REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the indexing system printed in Volume 22, Number 101, January 1968, page 212.

1 [2.00, 2.10, 2.20, 2.35, 3, 4].—FORMAN S. ACTON, *Numerical Methods that Work*, Harper & Row, Publishers, New York, 1970, xviii + 541 pp., 24 cm. Price $12.95.

This textbook is written for 'upper class students in engineering and physical sciences . . . whose motivations lie in the physical world'. It deals with techniques for using a computer—and one's own brain—to deal with standard numerical problems. The subject matter includes: locating zeros; quadrature; ordinary differential equations; eigenvalues of matrices; minimization and network problems.

This reviewer found the book easy to read, stimulating and highly amusing in places. Section headings such as 'What not to compute', 'How to seek and destroy singularities', 'Evil in recurrence relations' and even 'Minimum tree construction in Alabama' enticed me to read much more of the book than I had first intended. But, of course, being familiar with much of the subject matter, I am not the sort of person for whom the book is written. Whether a student in the throes of a life and death struggle with a new topic sees the point of the humor—or is simply annoyed by it—is a matter of conjecture.

The book is written in a fluent colloquial style. It is rather as if Mr. Acton had recorded his lectures, expanded the text and written the resulting speech as a book. This method of producing a book is not without drawbacks.

One of the main purposes of a lecture is to impart the enthusiasm and motivation of the lecturer to the student. The lecturer *talks* mainly about aspects of a subject that interest or stimulate him and he tends to leave out parts which he finds dull. In the course of his lecture, he may move rapidly from one topic to another as the momentum of his monologue demands.

However, what is a virtue in a lecture may be a defect in a textbook. Important aspects of a subject can be left out. And it may be very difficult to locate a particular result in a sea of expansive discussion. For example, the author gives an excellent description of quadrature, dealing in detail with all sorts of problems relating to singularities and infinite ranges. However, there is no reference whatever to Gauss-Laguerre, Gauss-Hermite or Gauss-Jacobi quadrature rules. And while advocating quite appropriately the use of elliptic functions, the author gives no definition of $sn(x)$, $cn(x)$ and $dn(x)$. Instead, he draws a picture and states some of the interrelations referring the reader for the definitions to a book which happens to use a different notation. Incidentally, my own view is that this sort of subject matter (which is conventionally omitted from numerical analysis textbooks) is extremely relevant, and I was happy to see it included. The omissions mentioned above stem from the general style and not from the choice of subject matter. By leaving out tiresome definitions and traditionally important techniques which may be tedious to discuss, Mr. Acton has made his book more enjoyable.

Thus, in the course of producing this book, Mr. Acton has possibly performed an additional service to the academic community. This is to focus attention on a more profound pedagogical problem. To what extent, if any, should a textbook be a written-out lecture? Should the book, or lecture, both or neither, be long-winded and expansive or concise or thorough—or even humorous?

Whatever the individual instructor's views on this might be, he should certainly enjoy reading this book himself. And if he agrees that a lecture and a textbook should be roughly equivalent, he now has at his disposal the means to put his own view to a practical test in class.

J. N. L.

2 [2.05].—A. Talbot, Editor, *Approximation Theory*, Proceedings of a Symposium held at Lancaster, July 1969, Academic Press, Inc., London, 1970, viii + 353 pp., 25 cm. Price 75s.

The advent of high-speed computers and the importance of approximation theory as a tool in computation have stimulated a great deal of recent research. *Approximation Theory* is a compendium of twenty-four papers presented at the International Symposium on Approximation Theory, held at the University of Lancaster, England, in July of 1969. The papers deal with a wide range of topics, both classical and modern, theoretical and practical. Of particular note is the inclusion of the first English account of the "method of functionals" developed by E. V. Voronovskaya. The papers give a good indication of the diversity and beauty of the field of approximation theory, and, hopefully, will help to stimulate more activity. The material is generally well presented and free of errors. This book should be a worthwhile addition to the library of anyone interested in approximation theory or numerical analysis. The table of contents follows:

I. Approximation by Polynomials

Error estimates for best polynomial approximations
  G. M. Phillips
Orthogonal polynomial approximation methods in numerical analysis
  J. C. Mason
Asymptotic properties and the convergence of numerical quadratures
  J. Miklosko
The method of functionals and its applications
  E. V. Voronovskaya
Some remarks on approximation by polynomials with integral coefficients
  L. B. O. Ferguson

II. Further Linear Approximation

Characterization of best spline approximations with free knots
  D. C. Handscomb
A note on interpolating periodic quintic spline functions
  F. Schurer
Non-negative interpolation formulas for harmonic and analytic functions
  P. J. Davis

Approximation relative to Hausdorff distance
  B. Sendov

III. Non-Linear Approximation

Tschebyscheff-approximation with sums of exponentials
  H. Werner
On the solubility of the Cauchy interpolation problem
  J. Meinguet
Rational approximation to analytic functions on an inner part of the domain of
  analyticity
  J. Szabados

