REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the indexing system printed in Volume 22, Number 101, January 1968, page 212.

16[2.05, 4, 5, 6].—MARTIN H. SCHULTZ, *Spline Analysis*, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1973, xiii + 156 pp., 24 cm. Price $10.50.

This book is intended as a unified, elementary introduction to polynomial spline functions and their applications in several important areas of numerical analysis, including the finite element method for approximating the solutions of differential equations. Since it is written for readers with a background in calculus and linear algebra, the approach is to illustrate these techniques by rigorously treating only simple model problems. Results in more general cases are left to the exercises and to further study. For example, only piecewise linear and cubic polynomials are used, and all the differential equations considered are of second order, defined on an interval, or a square.

Terminology and some basic results are provided in Chapter I. Chapters II, III and IV are concerned with spline interpolation, piecewise linear, piecewise cubic Hermite, and cubic spline, respectively. The one-dimensional case is considered first, and the existence, uniqueness and minimum energy properties of such interpolations are given. Local bases are developed for the piecewise linear and cubic Hermite cases. Unfortunately, however, the B-spline local basis for cubic splines is not introduced until later in the text. The two-dimensional (bivariate) interpolation problem on the unit square is then considered using Cartesian products of one-dimensional spline functions. A thorough discussion of error estimates is given.

Chapter V describes the method of degenerate kernels for approximating the solution of a Fredholm integral equation of Type II by the solution of a linear system of equations. Here, the kernel is approximated by a bivariate piecewise polynomial.

Chapter VI deals with least-squares approximation in one and two dimensions by spline functions. It is pointed out that, if the mesh spacing is uniform, and a normalized set of local basis functions is used, then the matrices involved are symmetric, positive definite, banded, and, in addition, have condition numbers that are uniformly bounded as the mesh spacing tends to zero. This latter fact should give least squares approximation by spline functions added impetus.

Chapters VII and VIII describe the Rayleigh-Ritz-Galerkin methods for approximating the solutions of boundary-value problems and eigenvalue problems. Chapter X deals with a Ritz procedure for the state regulator problem in optimal control. In each of these settings, the problem is first posed equivalently as finding the minimum of a real-valued functional over an infinite-dimensional function space. An approximation to the solution is obtained as the minimum of the functional over an appropriate space of piecewise polynomials (the finite element method)—or, equivalently, as the solution of a linear system of equations, involving positive-definite symmetric, sparse and well-conditioned matrices. In each case, existence, uniqueness and error estimates are given.

Chapter IX describes the Galerkin method for approximating the solution of a parabolic partial differential equation by the solution of a linear system of ordinary differential equations. The usual Padé approximation methods for solving this system are discussed, although somewhat tersely.

The only real criticism of the text is that, although discussed throughout, the practical aspects of spline methods are still not given adequate emphasis, especially, considering the intended audience. For example, numerical results are for the most part described only to motivate error estimates, and only for analytic objective functions. The treatment of the cubic spline interpolation problem is via continuity conditions rather than the more practical and easily generalized B-spline technique. And, finally, one of the most directly practical aspects of spline functions, their best estimation properties, is not treated. Aside from these relatively minor considerations, the book is a welcome addition to the literature in a very important and practical area of numerical analysis. It is the most readily accessible book on spline functions written to date, and the only text to treat the finite element methods in a unified elementary fashion.

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17 [2.10, 2.15, 2.20, 2.30, 2.35, 2.40, 2.55].—JOSEF STOER, *Einführung in die Numerische Mathematik*. I, Springer-Verlag, Berlin-Heidelberg-New York, 1972, ix + 250 pp., 21 cm. Price $4.70 paper bound.

The "Heidelberger Taschenbücher", of which this is volume 105, is a series of text books in mathematics and the physical sciences which among its authors includes some of the most distinguished names in the respective fields. The volume under review, the first of a two-volume introduction into numerical mathematics, continues in the same tradition of expository excellence. Written at about the beginning graduate level, it makes a serious attempt to treat in depth those numerical techniques which can readily be implemented on digital computers and which are proven to be useful and reliable for high-speed computation. Accordingly, essential parts of key algorithms are often given as short programs in ALGOL 60. In addition, and more importantly, a great deal of emphasis has been placed on questions of numerical stability. Concepts such as condition, algorithmic stability, and "good-naturedness" of algorithms are rightly considered by the author to belong to the very core of numerical mathematics.

