup> · 5 · 53 · 83 · 127	13212076 = 2 <sup>2</sup> · 31 · 47 · 2267
* 11252648 = 2 <sup>3</sup> · 11 · 71 · 1801	12101272 = 2 <sup>3</sup> · 67 · 107 · 211
11498355 = 3 <sup>4</sup> · 5 · 11 · 29 · 89	12024045 = 3 <sup>4</sup> · 5 · 11 · 2699
11545616 = 2 <sup>4</sup> · 19 · 163 · 233	12247504 = 2 <sup>4</sup> · 491 · 1559
11693290 = 2 · 5 · 7 · 167047	12361622 = 2 · 7 <sup>2</sup> · 13 · 31 · 313
11905504 = 2 <sup>5</sup> · 13 · 28619	13337336 = 2 <sup>3</sup> · 107 · 15581
12397552 = 2 <sup>4</sup> · 23 · 59 · 571	13136528 = 2 <sup>4</sup> · 359 · 2287
* 12707704 = 2 <sup>3</sup> · 17 · 41 · 43 · 53	14236136 = 2 <sup>3</sup> · 107 · 16631
* 13671735 = 3 · 5 · 7 <sup>2</sup> · 11 · 19 · 89	15877065 = 3 · 5 · 17 · 19 · 29 · 113
* 13813150 = 2 · 5 <sup>2</sup> · 13 · 79 · 269	14310050 = 2 · 5 <sup>2</sup> · 29 · 71 · 139
13921528 = 2 <sup>3</sup> · 19 · 67 · 1367	13985672 = 2 <sup>3</sup> · 19 · 101 · 911
14311688 = 2 <sup>3</sup> · 17 · 47 · 2239	14718712 = 2 <sup>3</sup> · 23 · 167 · 479
* 14426230 = 2 · 5 · 7 · 13 · 83 · 191	18087818 = 2 · 7 · 31 · 71 · 587
14443730 = 2 · 5 · 7 <sup>3</sup> · 4211	15882670 = 2 · 5 · 19 · 179 · 467
14654150 = 2 · 5 <sup>2</sup> · 7 · 149 · 281	16817050 = 2 · 5 <sup>2</sup> · 179 · 1879
15002464 = 2 <sup>5</sup> · 37 · 12671	15334304 = 2 <sup>5</sup> · 227 · 2111
* 15363832 = 2 <sup>3</sup> · 11 · 71 · 2459	16517768 = 2 <sup>3</sup> · 53 · 163 · 239
15938055 = 3 <sup>2</sup> · 5 · 7 · 19 · 2663	17308665 = 3 <sup>2</sup> · 5 · 11 · 73 · 479
* 16137628 = 2 <sup>2</sup> · 13 · 23 · 103 · 131	16150628 = 2 <sup>2</sup> · 13 · 31 · 43 · 233
16871582 = 2 · 7 <sup>2</sup> · 13 · 17 · 19 · 41	19325698 = 2 · 7 <sup>2</sup> · 19 · 97 · 107
* 17041010 = 2 · 5 · 7 · 31 · 7853	19150222 = 2 · 7 · 13 · 43 · 2447
* 17257695 = 3 · 5 · 7 · 13 · 47 · 269	17578785 = 3 · 5 · 7 · 23 · 29 · 251
17754165 = 3 <sup>2</sup> · 5 · 11 · 13 · 31 · 89	19985355 = 3 <sup>2</sup> · 5 · 13 · 127 · 269
17844255 = 3 <sup>2</sup> · 5 · 11 · 13 · 47 · 59	19895265 = 3 <sup>2</sup> · 5 · 13 · 71 · 479
17908064 = 2 <sup>5</sup> · 53 · 10559	18017056 = 2 <sup>5</sup> · 79 · 7127
* 18056312 = 2 <sup>3</sup> · 17 · 103 · 1289	18166888 = 2 <sup>3</sup> · 19 · 107 · 1117

TABLE 1 (Continued)