IV. Practical Techniques

The determination of H-sets for the inclusion theorem in nonlinear Tschebyscheff
  approximation
  L. Collatz
General purpose curve fitting
  J. R. Rice
On computing best L_1 approximations
  I. Barrodale
Mathematical programming and approximation
  P. Rabinowitz
One-sided approximations by linear combinations of functions
  G. Marsaglia
Approximation of non-linear inequalities on Banach spaces
  M. Sibony

V. Approximation in Abstract Linear Spaces

Minimal projections
  E. W. Cheney and K. H. Price
Interpolating subspaces in approximation theory
  D. A. Ault, F. R. Deutsch, P. D. Morris and J. E. Olson
Simultaneous approximation and interpolation with preservation of norm
  F. R. Deutsch and P. D. Morris

VI. Approximation by Linear Operators

On asymptotic approximation theorems for sequences of linear positive operators
  M. W. Müller
Related interpolation problems
  G. A. Read
Probabilistic methods in the theory of approximation of functions of several variables
    by linear positive operators
  D. D. Stancu

JOHN PIERCE

Department of Mathematics
University of Southern California
Los Angeles, California 90024

3 [2.05, 2.35, 2.55, 5, 6].—L. Collatz & H. Unger, Editors, *Funktionalanalytische Methoden der Numerischen Mathematik*, Birkhäuser Verlag, Basel, 1969, 143 pp., 25 cm. Price SFr. 24.-.

This volume contains the printed versions of lectures held in 1967 during a conference at the Mathematics Research Institute Oberwolfach (Black Forest). Topics from approximation theory predominate, but there are also lectures on functional equations in Banach space, iterative methods, theory of optimization, and interval arithmetic. A list of the authors and their titles follows.

H. Amann, Iterationsverfahren für die Hammersteinsche Gleichung

R. Ansorge, Zur Existenz verallgemeinerter Lösungen nichtlinearer Anfangswertaufgaben

J. Blatter, Approximative Kompaktheit verallgemeinerter rationaler Funktionen

B. Brosowski, Einige Bemerkungen zum verallgemeinerten Kolmogoroffschen Kriterium

F. Fazekas, Optimierungen mittels matrixalgorithmischer Methoden (MAM)

F. Fazekas, Funktionalanalytische Beziehungen bei Verallgemeinerungen des Vialzentrum-Problems

H.-P. Helfrich, Ein modifiziertes Newtonsches Verfahren

K.-H. Hoffmann, Über ein Eindeutigkeitskriterium bei der Tschebyscheff-Approximation mit regulären Funktionensystemen

H. Van Iperen, Beste Approximation mit Potenzen verallgemeinerter Bernsteinoperatoren

P. J. Laurent, Charakterisierung und Konstruktion einer Approximation in einer konvexen Teilmenge eines normierten Raumes

H. Leipholz, Über die Erweiterungen der Verfahren von Grammel und Galerkin und deren Zusammenhang

Ramon E. Moore, Functional Analysis for Computers

J. Nitsche, Zur Frage optimaler Fehlerschranken bei Differenzenverfahren

E. Schock, Beste Approximation von Elementen eines nuklearen Raumes

H. Werner, Der Existenzsatz für das Tschebyscheffsche Approximationsproblem mit Exponentialsummen

W. G.

4 [2.05, 3, 4, 5, 6].—P. L. Butzer & B. Szökefalvi-Nagy, Editors, *Abstract Spaces and Approximation*, Proceedings of a Conference held at the Mathematical Research Institute at Oberwolfach, Germany, July 18–27, 1968, Birkhäuser Verlag, Basel, 1969, 423 pp., 24 cm. Price SFr 68.-.

These proceedings contain an article on the life and work of Jean Farvard, to whose memory the volume is dedicated, a section on new and unsolved problems, and the following 39 papers.

I. Operator Theory

P. R. Halmos: Invariant subspaces...

I. I. Hirschman, Jr.: The asymptotic behavior of finite section Wiener-Hopf equations

R. G. Douglas: On the spectrum of Toeplitz and Wiener-Hopf operators........

R. S. Phillips: Scattering theory for hyperbolic equations......................

In this volume, special emphasis is placed on theoretical aspects of approximation theory and closely related topics of functional analysis. The volume also contains articles on other topics of functional analysis and on numerical analysis. The classical topics of approximation theory are well represented, while the active field of approximation by splines and other piecewise polynomial functions is discussed in only two of the papers. The influence of Jean Farvard is strongly felt in several contributions on saturation theory.

OLOF WIDLUND

Courant Institute of Mathematical Sciences
New York University
New York, New York 10012

5 [2.10, 7.00].—ROBERT PIESSENS, *Gaussian Quadrature Formulas for the Numerical Integration of Bromwich's Integral and the Inversion of the Laplace Transform*, Report TW1, Applied Mathematics Section, University of Leuven, June 1969, 10pp. + tables (48 unnumbered pp.), 27 cm. Copy deposited in UMT file.

If $F(p)$ is the Laplace transform of $f(t)$ and s is a positive parameter such that $p^s F(p)$ is analytic and has no branch point at infinity, then the tables in this report consist of 16S floating-point values of the coefficients A_k and p_k in the formula

$$f(t) = t^{s-1} \sum_{k=1}^{N} A_k (p_k/t)^s F(p_k/t),$$

such that it is exact when $F(p)$ is a linear combination of $p^{-(s+k)}$ for $k = 0(1)2N - 1$. As the author notes, such a parameter s may not exist for some Laplace transforms; nevertheless, even in such cases it is claimed that the formula gives good numerical results.