True to this spirit, the volume opens with a chapter on error analysis, developing the basic facts of machine arithmetic, rounding errors, and error propagation. It is here where the central concept of "good-natured algorithm" (due to F. L. Bauer) is introduced. Basically, this is a computing algorithm in which the influence of all intermediate rounding errors on the final result is not greater than the unavoidable error due to rounded input data. Examples of algorithms which enjoy "good-naturedness," and others which lack it, are given in this, as well as in subsequent, chapters. Chapter 2 takes up the problem of interpolation. It begins with the usual

facts on polynomial interpolation, and then proceeds with an excellent treatment of rational interpolation, including, in particular, the important Neville-type algorithms due to the author. This is followed by trigonometric interpolation of equidistant data, which in turn leads naturally to questions of computing discrete and continuous Fourier transforms. For the former, the author describes the algorithm of Goertzel, which unfortunately is not always stable, a "good-natured" variant due to Reinsch, and the very effective and popular algorithm of Cooley and Tukey (unfortunately without a discussion of its stability). For the latter, he presents the reviewer's theory of attenuation factors. The chapter ends with a short introduction to spline interpolation, covering the minimum curvature property of cubic spline interpolants, constructive methods, and convergence properties. Chapter 3 is a modern treatment of numerical integration. Formulas of the Newton-Cotes type, while briefly discussed, are clearly de-emphasized in favor of extrapolation algorithms. These, then, are treated with unusual care, not only for the integration problem, but also for other discretization algorithms (e.g., numerical differentiation). With the theory of Gaussian quadrature formulas the author then returns to more classical grounds, although here, too, he includes recent results (due to Golub and Welsch) concerning representation of Christoffel numbers. Surprisingly, no mention is made of the book by Stroud and Secrest [1], which is currently the standard reference work on the subject. There is also a brief discussion on how to deal with singularities. Chapter 4 is devoted to the solution of systems of linear algebraic equations by direct methods: Gauss elimination, triangular decomposition, Gauss-Jordan algorithm, and the Cholesky decomposition. (Iterative methods are to be discussed in the second volume.) There follows an introduction into the theory of norms, in preparation for discussing the condition of linear systems and a-posteriori error estimates. One finds an interesting result of Prager and Oettli which permits one to decide whether or not a solution is acceptable, given the size of the residual. A detailed analysis of rounding errors in Gauss elimination and in the triangular decomposition method then follows, and it is shown that Gauss elimination is indeed a "good-natured" algorithm if the triangular factors are not too large (in a specified sense). The author then turns to orthogonal reduction methods, in particular, to the reduction of a matrix to triangular form by a sequence of Householder transformations. He also discusses the related Gram-Schmidt orthogonalization and its numerical properties, being careful to point out the possible pitfalls in the absence of "reorthogonalizations." As a natural application, least squares approximation to overdetermined linear systems is considered, for which the basic theory is given and the elegant numerical solution by means of Householder reductions. There is also an interesting discussion of the condition of this problem, as well as a short outlook on nonlinear least squares approximation. Chapter 5 is devoted to nonlinear equations and systems thereof. The discussion first centers around iteration in finite-dimensional Euclidean space. The principle of contraction mapping is developed and a convergence theorem for the Newton-Raphson-Kantorovich method. A useful modification of Newton's method, involving ideas of steepest descent, is also developed and shown to converge globally under appropriate hypotheses. The remaining parts of the chapter concentrate on algebraic equations, presenting various versions of Newton's method, an interesting "double-step" Newton's method for polynomials with only real zeros, Maehly's ingenious device of successive deflation, the use of Sturm sequences in combination with successive bisection and Bairstow's method.

Questions of the condition of roots are also briefly considered. The chapter concludes with some interpolatory methods: the regula falsi, secant method, Muller's method, and with their convergence properties, and, finally, with convergence-accelerating processes such as those of Aitken and Steffensen.