* 18194715 = $3^2 \cdot 5 \cdot 7 \cdot 11 \cdot 59 \cdot 89$	22240485 = $3^2 \cdot 5 \cdot 31 \cdot 107 \cdot 149$
18655744 = $2^9 \cdot 83 \cdot 439$	19154336 = $2^5 \cdot 619 \cdot 967$
* 20014808 = $2^3 \cdot 11 \cdot 79 \cdot 2879$	21457192 = $2^3 \cdot 47 \cdot 149 \cdot 383$
20022328 = $2^3 \cdot 17 \cdot 23 \cdot 37 \cdot 173$	22823432 = $2^3 \cdot 1367 \cdot 2087$
20308995 = $3^3 \cdot 5 \cdot 7 \cdot 21491$	20955645 = $3^3 \cdot 5 \cdot 17 \cdot 23 \cdot 397$
21448630 = $2 \cdot 5 \cdot 7 \cdot 131 \cdot 2339$	23030090 = $2 \cdot 5 \cdot 19 \cdot 53 \cdot 2287$
* 22227075 = $3^3 \cdot 5^2 \cdot 13 \cdot 17 \cdot 149$	24644925 = $3^3 \cdot 5^2 \cdot 29 \cdot 1259$
22249552 = $2^4 \cdot 13 \cdot 41 \cdot 2609$	25325528 = $2^3 \cdot 587 \cdot 5393$
22508145 = $3^3 \cdot 5 \cdot 11 \cdot 23 \cdot 659$	23111055 = $3^3 \cdot 5 \cdot 11 \cdot 79 \cdot 197$
22608632 = $2^3 \cdot 19 \cdot 23 \cdot 29 \cdot 223$	25775368 = $2^3 \cdot 1439 \cdot 2239$
23358248 = $2^3 \cdot 23 \cdot 37 \cdot 47 \cdot 73$	25233112 = $2^3 \cdot 37 \cdot 85247$
23389695 = $3^3 \cdot 5 \cdot 7 \cdot 53 \cdot 467$	25132545 = $3^3 \cdot 5 \cdot 17 \cdot 47 \cdot 233$
* 23628940 = $2^2 \cdot 5 \cdot 37^2 \cdot 863$	27428276 = $2^2 \cdot 17 \cdot 251 \cdot 1607$
* 24472180 = $2^2 \cdot 5 \cdot 17 \cdot 167 \cdot 431$	30395276 = $2^2 \cdot 47 \cdot 107 \cdot 1511$
25596544 = $2^7 \cdot 311 \cdot 643$	25640096 = $2^5 \cdot 67 \cdot 11959$
25966832 = $2^4 \cdot 29 \cdot 191 \cdot 293$	26529808 = $2^4 \cdot 47 \cdot 35279$
26090325 = $3^2 \cdot 5^2 \cdot 17 \cdot 19 \cdot 359$	26138475 = $3^2 \cdot 5^2 \cdot 11 \cdot 59 \cdot 179$
* 28118032 = $2^4 \cdot 47 \cdot 139 \cdot 269$	28128368 = $2^4 \cdot 59 \cdot 83 \cdot 359$
* 28608424 = $2^3 \cdot 13 \cdot 139 \cdot 1979$	29603576 = $2^3 \cdot 23 \cdot 349 \cdot 461$
30724694 = $2 \cdot 7 \cdot 11 \cdot 13 \cdot 103 \cdot 149$	32174506 = $2 \cdot 7 \cdot 13 \cdot 17 \cdot 10399$
30830696 = $2^3 \cdot 13 \cdot 521 \cdot 569$	31652704 = $2^5 \cdot 449 \cdot 2203$
31536855 = $3^2 \cdot 5 \cdot 7 \cdot 53 \cdot 1889$	32148585 = $3^2 \cdot 5 \cdot 7 \cdot 102059$
* 31818952 = $2^3 \cdot 11 \cdot 41 \cdot 8819$	34860248 = $2^3 \cdot 97 \cdot 167 \cdot 269$
32205616 = $2^4 \cdot 17 \cdot 167 \cdot 709$	34352624 = $2^4 \cdot 2147039$
* 32642324 = $2^2 \cdot 11 \cdot 13 \cdot 149 \cdot 383$	35095276 = $2^2 \cdot 17 \cdot 47 \cdot 79 \cdot 139$
32685250 = $2 \cdot 5^3 \cdot 13 \cdot 89 \cdot 113$	34538270 = $2 \cdot 5 \cdot 13 \cdot 379 \cdot 701$
* 33501825 = $3^2 \cdot 5^2 \cdot 7 \cdot 89 \cdot 239$	36136575 = $3^2 \cdot 5^2 \cdot 19 \cdot 79 \cdot 107$
34256222 = $2 \cdot 7 \cdot 11 \cdot 13 \cdot 71 \cdot 241$	35997346 = $2 \cdot 7 \cdot 11 \cdot 23 \cdot 10163$
* 34364912 = $2^4 \cdot 43 \cdot 199 \cdot 251$	34380688 = $2^4 \cdot 47 \cdot 131 \cdot 349$
34765731 = $3^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 227$	36939357 = $3^2 \cdot 7 \cdot 13 \cdot 23 \cdot 37 \cdot 53$
* 35115795 = $3^3 \cdot 5 \cdot 11 \cdot 13 \cdot 17 \cdot 107$	43266285 = $3^3 \cdot 5 \cdot 53 \cdot 6047$
35361326 = $2 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 1039$	40117714 = $2 \cdot 7 \cdot 13 \cdot 53 \cdot 4159$