The coefficients p_k and A_k are complex numbers, occurring in conjugate complex pairs; accordingly, it suffices to tabulate only the p_k with positive imaginary part, together with the corresponding A_k, and this is done here for the ranges $N = 6(1)12$ and $s = 0.1(0.1)3(0.5)4$,

$$\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, \frac{5}{3}, \frac{7}{3}, \frac{8}{3}, \frac{10}{3}, \frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{7}{4}, \frac{9}{4}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \frac{1}{9}.$$

The numbers p_k were calculated as the zeros of generalized Bessel polynomials $P_{N,s}(p^{-1})$ by Newton-Raphson iteration, and the coefficients A_k were then calculated from the relation

$$A_k = (-1)^{N-1} \frac{(N-1)!}{\Gamma(N+s-1)Np_k^2} \left[\frac{2N+s-2}{P_{N-1,s}(p_k^{-1})} \right]^2 .$$

All calculations were performed on the IBM 1620 system at the Computing Centre of the University of Leuven.

Application of the tables is illustrated by four diversified examples.

In his description of previous tabulations of this type the author includes references to tables by Salzer [1], [2] and by Stroud & Secrest [3], wherein s is restricted to the special value 1, and to more general tables by Skoblia [4] and by Krylov & Skoblia [5].

<div align="right">J. W. W.</div>

1. H. E. SALZER, "Orthogonal polynomials arising in the numerical evaluation of inverse Laplace transforms," *MTAC*, v. 9, 1955, pp. 164–177.

2. H. E. SALZER, "Additional formulas and tables for orthogonal polynomials originating from inversion integrals," *J. Math. and Phys.*, v. 40, 1961, pp. 72–86.

3. A. STROUD & D. SECREST, *Gaussian Quadrature Formulas*, Prentice-Hall, Englewood Cliffs, N. J., 1966.

4. N. S. SKOBLIA, *Tables for the Numerical Inversion of Laplace Transforms*, Academy of Sciences of USSR, Moscow, 1964. (Russian) (For a review, see *Math. Comp.*, v. 19, 1965, pp. 156–157, RMT **15**.)

5. V. I. KRYLOV & N. S. SKOBLIA, *Handbook on the Numerical Inversion of the Laplace Transform*, Izdat. Nauka i Teknika, Minsk, 1968.

6 [2.20, 2.35].—A. S. HOUSEHOLDER, *KWIC Index for the Numerical Treatment of Nonlinear Equations*, Oak Ridge National Laboratory, Oak Ridge, Tennessee, 1970, vii + 129 pp., 28 cm. Available from U. S. Department of Commerce, Springfield, Va. 22151. Price: printed copy $3.00; microfiche $0.65.

The author of this index has a reputation not only for his numerous original research contributions but also for his many excellent bibliographies. For example, his extensive bibliography on numerical analysis in his book, *Principles of Numerical Analysis* [McGraw-Hill, 1953; continued in *J. Assoc. Comput. Mach.*, v. 3, 1956, pp. 85–100] has been one of the earliest reference collections in that field and as such has had considerable influence. More recently appeared a *KWIC Index for Matrices in Numerical Analysis* (Oak Ridge National Lab. Report ORNL-4418] and the present index on nonlinear equations is described as being in part a supplement to it. In the words of the preface, this is to mean that "many items, especially textbooks and treatises, relate to both topics, but (that), apart from accidental duplication, each item is listed in only one place".

The present index contains 1182 items and thus is probably the most extensive reference collection on nonlinear equations. At the same time, no claim for completeness can be or is being made. The preface specifically observes that "for the older literature, no exhaustive historical search was attempted" and that on the topic of solving systems of equations "the list is far from exhaustive", since "this area merges imperceptibly into the far more general one of solving functional equations, on the one hand, and on the other of mathematical programming, where the literature is vast". This latter point probably accounts for the fact that, for instance, a bibliography of Ortega and Rheinboldt [*Iterative Solution of Nonlinear Equations in Several Variables*, Academic Press, 1970] contains a number of titles which would appear to have been natural candidates for inclusion in this index. Also, the earlier

bibliography of Traub [*Iterative Methods for the Solution of Equations*, Prentice-Hall, 1964] has some entries not subsumed here.

A KWIC index certainly increases the information content of any bibliography, since it provides ready access to sources on specific topics—as far as the titles contain the appropriate keywords. The choice of the "trivial words" in any KWIC index is almost always a matter for some debate. In this case, it appears to be surprising that such words as "system(s)" were excluded while "equation(s)" was not—resulting in over 400 entries in the index.

All in all, in the face of the explosive growth of the scientific literature, we can only be thankful when a man with the wide knowledge and critical facility of Professor Householder undertakes the large task of collecting a bibliography in an area of his interest. It would be highly desirable, however, if such bibliographies could find a more permanent place of their own in the printed literature which would assure them of a wider, continued distribution than is possible for mimeographed reports.