Each chapter is followed by a good set of exercises and by a list of selected references. The text makes occasional references to the items in these bibliographies, but additional notes on some of the sources would have been helpful. The index at the end of the book, on the whole, seems adequate, but the term "good-natured algorithm" is conspicuously missing.

Unfortunately, the book is not error-free; in fact, there are quite a few of them. However, most of the errors are of a trivial nature and can easily be rectified by an alert reader.

From the above outline of content, it should be apparent that we have here before us a text which is thoroughly up-to-date, original both in the selection of topics and in their mathematical treatment, a book, in short, which contains the essence of a good many years of experience in computing. Adding to this the extreme clarity and conciseness of exposition makes this indeed one of the outstanding introductory texts in numerical analysis. It is only to be hoped that an English translation will be available in the not too distant future.

W. G.

1. A. H. Stroud & D. Secrest, *Gaussian Quadrature Formulas*, Prentice-Hall, Englewood Cliffs, N. J., 1964.

18[2.10].—James L. Phillips & Richard J. Hanson, *Gauss Quadrature Rules with B-Spline Weight Functions*, 28 pages of tables and 4 pages of explanatory text, reproduced on the microfiche card attached to this issue.

The abscissas and weights of n-point Gaussian quadrature rules for integrals

$$\int_{-1}^{1} N(i, k; t)y(t)\, dt$$

are tabulated to 14S for $n = 1(1)17$, $k = 2, 4$, $i = 1(1)k$. Here $N(i, k; t)$ is a normalized B-spline of order k (degree $k - 1$) with support on $(-1, 1)$. Translates and reflections of the k B-splines $N(1, k; t), \cdots, N(k, k; t)$ provide a basis for the space of splines of order k defined on an interval $[a, b]$ with respect to a partition of equally spaced interior knots and end knots of multiplicity k.

The first 17 coefficients in the three term recurrence formula for polynomials orthonormal on $(-1, 1)$ with respect to the weight function $N(i, k; t)$ are given to 14S for the same values of i and k.

Details of the underlying calculations on an IBM 360/67 at Washington State University are also furnished.

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19[2.30, 7].—W. A. BEYER & M. S. WATERMAN, *Decimals and Partial Quotients of Euler's Constant and* ln 2, ms. of 28 computer sheets deposited in the UMT file.

Herein are tabulated Euler's constant γ and ln 2 to 7114D and 7121D, respectively, as well as the first 6922 and 6890 partial quotients of the corresponding simple continued fractions. Details of the calculations are presented in a paper [1] by the same authors in this issue.

It may be of interest to record here that in the first 6922 partial quotients for γ a total of nine exceed 1000; namely, $a_{528} = 2076$, $a_{1245} = 1168$, $a_{1273} = 1672$, $a_{1553} = 1925$, $a_{2286} = 1012$, $a_{3079} = 1002$, $a_{3751} = 1095$, $a_{4802} = 2254$, and $a_{5428} = 4351$. This count is in excellent agreement with the Gauss-Kuzmin law, which predicts for almost all real numbers a count of 10 such quotients for this sample size. On the other hand, of the first 6890 partial quotients for ln 2 only six exceed 1000; namely, $a_{501} = 3377$, $a_{1271} = 2745$, $a_{3137} = 1247$, $a_{3915} = 2158$, $a_{5262} = 2765$, and $a_{6803} = 1350$.

In [1] the authors tabulate the individual relative frequencies of those partial quotients among the first 3470 for γ that do not exceed 10 in magnitude.

J. W. W.

1. W. A. BEYER & M. S. WATERMAN, "Error analysis of a computation of Euler's constant," *Math. Comp.*, v. 28, 1974, pp. 599–604.

20[2.60].—RAYMOND E. MILLER & JAMES W. THATCHER, Editors, *Complexity of Computer Computations*, Plenum Publishing Corporation, New York, 1972, 225 pp., 25 cm. Price $16.50.