TABLE 1 (Continued)

35373195 = $3^2 \cdot 5 \cdot 11 \cdot 13 \cdot 23 \cdot 239$	40105845 = $3^2 \cdot 5 \cdot 13 \cdot 179 \cdot 383$
35390008 = $2^3 \cdot 19 \cdot 23 \cdot 53 \cdot 191$	39259592 = $2^3 \cdot 71 \cdot 69119$
35472592 = $2^4 \cdot 43 \cdot 47 \cdot 1097$	36415664 = $2^4 \cdot 53 \cdot 42943$
* 37363095 = $3^2 \cdot 5 \cdot 7 \cdot 11 \cdot 41 \cdot 263$	45663849 = $3^2 \cdot 7 \cdot 11 \cdot 131 \cdot 503$
* 37784810 = $2 \cdot 5 \cdot 7 \cdot 539783$	39944086 = $2 \cdot 7 \cdot 13 \cdot 41 \cdot 53 \cdot 101$
37848915 = $3^2 \cdot 5 \cdot 13 \cdot 23 \cdot 29 \cdot 97$	39202605 = $3^2 \cdot 5 \cdot 13 \cdot 19 \cdot 3527$
* 38400512 = $2^9 \cdot 179 \cdot 419$	38938288 = $2^4 \cdot 83 \cdot 109 \cdot 269$
* 38637016 = $2^3 \cdot 11 \cdot 359 \cdot 1223$	40678184 = $2^3 \cdot 29 \cdot 271 \cdot 647$
38663950 = $2 \cdot 5^2 \cdot 13 \cdot 17 \cdot 3499$	43362050 = $2 \cdot 5^2 \cdot 59 \cdot 14699$
* 38783992 = $2^3 \cdot 13 \cdot 37 \cdot 10079$	41654408 = $2^3 \cdot 47 \cdot 139 \cdot 797$
38807968 = $2^5 \cdot 37 \cdot 73 \cdot 449$	40912232 = $2^3 \cdot 37 \cdot 89 \cdot 1553$
* 43096904 = $2^3 \cdot 17 \cdot 41 \cdot 59 \cdot 131$	46715896 = $2^3 \cdot 53 \cdot 239 \cdot 461$
* 44139856 = $2^4 \cdot 29 \cdot 251 \cdot 379$	44916944 = $2^4 \cdot 83 \cdot 149 \cdot 227$
45263384 = $2^3 \cdot 17 \cdot 59 \cdot 5641$	46137016 = $2^3 \cdot 19 \cdot 433 \cdot 701$
* 46237730 = $2 \cdot 5 \cdot 7 \cdot 11^2 \cdot 53 \cdot 103$	61319902 = $2 \cdot 7 \cdot 83 \cdot 113 \cdot 467$
* 46271745 = $3^2 \cdot 5 \cdot 13 \cdot 19 \cdot 23 \cdot 181$	49125375 = $3^2 \cdot 5^3 \cdot 13 \cdot 3359$
* 46521405 = $3^3 \cdot 5 \cdot 7 \cdot 19 \cdot 2591$	53011395 = $3^3 \cdot 5 \cdot 31 \cdot 53 \cdot 239$
* 46555250 = $2 \cdot 5^3 \cdot 7 \cdot 37 \cdot 719$	55880590 = $2 \cdot 5 \cdot 103 \cdot 227 \cdot 239$
* 46991890 = $2 \cdot 5 \cdot 11 \cdot 29 \cdot 14731$	48471470 = $2 \cdot 5 \cdot 19^2 \cdot 29 \cdot 463$
* 48639032 = $2^3 \cdot 13 \cdot 29 \cdot 16127$	52967368 = $2^3 \cdot 59 \cdot 293 \cdot 383$
48641584 = $2^4 \cdot 29 \cdot 104831$	48852176 = $2^4 \cdot 47 \cdot 167 \cdot 389$
* 49215166 = $2 \cdot 7 \cdot 11 \cdot 13^2 \cdot 31 \cdot 61$	55349570 = $2 \cdot 5 \cdot 31 \cdot 61 \cdot 2927$
* 50997596 = $2^2 \cdot 13 \cdot 19 \cdot 71 \cdot 727$	51737764 = $2^2 \cdot 13 \cdot 23 \cdot 181 \cdot 239$
52695376 = $2^4 \cdot 17 \cdot 151 \cdot 1283$	56208368 = $2^4 \cdot 3513023$
56055872 = $2^6 \cdot 79 \cdot 11087$	56598208 = $2^6 \cdot 383 \cdot 2309$
* 56512610 = $2 \cdot 5 \cdot 7 \cdot 11 \cdot 23 \cdot 3191$	75866014 = $2 \cdot 7^2 \cdot 774143$
56924192 = $2^5 \cdot 13 \cdot 193 \cdot 709$	64562488 = $2^3 \cdot 283 \cdot 28517$
* 58580540 = $2^2 \cdot 5 \cdot 23 \cdot 347 \cdot 367$	70507972 = $2^2 \cdot 23 \cdot 521 \cdot 1471$
* 59497888 = $2^5 \cdot 41 \cdot 101 \cdot 449$	61953512 = $2^3 \cdot 29 \cdot 97 \cdot 2753$
* 63560025 = $3^3 \cdot 5^2 \cdot 17 \cdot 29 \cdot 191$	65003175 = $3^3 \cdot 5^2 \cdot 23 \cdot 53 \cdot 79$
63717615 = $3^2 \cdot 5 \cdot 13 \cdot 17 \cdot 43 \cdot 149$	66011985 = $3^2 \cdot 5 \cdot 13 \cdot 19 \cdot 5939$
66595130 = $2 \cdot 5 \cdot 7 \cdot 31 \cdot 30689$	74824390 = $2 \cdot 5 \cdot 31 \cdot 59 \cdot 4091$