W. RHEINBOLDT

Computer Science Center
University of Maryland
College Park, Maryland 20742

7 [5].—S. D. ĔĬDEL'MAN, *Parabolic Systems*, North-Holland Publishing Co., Amsterdam, and Wolters-Noordhoff Publishing, Groningen, 1969, ii + 469 pp., 23 cm. Price $18.20.

This is a translation of a research monograph which appeared in Russian in 1964. It essentially concentrates on partial differential equations which are parabolic in the sense of Petrovski. The mathematical techniques are mostly classical—Fourier-Laplace transforms, the parametric approach developed by E. E. Levi in 1907, etc. Much of the work in this active field has been done by Russian mathematicians inspired by a famous paper of Petrovski's in 1938.

The translation has a certain awkwardness in the choice of terminology, order of words, etc., which is common in translations. However, no one who is mathematically prepared to read the book should have great difficulty in understanding the text.

There are four chapters and two appendices. In the first chapter, the author derives bounds for the fundamental solution in the Cauchy case. He first discusses the second-order case, then general Petrovski parabolic systems with bounded, Hölder-continuous coefficients. These results are generalized to problems with unbounded coefficients and with less restrictive assumptions on the smoothness of the coefficients. Among other topics, the work of Nash is discussed, as well as the behavior in the neighborhood of singularities and the relation between the fundamental solutions of elliptic and parabolic systems.

The second chapter is devoted to interior Schauder estimates, a discussion of hypoellipticity and Liouville type results.

In the third chapter, the fundamental solution is used to derive existence and uniqueness theorems, in the Cauchy case, for classes of initial data characterized by sharp growth conditions. Other topics include local solvability and continuation

results for nonlinear problems and the behavior of solutions when $t \to \infty$.

The main new contribution of the book is contained in Chapter 4. Mixed initial boundary value problems are treated first for problems with constant coefficients in a halfspace and then for the case of variable coefficients in a general cylindrical domain.

The two appendices discuss the well-posedness of more general parabolic equations and another class of initial-value problems.

OLOF WIDLUND

Courant Institute of Mathematical Sciences
New York, University
New York, New York, 10012

8 [5].—A. R. MITCHELL, *Computational Methods in Partial Differential Equations*, John Wiley & Sons Ltd., Aberdeen, 1969, xiii + 255 pp., 23 cm. Price $11.00.

This book is concerned with the numerical solution of partial differential equations by finite-difference methods. There are six chapters: a review of basic linear algebra; parabolic equations; elliptic equations; hyperbolic systems; hyperbolic equations of second order; applications in fluid mechanics and elasticity. Basically, two classes of problems are considered. One class involves elliptic equations for bounded regions, together with various types of boundary conditions. These lead to systems of linear algebraic equations. The other class of problems involves parabolic or hyperbolic equations. There is a time variable, t, as well as one or more space variables. The desired function satisfies prescribed conditions in a region of the space variables for $t = 0$, and also on the boundary of the region for $t \geqq 0$. Such problems can sometimes be solved stepwise with respect to time by explicit methods. However, considerations of stability and accuracy often make the use of implicit methods desirable. With implicit methods one is faced with the solution of a system of linear algebraic equations at each time step.

In the case of parabolic problems the Crank-Nicolson implicit method is often used. In the case of one space variable, the method can be carried out by solving a tridiagonal system. However, in the case of two space variables, one must solve a more general linear system. One method for doing this is to use the successive over-relaxation method (S. O. R. method) which is analyzed in detail. The author states that the analysis carries over directly to elliptic problems, but does not give the details, particularly concerning the choice of the relaxation factor. Another class of methods considered includes various alternating direction implicit methods (A.D.I. methods) including the Peaceman-Rachford method, the Douglas-Rachford method, D'yakonov's method, and others. These methods are used for parabolic, elliptic, and hyperbolic problems. Still another class of methods, called "locally one-dimensional methods" (L.O.D. methods) are considered. These methods were developed primarily by Russian authors including D'yakonov and others, and the author's treatment appears to be one of the first accounts given in a textbook written in English. Explicit and implicit L.O.D. methods are used for parabolic, elliptic, and hyperbolic problems. Other methods used include the use of "split operators" and "locally A.D.I. methods," both of which are applied to hyperbolic problems.

For parabolic and hyperbolic problems, the question of stability is studied using the von Neumann method, which is based on a harmonic decomposition of the error. Other methods used for stability analysis include a matrix method for parabolic problems and a method due to Courant, Friedrichs, and Lewy for hyperbolic problems.

The author states that the book is aimed at science and engineering students in the second or third year of their undergraduate studies. No specialized knowledge of mathematics is assumed beyond undergraduate courses in calculus and in matrix theory. In particular, a knowledge of the theory of partial differential equations is not assumed. It seems, however, that in this country the book would be more appropriate for a student at a more advanced level with a good first course in numerical analysis.