This book is the proceedings of a symposium held at the IBM Thomas J. Watson Research Center in March 1972. It contains the fourteen presented papers plus an account of the panel discussion session. There was considerable attention given in the panel discussion to the field of the symposium. There was no agreement on a suitable name although "computational complexity", "computability", "theory of algorithms" and "concrete computational complexity" were suggested. Neither was there good agreement on the content of the field, but the symposium (and this book) itself serve admirably to delineate the field. That is, the content of this field (whatever it is called) is that which the people in the field are doing.

There are two branches to the field, one numerical and the other combinatorial in nature. Space precludes presenting a review of each of the fourteen papers, but, since the nature of this field is a prime question at this time, a very short description is given for each paper. The order is that of the book.

NUMERICAL COMPUTATIONS

1. V. STRASSEN. Analysis of the number of arithmetic operations required to evaluate a rational function.

2. M. O. RUBIN. Analysis of the effort to solve a system of n linear equations using only scalar product computations. At least $n(n + 1)/2 - 1$ inner products must be used.

3. E. M. REINGOLD & A. I. STOCKS. New and more elementary proofs of the lower bounds on the number of arithmetic operations required to evaluate a polynomial.

4. C. M. FIDUCCIA. An analysis of fast matrix multiplication algorithms which involves a new representation/interpretation of the situation.

5. M. S. PATERSON. Applies ideas from the study of the efficiency of solving a nonlinear equation to the problem of evaluating an algebraic number (solving a polynomial equation).

6. S. WINOGRAD. An analysis of the behavior of parallel algorithms for solving a nonlinear equation. Parallelism does not pay off.

7. R. BRENT. Analysis of local iterative methods (which use no derivatives) for the solution of systems of nonlinear equations. A conjecture is made about the optimum efficiency.

8. M. SCHULTZ. Demonstration that, in a certain reasonable sense, the numerical solution of an elliptic partial differential equation is as efficient as the tabulation of the solution from a known closed form formula.

COMBINATORIAL COMPUTATIONS

9. R. M. KARP. Presentation of a systematic method of establishing the equivalence of problems in complexity. A large number are shown to be equivalent although their complexity is still unknown.

10. R. W. FLOYD. Presents bounds on the work required to rearrange information (in pages and records) in a slow memory by bringing pairs of pages into a fast memory where records may be rearranged.

11. V. R. PRATT. Analysis of the effort in defining a computer library given the probability of accessing the ith program after the jth one. Results are given in some special cases.

12. D. C. VANVOORHIS. Study of the smallest number of components required for constructing a sorting network.

13. J. E. HOPCROFT & R. E. TARJAN. Presentation of an algorithm to determine if two planar graphs with n vertices are isomorphic. It uses $O(n \log n)$ operations.

14. M. J. FISHER. Proof of a new upper bound on the computation required to obtain the finest partition of a set consistent with a given set of equivalence relations.

The panel discussion centered on two questions: "Is there an emerging unity in this field?" and "Are real computations improved as a result of studies in this field?" There was some optimism and considerable doubt expressed about the unity of this field. The diversity in the field is underscored by the fact that few people will feel comfortable with all the papers presented. The case for stating that these studies have had an effect on real computation is weak. However, several people expressed the opinion that the effect will be felt in the future as this field provides the proper framework to think about computation. The reviewer agrees with this opinion and even with the one that this field and classical numerical analysis will merge at some future time.

In summary, this book provides a good snapshot of the "complexity of computer computations" field as of 1972. A number of significant research results are presented and the panel discussion transcript allows one to obtain a feel for the thinking of some of the leaders in this field. The book is overpriced in view of its length and the lack of typesetting or royalty expenses to the publisher.

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21 [3].—J. K. REID, Editor, *Large Sparse Sets of Linear Equations*, Academic Press, London-New York, 1971, ix + 283 pp., Price $16—.

DONALD J. ROSE, RALPH A. WILLOUGHBY, Editors, *Sparse Matrices and their Applications*, Plenum Publishing Corporation, New York-London, 1972, xii + 215 pp., 26 cm. Price $11.50.