TABLE 1 (Concluded)

* 66854710 = 2 · 5 · 13 <sup>3</sup> · 17 · 179	71946890 = 2 · 5 · 17 · 83 · 5099
67729064 = 2 <sup>3</sup> · 13 · 431 · 1511	69439576 = 2 <sup>3</sup> · 23 · 107 · 3527
67738268 = 2 <sup>2</sup> · 13 · 17 · 19 · 37 · 109	79732132 = 2 <sup>2</sup> · 19 · 263 · 3989
68891992 = 2 <sup>3</sup> · 13 · 23 · 83 · 347	78437288 = 2 <sup>3</sup> · 587 · 16703
71015260 = 2 <sup>2</sup> · 5 · 23 · 263 · 587	85458596 = 2 <sup>2</sup> · 41 · 47 · 11087
71241830 = 2 · 5 · 11 · 19 · 89 · 383	78057370 = 2 · 5 · 17 · 359 · 1279
* 72958556 = 2 <sup>2</sup> · 11 · 19 · 197 · 443	74733604 = 2 <sup>2</sup> · 11 · 53 · 73 · 439
73032872 = 2 <sup>3</sup> · 11 · 71 · 11689	78469528 = 2 <sup>3</sup> · 59 · 83 · 2003
74055952 = 2 <sup>4</sup> · 23 · 61 · 3299	78166448 = 2 <sup>4</sup> · 197 · 24799
* 74386305 = 3 <sup>2</sup> · 5 · 7 · 17 · 29 · 479	87354495 = 3 <sup>2</sup> · 5 · 19 · 71 · 1439
* 74769345 = 3 <sup>3</sup> · 5 · 7 <sup>2</sup> · 89 · 127	82824255 = 3 <sup>3</sup> · 5 · 17 · 151 · 239
75171808 = 2 <sup>5</sup> · 53 · 127 · 349	77237792 = 2 <sup>5</sup> · 479 · 5039
75226888 = 2 <sup>3</sup> · 11 · 59 · 14489	81265112 = 2 <sup>3</sup> · 53 · 137 · 1399
* 78088504 = 2 <sup>3</sup> · 13 · 31 · 53 · 457	88110536 = 2 <sup>3</sup> · 167 · 65951
* 78447010 = 2 · 5 · 17 · 19 · 149 · 163	80960990 = 2 · 5 · 11 · 491 · 1499
* 79324875 = 3 <sup>2</sup> · 5 <sup>3</sup> · 7 <sup>2</sup> · 1439	87133365 = 3 <sup>2</sup> · 5 · 11 · 103 · 1709
80422335 = 3 <sup>3</sup> · 5 · 7 · 85103	82977345 = 3 <sup>3</sup> · 5 · 11 · 71 · 787
82633005 = 3 <sup>2</sup> · 5 · 7 · 13 · 17 · 1187	104177619 = 3 <sup>2</sup> · 7 · 13 · 131 · 971
83135650 = 2 · 5 <sup>2</sup> · 13 · 79 · 1619 <sup>*</sup>	85603550 = 2 · 5 <sup>2</sup> · 19 · 251 · 359
* 84521745 = 3 <sup>3</sup> · 5 · 7 · 11 · 47 · 173	107908335 = 3 <sup>3</sup> · 5 · 383 · 2087
* 84591405 = 3 <sup>3</sup> · 5 · 17 · 29 · 31 · 41	89590995 = 3 <sup>3</sup> · 5 · 13 · 71 · 719
86158220 = 2 <sup>2</sup> · 5 · 41 · 105071	99188788 = 2 <sup>2</sup> · 23 · 43 · 25073
* 87998470 = 2 · 5 · 7 · 29 · 67 · 647	102358010 = 2 · 5 · 47 · 89 · 2447
* 88144630 = 2 · 5 · 7 <sup>2</sup> · 29 · 6203	102814490 = 2 · 5 · 37 · 269 · 1033