The book is clearly written and provides a good introduction to the subject. However, probably because of the attempt to treat a large amount of material in a short space, certain criticisms are perhaps inevitable. First, there are so many methods presented that the reader is likely to become confused. It might have been better to have presented fewer methods and given more comparative evaluations and numerical results. More discussion of iterative methods for solving large linear systems would have been helpful—for instance, semi-iterative methods. Also, the treatment of the S.O.R. method for elliptic problems is very brief; in particular, there is very little discussion on the use of the method in practical cases. Only a very limited class of hyperbolic equations are considered. These involve regions with boundaries parallel to a coordinate axis. Also, a brief summary of the properties of characteristics would have been helpful. For elliptic equations some mention of the existing knowledge of discretization errors (e.g., Gershogrin's results and more recent work) would seem appropriate. Finally, it should be mentioned that the chapter on linear algebra has some errors (for example, it is stated that if two matrices have the same eigenvalues then they are similar).

<div align="right">
David M. Young, Jr.
David R. Kincaid
</div>

Center for Numerical Analysis
The University of Texas at Austin
Austin, Texas 78712

9 [7].—L. S. Bark, N. I. Dmitrieva, L. N. Zakhar'ev & A. A. Lemanskiĭ, *Tablitsy Sobstvennykh Znachenii Uravneniia Mat'e* (*Tables of Characteristic Values of the Mathieu Equation*), Computing Center, Acad. Sci. USSR, Moscow, 1970, xi + 151 pp., 27 cm. Price 1.81 rubles.

For the Mathieu equation in the canonical form $d^2y/dx^2 + (p - 2q \cos 2x)y = 0$, these tables give to 7S (in floating-point form) the characteristic values for both the even and odd periodic solutions, for a more extensive range of the parameter q than heretofore.

More explicitly, the first table (pp. 3–86) contains the characteristic values a_{2n}, a_{2n+1} (associated with the even periodic solution) for $n = 0(1)15$ and $q = 0.1(0.1)100$. A continuation of this table (pp. 87–111) gives these characteristic

values for $n = 16(1)50$ and $q = 1(1)100$.

The characteristic values b_{2n}, b_{2n+1} (associated with the odd periodic solution) are tabulated on pp. 112–150 for $q = 0.1(0.1)100$ and $n = 0(1)m$, where m increases from 1 to 9 for b_{2n} and from 0 to 8 for b_{2n+1}, with increasing values of q.

Immediate comparison of these tables is possible with the considerably abridged 8D table (Table 20.1) in the NBS *Handbook* [1]. Such a comparison has revealed to this reviewer that the Russian values were simply chopped at seven significant figures, thereby resulting in last-figure tabular errors approaching a unit. It might be pointed out here that greater precision (over the more restricted ranges of $n \leq 7$ and $q \leq 25$) can also be obtained from another NBS publication [2], in conjunction with the relations $a_n = be_n - 2q$, $b_n = bo_{n+1} - 2q$, and $s = 4q$.

An introduction to the present tables describes their contents and preparation and includes three illustrative examples of the application of appropriate interpolative procedures. The appended list of seven references does not include any of the pertinent NBS publications, which contain extensive bibliographies relating to Mathieu functions.

Despite such defects, these tables contain much new numerical information, constituting a valuable addition to that available in previous tables of the characteristic values of the Mathieu equation.

<div align="right">J. W. W.</div>

1. MILTON ABRAMOWITZ & IRENE STEGUN, Editors, *Handbook of Mathematicial Functions with Formulas, Graphs, and Mathematical Tables*, National Bureau of Standards, Applied Mathematics Series, No. 55, U. S. Government Printing Office, Washington, D. C., 1964.

2. *Tables Relating to Mathieu Functions*, National Bureau of Standards, Applied Mathematics Series, No. 59, U. S. Government Printing Office, Washington, D. C., 1967.

10 [7].—YUDELL L. LUKE, *The Special Functions and Their Approximations*, Academic Press, New York, 1969, vol. I, xx + 349 pp., vol. II, xx + 485 pp., 23 cm. Price $19.50 per volume.

The special functions of mathematical physics are, simply, those functions which arise most frequently in the classical problems of applied mathematics and physics. Setting aside the elementary functions (logarithmic, exponential, circular and hyperbolic), there remain a number of well-known, named functions which are of very wide application. They are usually functions of more than one variable, satisfying fairly simple linear differential equations from which many basic properties may be derived. Such properties have been explored by many authors; Y. L. Luke's contribution has been the development of a unified basis for these special functions with special attention to methods for their computation. They are all treated as special cases of the hypergeometric functions.

Treatment of the Gaussian hypergeometric function $_2F_1$ embraces the Legendre functions, the incomplete beta function, the complete elliptic functions of the first and second kinds, and the familiar systems of orthogonal polynomials. The confluent hypergeometric function $_1F_1$ includes the Bessel functions and their relatives, and the incomplete gamma function. Connections with a still wider class of functions are demonstrated in a discussion of Meijer's G-function, a generalization of the hypergeometric function. It is shown that each of the most commonly used functions

of analysis is a special case of the G-function, which also has relevance to many functions which are not hypergeometric in character.

