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Gaussian Quadrature Formulas for the Integration of Oscillating Functions, see review #24, p. 479. Robert Piessens

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The editorial committee would welcome readers' comments about this microfiche feature. Please send comments to Professor Eugene Isaacson, Mathematics of Computation, Courant Institute of Mathematical Sciences, New York University, 251 Mercer Street, New York, New York 10012.
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