89477984 = 2 <sup>5</sup> · 59 · 83 · 571	92143456 = 2 <sup>5</sup> · 1637 · 1759
90437150 = 2 · 5 <sup>2</sup> · 19 · 23 · 4139	94372450 = 2 · 5 <sup>2</sup> · 23 · 137 · 599
* 91996816 = 2 <sup>4</sup> · 29 · 331 · 599	93259184 = 2 <sup>4</sup> · 79 · 89 · 829
93837808 = 2 <sup>4</sup> · 19 · 83 · 3719	99899792 = 2 <sup>4</sup> · 1399 · 4463
95629904 = 2 <sup>4</sup> · 37 · 67 · 2411	97580944 = 2 <sup>4</sup> · 67 · 227 · 401
95791430 = 2 · 5 · 7 · 17 · 101 · 797	115187002 = 2 · 7 · 17 · 113 · 4283
* 96304845 = 3 · 5 · 7 <sup>2</sup> · 13 · 10079	96747315 = 3 · 5 · 7 <sup>2</sup> · 23 · 59 · 97
97041735 = 3 <sup>2</sup> · 5 · 7 · 71 · 4339	97945785 = 3 <sup>2</sup> · 5 · 7 · 239 · 1301

TABLE 2  
NEW SOCIABLE GROUPS

	$1264460 = 2^2 \cdot 5 \cdot 17 \cdot 3719$		$2115324 = 2^2 \cdot 3^2 \cdot 67 \cdot 877$
(1)	$1547860 = 2^2 \cdot 5 \cdot 193 \cdot 401$	(2)	$3317740 = 2^2 \cdot 5 \cdot 165887$
	$1727636 = 2^2 \cdot 521 \cdot 829$		$3649556 = 2^2 \cdot 107 \cdot 8527$
	$1305184 = 2^5 \cdot 40787$		$2797612 = 2^2 \cdot 331 \cdot 2113$
	$2784580 = 2^2 \cdot 5 \cdot 29 \cdot 4801$		$4938136 = 2^3 \cdot 7 \cdot 109 \cdot 809$
(3)	$3265940 = 2^2 \cdot 5 \cdot 61 \cdot 2677$	(4)	$5753864 = 2^3 \cdot 23 \cdot 31271$
	$3707572 = 2^2 \cdot 11 \cdot 84263$		$5504056 = 2^3 \cdot 17 \cdot 40471$
	$3370604 = 2^2 \cdot 23 \cdot 36637$		$5423384 = 2^3 \cdot 53 \cdot 12791$
	$7169104 = 2^4 \cdot 17 \cdot 26357$		$18048976 = 2^4 \cdot 11 \cdot 102551$
(5)	$7538660 = 2^2 \cdot 5 \cdot 376933$	(6)	$20100368 = 2^4 \cdot 919 \cdot 1367$
	$8292568 = 2^3 \cdot 59 \cdot 17569$		$18914992 = 2^4 \cdot 37 \cdot 89 \cdot 359$
	$7520432 = 2^4 \cdot 127 \cdot 3701$		$19252208 = 2^4 \cdot 1203263$
	$18656380 = 2^2 \cdot 5 \cdot 932819$		$28158165 = 3^3 \cdot 5 \cdot 7 \cdot 83 \cdot 359$
(7)	$20522060 = 2^2 \cdot 5 \cdot 13 \cdot 17 \cdot 4643$	(8)	$29902635 = 3^3 \cdot 5 \cdot 7 \cdot 31643$
	$28630036 = 2^2 \cdot 19 \cdot 449 \cdot 839$		$30853845 = 3^3 \cdot 5 \cdot 11 \cdot 79 \cdot 263$
	$24289964 = 2^2 \cdot 97 \cdot 62603$		$29971755 = 3^3 \cdot 5 \cdot 11 \cdot 20183$
			$46722700 = 2^2 \cdot 5^2 \cdot 47 \cdot 9941$
		(9)	$56833172 = 2^2 \cdot 11 \cdot 53 \cdot 24371$
			$53718220 = 2^2 \cdot 5 \cdot 2685911$
			$59090084 = 2^2 \cdot 43 \cdot 343547$