Volume I contains eight chapters and an extensive bibliography. Chapter 1 is a very brief introduction to asymptotic expansions and Watson's lemma, and is followed by chapters headed: 2. The gamma function and related functions, 3. Hypergeometric functions, 4. Confluent hypergeometric functions, 5. The generalized hypergeometric function and the G-function, 6. Identification of the $_pF_q$ and G-functions with the special functions of mathematical physics, 7. Asymptotic expansions of $_pF_q$ for large parameters and 8. Orthogonal polynomials.

Volume II includes: 9. Expansions of generalized hypergeometric functions in series of functions of the same kind, 10. The τ-method, 11. Polynomial and rational approximations to generalized hypergeometric functions, 12. Recursion formulas for polynomials which occur in infinite series and rational approximations to generalized hypergeometric functions, 13. Polynomial and rational approximations for $E(z) = {}_2F_1(1, \sigma; \rho + 1; -1/z)$, 14. Polynomial and rational approximations for the incomplete gamma function, 15. Trapezoidal rule integration formulas, 16. Applications and 17. Tables of coefficients. These chapters are followed by a copy of the bibliography appearing in Volume I.

There is a vast amount of information contained in this work, which explores thoroughly the interconnections between the functions considered. Asymptotic series for large absolute values of the argument are developed in all appropriate cases, and series in terms of orthogonal polynomials, notably Chebyshev polynomials of the first kind, are also explored. Most of this groundwork is laid in the first volume, while in the second the emphasis is on computation, with discussion of general methods of approximation and treatment of particular numerical approximations for the special functions.

The author says that his book is primarily intended as a reference tool. As such, it can be welcomed as a significant addition to the literature. It is evident that the tables of Chapter 17 have been constructed with considerable care, and they will undoubtedly be used widely and often. The comprehensive listing of the various relations between the special functions and the hypergeometric functions, and of the important asymptotic series will also be consulted frequently. The background to the methods of approximation is a valuable companion to the approximations themselves.

There is also, however, a suggestion that the book might be used as a text for undergraduate or graduate courses in special functions. Such use should be implemented with caution. Most of the mathematical development is very terse, and much of it sketchy. Moreover, it is not uniformly reliable. There is, for instance, a Theorem 5 in Chapter 8 (Section 8.5.3) which states that "If $Q_n(x)$ is the best polynomial approximation to $f(x)$ of degree n, then $f(x) - Q_n(x) = \omega T_{n+1}(x)$,

$$\omega = \max_{a \leq x \leq b} |f(x) - Q_n(x)|.\text{"}$$

One must, of course, expect a few slips in a work of such magnitude, but it is difficult here even to guess what was intended. Fortunately (though perhaps confusingly for the student), this theorem is contradicted by the next sentence.

The blemishes will be revealed, and no doubt removed, in time. They cannot

obscure the value of a reference book which no practical mathematician can afford to overlook.

C. W. CLENSHAW

Mathematics Division
National Physics Laboratory
Teddington, Middlesex
England

11 [7].—KENNETH L. MILLER, PAUL MOLMUD & WILLIAM C. MEECHAM, *Tables of the Functions*

$$G(x + iy) = \int_0^\infty \frac{e^{-t^2}}{t - (x + iy)}\, dt \quad and \quad F(x + iy) = \int_0^\infty \frac{t^4 e^{-t^2}}{t - (x + iy)}\, dt,$$

Report 6121-6249-RU-000, Space Technology Laboratories, Redondo Beach, California, 8 February 1963, 17 pp. + tables (consisting of 102 unnumbered pp.), deposited in the UMT file.

The tables in this report, only recently submitted for deposit in the UMT file, consist of 6S decimal values in floating-point form of the infinite integrals identified in the title, for the ranges $x = -10(0.2)10$ and $y = 0(0.2)10$. As the authors note, values of these integrals in the lower half-plane are immediately obtainable by taking the complex conjugates of the corresponding values in the upper half-plane.

In the introduction to these tables it is shown that $F(\zeta)$ is expressible in terms of $G(\zeta)$, where $\zeta = x + iy$; consequently, only the properties of $G(\zeta)$ are discussed in detail. In particular, the relation of this function to the complex error function and the complex exponential integral is set forth.

Computation of the tables was performed on an IBM 7090 system, using single-precision floating-point arithmetic. Except for the range $x \geq 0$, $0 \leq y < 0.8$, the function $G(\zeta)$ was evaluated from its continued-fraction representation, derived by the quotient-difference algorithm; in the remaining range of the tabular arguments the function was evaluated by numerical integration of the first-order linear differential equation that it satisfies. The corresponding values of $F(\zeta)$ were then deduced by means of the stated identity relating the two functions.

Asymptotic series are presented for the calculation of these integrals outside the range of the tables. Also, calculation of intermediate values by bilinear interpolation and by Taylor's series is briefly discussed.

The bibliography, consisting of nine references, omits a pertinent paper of Goodwin & Staton [1], which contains a 4D table of $G(\zeta)$ for $-x = 0(0.02)3(0.1)10$, $y = 0$.

In addition to their immediate use in the theoretical determination of the alternating current electrical conductivity of weakly ionized gases, these tables can also be applied, as the authors note, in the theory of plasma oscillations and also in the theory of the thermoelectric properties of metals and semiconductors.