We have here the proceedings of two conferences, one in Oxford in 1971, the other at the IBM Research Center in Yorktown Heights in 1972. They are more important than most sets of proceedings because, for a while, they will be *the* books in this new area called sparse matrix technology. Although attempts to exploit the zero elements in a matrix date back more then twenty years, the first explicit move to coordinate these efforts was Willoughby's first sparse matrix conference in 1968. Both of the books under review have been carefully edited and there is little overlap between them. They make a good introduction to the subject.

Let us begin with a few words about the subject itself. What are all these research workers trying to do? Mostly, they are trying to solve $Ax = b$ and to update A^{-1} after modifying A. Amazing. Can people still find something new to say on these corny old subjects? The answer is yes, and it is interesting to see how this comes about.

In the beginning came the existence of a solution, when A^{-1} existed, and a formula for the solution in terms of the data (Cramer's rule). Then there was Gaussian elimination for those unfortunates who actually had to find a solution. The advent of digital computers after World War II led people to apply elimination automatically to big 30×30 matrices and this brought on a new worry, namely its stability and then a detailed discussion of minor variations. During the 50's came the first analyses of iterative methods, and a full understanding of the stability question for triangular factorization. However, the difficulties in finding a proper scaling both for equations and unknowns were not yet appreciated. George Forsythe wrote a paper entitled "Solving linear equations can be interesting" which sought to explain this flurry of activity to the rest of the mathematical community. Yet few of the issues confronted in these conferences were discussed in that article. What more could there be to say on the subject?

A basic error to be avoided is the assumption that a numerical analyst is simply devoting his talents to help users save money. What if a computation that used to take 1 hour now takes 10 seconds? The answer is simple. Advances in computers and numerical methods have made *infinite* improvements; they have permitted the accomplishment of tasks that were previously infeasible. That *is* significant. It is this pressure to solve bigger and more complex problems that has led people to return again and again to look in ever increasing detail at such basic tools as a linear equations solver. Once standard algorithms were in good shape, the limitations of storage and execution time came to the fore.

A matrix of order one thousand has a million elements and most computers have fast, random access memories of fewer than seventy thousand cells. If few of the elements are zero, then it is necessary to use a much slower back up storage and one is faced with the problem of how to reorganize the calculation. Moreover, a machine which can perform a million multiplications in a second would still take more than five hours to solve a single system by Gaussian elimination. In many applications,

the system is to be solved hundreds if not thousands of times and the project would not be feasible. However, most elements of the big matrices which arise in practice are zero. The problem becomes tractable if the conventional storage of the matrix (one cell for each element) is abandoned and zero elements are not stored.

Band matrices are easy to deal with. In such matrices, the (i, j) element vanishes if $|i - j|$ exceeds some number w which is small compared with the order. More difficult are those matrices in which the few nonzero elements are scattered randomly in the matrix. What is the most efficient way of representing such an array in the memory? This is a nontrivial problem in what is called data structures in computer science departments. How does the array representation affect the implementation of triangular factorization or iterative techniques?

So the reader unfamiliar with sparse matrices will begin to see that the new problems lie less in the domain of traditional numerical linear algebra and more in the realm of software, computer architecture, and graph theory. Naturally, much of the pioneering work has been done by those who had real problems to solve. It is perhaps worth mentioning that most of these large matrices are themselves generated in the computer by other programs and the results used immediately in yet other programs, no part of the process necessarily being seen by the user.

In the Oxford conference, there were papers showing the particular form of sparseness that arises in linear programming, in the static analysis of stresses and strains in structures, in geodesy, in high voltage power systems and in other network flows. There is a review of direct and iterative methods and, inevitably, some of these techniques are described in almost every paper. Mercifully, matrix notation is now universal and this itself indicates how the study of numerical methods has matured. Typical points which are taken up are (i) what is the best way of representing an inverse as a product of elementary factors (the PFI versus the EFI, for those in the trade), (ii) how should the equations be ordered to minimize bandwidth (let the academics worry about stability?), (iii) can standard iterative techniques, like over-relaxation or clever adaptations of the conjugate gradient method, compete with these specialized direct methods? In addition, there was a paper showing the utility of graph theory for discussing sparseness structure and the effect of pivot selection (that is the ordering of the rows and/or columns) on the fill-in of previously zero elements as the elimination proceeds. Another paper describes a programming system, written in FORTRAN, to facilitate the management of the linked list structure used to store the matrix.