*Note Added.* After the first version of this paper was submitted to *Math. Comp.*, I was informed that Paul Bratley, John McKay, and Fred Lunnon had independently computed the amicable pairs from  $10^7$  to  $10^8$ . Their 128 pairs agree exactly with mine.

*Results on Sociable Numbers.* Until now only two groups of sociable numbers were known, respectively of order 5 and 28; both were found by Poulet [8]. I have made an exhaustive search for sociable groups of order  $t \leq 10$  of which the lesser number is smaller than  $6 \cdot 10^7$ . This search has yielded 9 new groups, which interestingly enough are all of order 4. They are given in Table 2.

This relative abundance of order 4 sociables compared with other orders is rather surprising and calls for some comments.

Let us say that a sociable group is a *regular* group of order  $t$  if it is of the form  $(a \cdot n_1, \dots, a \cdot n_t)$  with each  $n_i$  prime to  $a$  for  $1 \leq i \leq t$  and  $n_1, \dots, n_t$  have no common factor. Then a theorem of Dickson [10], states that there are no regular groups of odd order  $> 1$ . On the other hand, of the 236 amicable pairs up to  $10^8$ , 193 are regular, and of the 9 sociables of order 4, 7 are regular. Regular groups thus seem to form the large majority of groups of even order 2 and 4, so Dickson's theorem can

explain, at least partly, why only one group has been found of odd order  $> 1$ . It does not explain why no groups of order 6, 8 or 10 have been found.

*Results on Unbounded Sequences.* It has been conjectured by Catalan (see revision by Dickson [10]) that the aliquot series of  $n$  is never unbounded. It is known to be bounded for  $2 \leq n \leq 275$ . The smallest  $n$  for which the behavior is not known is 276. G. A. Paxson [9] has calculated 67 terms of this sequence. I have extended this to 119 terms and found:

$$a_{118}(276) = 2133148752623068133100.$$

*Conclusion.* From these results a number of conjectures can be made.

Let  $A(x)$  be the number of amicable pairs of which the smaller number is less than  $x$ ; then empirically one can conjecture:

*Conjecture 1.* There exists  $\beta > 0$  such that

$$\text{Log } A(x) \sim \beta \cdot \text{Log}(x).$$

This conjecture of course implies the as yet unknown fact that there exists an infinity of amicable pairs.

From Table 1 and preceding tables a least square method gives

$$\beta = 0.29 \dots$$

A heuristic computation of  $\beta$  would be welcome.

*Conjecture 2.* There exists an infinity of sociable groups of order 4.

This is a particular case of a general conjecture of Erdős [11]. Furthermore in the same paper Erdős states that the density of sociable groups of any order is 0. Combining this with Catalan's conjecture as revised by Dickson one obtains:

*Conjecture 3.* For almost all  $n$  (i.e. with density 1) the associated sequence converges. These conjectures seem very difficult to prove.

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Institut de Biologie Moléculaire  
C.N.R.S.  
9 Quai Saint Bernard  
Paris (5<sup>e</sup>), France

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