J. W. W.

1. E. T. GOODWIN & J. STATON, "Table of $\int_0^\infty e^{-u^2}/(u + x)\, du$," *Quart. J. Mech. Appl. Math.*, v. 1, 1948, pp. 319–326.

12 [7].—H. P. Robinson & Elinor Potter, *Mathematical Constants*, Report UCRL-20418, Lawrence Radiation Laboratory, University of California, Berkeley, California, March 1971, iii + 183 pp. Available from National Technical Information Service, Operations Division, Springfield, Virginia 22151. Price $3.00 (printed copy), $0.95 (microfiche).

The unique collection of miscellaneous mathematical constants comprising the body of this report has been arranged in four tables.

Table I consists of 2498 constants tabulated to 20D (with a few exceptions given to less precision) and arranged unconventionally in ascending magnitude of the decimal part of each number.

Table II consists mainly of 20D values of the positive irrational roots of 338 selected quadratic equations, such that the positive integral leading coefficient of each equation does not exceed 5 and the remaining two nonzero integral coefficients do not exceed 10 numerically. Interlarded among these roots are 126 constants that properly belong in Table I.

Table III lists a variety of characterizations of each of the first 1000 positive integers; for example, their binary and ternary representations, their factorizations, representations as sums of two and three squares and of a similar number of cubes, also as differences of squares and of cubes, as well as other representations, including linear combinations of factorials. The special symbols used in this table are defined in an introductory section on pp. 80–81.

Table IV consists of 260 sets of coefficients of the first 10 terms of the Maclaurin series of selected algebraic and transcendental functions and combinations thereof, which head each tableau. The leading coefficient of each series has been arbitrarily taken as unity, and the remaining coefficients are presented in both rational form and as 10D approximations.

The authors explain in their introduction that the unusual arrangement of data in Tables I and II accomplishes a twofold purpose; namely, it permits the user to improve to 20D the accuracy of a tabulated constant known to him to lower precision, and it also permits the possible identification of a number obtained empirically or as the result of a calculation. Similarly, Table IV enables the user to obtain closed-form expressions for the sums of a large number of power series.

In the space of this review it is impossible adequately to describe the variety of mathematical constants in Table I. Suffice it to say that a partial enumeration of the contents includes: square roots, cube roots, and reciprocals of selected integers (and sums, differences, and rational multiples thereof); powers and combinations of powers of π, e, and γ (including submultiples); special values of such functions as trigonometric and hyperbolic functions (both direct and inverse), logarithms (both natural and common), Bessel functions (including zeros thereof), Riemann zeta function and related functions, gamma and psi functions, probability integral and Dawson's integral, sine and cosine integrals; as well as the real roots of selected algebraic and transcendental equations.

The authors calculated anew the values of about half the entries in Table I, using double-precision arithmetic on a Wang 720 programmable desk calculator. Furthermore, this reviewer has independently checked more than 70 percent of the contents of Table I, and discovered that only about one percent of the tabulated values that

he examined require any correction. These corrections are separately listed in this issue.

The authors have presented herein a valuable collection of numerical material apparently not available in any other single source. It is to be hoped that they will issue a revised, enlarged edition in the near future.

<div align="right">J. W. W.</div>

13 [7].—MELVIN KLERER & FRED GROSSMAN, *A New Table of Computer Processed, Indefinite Integrals*, Dover Publications, Inc., New York, 1971, xiii + 198 pp., 24 cm. Price $3.00 (paperbound).

The following is quoted from the introduction to this book:

"This volume is a product of a computer science research program begun by M. Klerer at Hudson Laboratories, Columbia University and continued at the School of Engineering and Science, New York University. During 1963, M. Klerer and J. May implemented a programming system that accepted mathematical expressions typed in normal (pre-computer) textbook notation. The typing was done on a modified computer input-output typewriter terminal. Individual characters could be typed in any order and mathematical symbols, such as integral operators, could be constructed in any size. The typing did not have to be neat or symmetric, and mistakes could be erased by backspacing and over-typing or by pressing an "erase" key which produced a special code tagging the particular location for subsequent computer processing. Two-dimensional positioning of the paper to permit the typing of subscripts, super-scripts, or fraction expressions was done by keyboard control using the space, back-space, sub (half-line down), and sup (half-line up) keys. Besides eliminating a good deal of the effort usually required in translating mathematical expressions into FORTRAN code, this system, since it was entirely free format, allowed easy input by unskilled typists.

"Since this new programming system was part of a long-range effort directed toward the automation of applied mathematics, it was natural to consider the feasibility of making mathematical tables accessible to this computer system."

The introduction describes in detail the process of culling integrals from known tables and processing them to produce the final table of integrals. More than seven years have passed since the project began and, no doubt, current techniques are much improved. Nonetheless, the final output is readable and pleasing to the eye.

The tables themselves differ little from several already available tables. Integrals are divided into eight basic categories which are rational, algebraic, irrational, trigonometric, inverse trigonometric, exponential, logarithmic, hyperbolic and inverse hyperbolic.