A nice feature of the book is the inclusion of the discussions which followed the presentation of each paper.

The second conference, at Yorktown Heights, took advantage of the work done at the previous two conferences and omitted surveys of the major fields of application. The first chapter, by the editors, is a very useful discussion of the whole conference including comments on all of the papers. These were divided into the following categories: Computational Circuit Design, Linear Programming (problems with 7000 rows have been solved), Partial Differential Equations (particularly finite element matrices, whose orders can approach 100,000, and the generation, storage, and fetching of their elements), Special Topics (applications to Photogrammetry, Data Base Systems), Combinatorics and Graph Theory (optimal orderings, minimization of bandwidth), and a Bibliography.

The role of storage recurs throughout the book; what different "levels" of memory are available, how are they related, how can each hierarchy be best exploited?

The work of Gustavson and his colleagues is described here. They began in 1966 with the clever idea of a program whose output is not the solution to $Ax = b$ but instead a loop-free machine language code to compute x taking explicit advantage of the sparseness structure of A. This is dramatically efficient for the common case in which many problems are to be solved but all sharing the same sparseness structure. The personalized compiler has arrived.

B. N. P.

22 [5, 13.20].—F. BAUER, P. GARABEDIAN & D. KORN, *Supercritical Wing Sections*, Springer-Verlag, Berlin, Heidelberg, New York, 1972, v + 211 pp., 26 cm. Price $6.40.

When aircraft fly at high subsonic velocities, an increasing aerodynamic resistance or drag is experienced as the speed of sound is approached. Initially, it was thought that the drag was unbounded at Mach 1 and, hence, that a "sonic barrier" had been discovered. Now we know that the barrier is only a local maximum, that supersonic flight is possible on one side, and that supercritical subsonic flight is possible on the other side. In the latter region, local regions of supersonic flow appear on wings and bodies, and special designs are required to avoid strong shock waves and accompanying adverse effects. As stated in the preface of *Supercritical Wing Sections* "The purpose of this report is to make available to the engineering public mathematical methods for the design of supercritical wings." These are the methods which have been used by the authors to design and analyze two-dimensional shock free supercritical airfoils. The publication, written in the style of a technical report, is divided into three parts giving the mathematical theory, a users' manual for the listed computer programs, and calculated results and examples.

Part I reviews the theoretical formulations with primary emphasis on the airfoil design techniques using the method of complex characteristics. In this ingenious method, the mixed elliptic-hyperbolic partial differential equation is made linear by a hodograph transformation, and, then, all variables are analytically continued from real 2-space to complex 4-space. The resulting equation is purely hyperbolic with distinct eigenvalues except at the sonic locus. The problem posed is of the inverse variety, that is, to find suitable initial data which yield a solution with a desirable body shape and pressure distribution, with the correct flow at infinity and which is shock free (no external limit lines). Selecting the proper initial data is complicated, and a general form is deduced based on knowledge of the incompressible flow past an ellipse and on experience. The singular part of the solution may be written as an integral formula, and the remainder is obtained by numerical integration of linear nonhomogeneous characteristic equations.

Two other theoretical methods which are used are briefly discussed in Part I. An integral boundary layer theory due to Nash and MacDonald is incorporated in the design method to predict the separation point and to make a displacement thickness correction for the airfoil shape. Solutions for off-design conditions are

obtained from the analysis program which uses relaxation methods to solve the mixed elliptic-hyperbolic partial differential equation for flow about a specified shape (direct problem). It is assumed that embedded shock waves will be weak enough to introduce negligible vorticity and, hence, a potential flow model is used. The solution procedure is quite efficient, but numerical instabilities may be encountered for large regions of supersonic flow. In addition, the shock wave jump may be in error since the difference equations are not in conservative form.

Part II of the report is a users' manual describing the input procedures and execution of the program. Program listings and an example case are given. It is intended that the programs may be implemented without understanding the theory of Part I, and experience will tell if this is realizable. There are several aspects of the design program which require "individual skill and ingenuity" for "success." The programs are written in ANSI Fortran IV and designed for a teletype time-sharing system. Several computed outputs are given in Part III to illustrate the versatility of the techniques.

In general, the publication is a laudable effort to implement theoretical research into the design process. One weakness in this respect could be that only sparse information is provided on the experience with other users of the programs or with the limitations of the methods. Future improvements in this regard and in the boundary layer and analysis methods will undoubtedly be forthcoming.

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23 [7].—D. S. MITRINOVIC, *Uvod u Specijalne Funkcije* (*Introduction to the Special Functions*) (in Serbo-Croatian), Izdavacko Preduzeće Gradevinska Knjiga, Beograd, 1972, xi + 188 pp., 24 cm.

At a first glance of the volume under consideration, it would appear difficult to render a faithful review since the book is written in a totally unfamiliar language. By the same token, it would seem that the volume would be of little use to potential readers not acquainted with this language. On closer examination, the language barrier is considerably softened because there is little text, and what little there is is such that the meaning can usually be identified by the closeness of the words to their English counterparts and by the mathematical equations and notation surrounding the text.

The volume, which is divided into eight chapters, is essentially in the form of a handbook, though some proofs are given in sketchy form. On the other hand, numerous results are stated without proof in the principal part of the text or are given at the end of each chapter as a problem. Thus, the tome is ideally suited for self-study or as a pedagogical aid for classroom instruction. The results given are fundamental to the subject and are those which one would normally expect in an introductory text on the subject. A wealth of material is covered. The chapter titles are (1) Gamma Function and Beta Function, (2) Legendre Polynomials, (3) Laguerre Polynomials, (4) Hermite Polynomials, (5) Chebyshev Polynomials, (6) Bessel Functions, (7)

Orthogonal Functions, (8) Laplace Partial Differential Equation and Special Functions, (9) Examination Questions in Special Functions and (10) Tables of Special Functions.

Further comments on the first eight chapters are not required in view of our previous remarks. Chapter 9 is a list of exercises taken from examinations given to students by the Electrotechnical Faculty of the University of Beograd. Chapter 10 contains 5D tables of the basic functions pertinent to the material of Chapters 1–8. Thus, there are tables of the gamma function and its logarithmic derivative, the classical orthogonal polynomials and the various Bessel functions.

A bibliography and notation index enhance the usefulness of the volume.

Y.L.L.

24 [7].—D. S. MITRINOVIC, with the assistance of D. D. TOSIC & R. R. JANIC, *Specijalne Funkcije—Zbornik Zadataka i Problema* (*A Collection of Exercises and Problems*) (in Serbo-Croatian), Naucna Prjiga, Beograd, 1972, xii + 158 pp., 24 cm.

This work contains 375 problems. It can be considered a companion volume to the above reviewed *Special Functions* by the same author. The general remarks made there also pertain here. The first six chapters in both volumes have the same titles. Here, Chapter 7 is a collection of miscellaneous problems.

Except for Chapter 7, each chapter is in two parts. The first part states basic definitions and the second gives problems, all of which can be solved by use of the data in the first part. For the more difficult problems, hints are given and, in certain instances, there are references to the literature. Many of the problems are taken from the problem sections of such journals as Matematicki Vesnik, American Mathematical Monthly and Mathematical Gazette.

Y. L. L.

25 [7].—C. J. TRANTER, *Bessel Functions with Some Physical Applications*, Hart Publishing Co., Inc., New York, 1969, ix + 148 pp., 24 cm. Price $10.00.

I quote the first paragraph from the author's preface: "The classic work on Bessel functions is G. N. Watson's monumental treatise. This great work was completed in 1922 and therefore lacks references to developments in the subject during the last forty-five years. Its high standard of rigour and great size also make it somewhat forbidding to the scientist who is only interested in applications to physical problems. I have consequently attempted in the present book to provide a short up-to-date account of Bessel functions which will be useful to the increasing number of scientists and engineers who encounter these functions in their work."