The integrals were taken from eight commonly used and well-known tables. In the culling process, the authors made a study of the reliability of these tables and the results are reported in the introduction. On the basis of the discussion given there, one might expect that the present tables should not be faulty and that they are free of errors. Such is not the case and glaring inconsistency abounds throughout. In the illustration on p. 2, there appear the entries

$$\int \frac{DX}{X} = \text{LN} \, |X| \quad \text{and} \quad \int \frac{DX}{1-X} = -\text{LN}(1-X).$$

In a table of indefinite integrals, there is no need for absolute-value signs. Further, it is unusual to assume that parameters and variables are real. The above is not an isolated instance. In some cases, when absolute value signs are not used, sufficient conditions are given, as in

$$\int \frac{D\,X}{X(A - BX)} = \frac{1}{2A} \, \text{LN}\!\left(\frac{X}{BX^2 - A}\right),$$

if $A < 0$ and $B < 0$ and $X > (A/B)^{1/2}$.

In summary, the principal virtue of the project is that computer techniques could be used to produce the table. This is currently an important consideration in view of cost of publication. The tables are useful though they are not new. Eventually, one must forever be mindful of the possibility of human error.

<div align="right">Y. L. L.</div>

14 [7,10].—H. W. Gould, *Research Bibliography of Two Special Number Sequences,* Number 12 of *Mathematica Monongaliae,* Department of Mathematics, West Virginia University, Morgantown, West Virginia, May 1971, iv + 25 pp. One copy deposited in the UMT file.

Herein are presented two definite bibliographies: the first, of 137 items, relates to the Bell (or exponential) numbers; the second, of 243 items, to the Catalan numbers (also studied originally by Euler, Fuss, and Segner, as noted by the author).

An introduction of four pages includes the various definitions of these two integer sequences, their generating functions, recurrence relations, and their relations to other numbers such as binomial coefficients and Stirling numbers of the second kind. This is supplemented by pertinent historical information and references to the numerous combinatorial interpretations of these numbers.

The present work supersedes earlier, related bibliographies, of which the most extensive are included in expository papers by Rota [1] and Brown [2] cited by the author. The most extensive table of Bell numbers appears to be that in a paper by Levine & Dalton [3], also included in the present bibliography.

This scholarly report should be of special value to researchers in such fields as combinatorial analysis and graph theory.

<div align="right">J. W. W.</div>

1. Gian-Carlo Rota, "The number of partitions of a set," *Amer. Math. Monthly,* v. 71, 1964, pp. 498–504.
2. W. G. Brown, "Historical note on a recurrent combinatorial problem," *Amer. Math. Monthly,* v. 72, 1965, pp. 973–977.
3. Jack Levine & R. E. Dalton, "Minimum periods, modulo *p,* of first-order Bell exponential integers," *Math. Comp.,* v. 16, 1962, pp. 416–423.

15 [9].—Hideo Wada, "A table of fundamental units of purely cubic fields," *Proc. Japan Acad.,* v. 46, 1970, pp. 1135–1140.

The table gives the fundamental unit ϵ of the cubic field $Q(m^{1/3})$ for all such fields with $m < 250$ in the form

(1) $\epsilon = (A + B\alpha + C\alpha^2)/n$

with $\alpha = m^{1/3}$ and $n = 1, 2, 3$, or 6. Reference is made to Markoff's table to $m \leq 70$ which is reproduced in [1]. The present table is much to be preferred. We may discount Markoff because of typographical and other errors, and because his units are not given uniformly as in (1), but in a variety of forms:

$$55 + 24 \cdot 12^{1/3} + 21 \cdot 18^{1/3},$$

$$5/(5 - 2 \cdot 15^{1/3})^3,$$

$$(1 + 23^{1/3})^6/9(3 - 23^{1/3})^9,$$

etc. The form (1) has several advantages including a quicker, more accurate way of estimating the regulator.

The brief text mentions the methods of Voronoi and Billebič but gives no clue how the present table was computed. It required five hours of computer time. For $m = 239$, A is 188 digits long (and so $Q(239^{1/3})$ presumably has class number 1). These digits are printed 79 to a line with no spacing. I would hate to proofread it.

D. S.

1. B. N. Delone & D. K. Faddeev, *Theory of Irrationalities of the Third Degree*, Transl. Math. Monographs, vol. 10, Amer. Math. Soc., Providence, R. I., 1964, p. 304.

16 [12].—Clive B. Dawson & Thomas C. Wool, *From Bits to If's, An Introduction to Computers and Fortran* IV, Harper & Row, Publishers, New York, 1971. xii + 157 pp., 21 cm. Price $2.50.

This pocket book paperback makes for light, informative reading. It introduces the Fortran IV language in a somewhat breezy manner and the two authors are to be congratulated for not getting bogged down by too much technical matter.

The repertoire of Fortran IV is treated in a methodical manner and each of the nine chapters contains a set of exercises, with answers included for some of them. Programming techniques, per se, are not included. As the authors point out, the book is intended to serve as a basis and its object is to whet the appetite of the novice. This it should do admirably.

Henry Mullish

Courant Institute of Mathematical Sciences
New York University
251 Mercer Street
New York, New York 10012