The volume is divided into eight easily read chapters. The chapter titles are indicative of the material covered and are as follows: 1. The solution of Bessel's and associated equations. 2. Some indefinite integrals, expansions and addition

theorems. 3. Integral representations and asymptotic expansions. 4. Zeros of Bessel functions, Fourier-Bessel series and Hankel transforms. 5. Some finite and infinite definite integrals containing Bessel functions. 6. Dual integral and dual series equations. 7. The equations of mathematical physics: solution by separation of variables. 8. The equations of mathematical physics: solution by integral transforms.

Contrary to what might be inferred from the second sentence of the preface quoted, for the most part one can only claim that Chapters 6–8 present material not given by Watson. I find the lack of references disturbing. The bibliography consists of only nine books. On p. 82 and p. 84, reference is made to V. G. Smith's formula and to a study of certain integrals by H. F. Willis, respectively, but the sources are not given.

To the novice who wants to get at some tools quickly, the volume will be useful. However, except for the new material noted above, I would much prefer to use Watson. On the plus side of the ledger, each chapter contains a number of exercises which should prove useful for self-study purposes. Chapters 6–8 are enhanced by inclusion of physical applications.

Y. L. L.

26 [7].—SERGE COLOMBO & JEAN LAVOINE, *Transformations de Laplace et de Mellin*, Gauthier-Villars, Paris, 1972, xiii + 170 pp., 24 cm. Price F 96.— (paper bound).

There are several tables of integrals of transforms available. These are all essentially of the same kind since the integrals are defined in the sense of Riemann with the further proviso that we also include integrals of the Cauchy principal value type. The present volume is distinctive in that it contains material not found in previous compilations.

In rather recent times, items such as distributions, modified distributions and pseudo-functions have received considerable attention. The terminology "generalized functions" is often used. It is not our purpose to define these concepts, but an example is useful for the review at hand. With sufficient regularity conditions on $g(t)$, $G(\nu) = \int_\alpha^\beta t^{\nu'+\nu} g(t)\, dt$ is meromorphic in the half plane $R(\nu) > -R(\nu') - 1$. Let Pf stand for pseudo-function. Then $Pf \int_\alpha^\beta g(t)\, dt$ equals $G(-\nu')$ if $-\nu'$ is a regular point, and equals the constant term in the Laurent expansion of $G(\nu)$ about $-\nu'$, if $-\nu'$ is a pole. This is Hadamard's finite part of the integral. Thus, in a table of Laplace transforms $\int_0^\infty e^{-pt} g(t)\, dt$, the Laplace transform of $g(t) = t^{-1/2}$ is $(\pi/p)^{1/2}$, and the corresponding pseudo-function for $g(t) = t^{-1}$ is $-(\gamma + \ln p)$.

The volume at hand is in two parts. The first is a general discussion of integral transforms as ordinarily conceived, generalized functions and their transforms with special emphasis on Laplace transforms (both one-sided and two-sided), Mellin transforms and their inverses. The second is a list of particular transforms of the above type, including ordinary as well as the corresponding pseudo-functions.

This useful tome contains a fairly complete bibliography. A table of notations is also included.

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27 [8].—S. JOHN, *Critical Values for Inference about Normal Dispersion*, The Australian National University, Canberra. Ms. of 17 computer sheets deposited in the UMT file.

If $f_N(\cdot)$ represents the density function of χ_N^2, this table consists of values of C_1 and C_2 to 3D (generally 5 or 6S) satisfying the twin conditions $\Pr(C_1 \leqq \chi_N^2 \leqq C_2) = 1 - \alpha$ and $f_{N+2}(C_1) = f_{N+2}(C_2)$ for $\alpha = 0.2, 0.1, 0.05, 0.01, 0.005, 0.001$, and $N = 1(1)350$. The conditions $\Pr(C_1 \leqq \chi_N^2 \leqq C_2) = 1 - \alpha$, $E(\chi_N^2 \mid C_1 \leqq \chi_N^2 \leqq C_2) = N$ are jointly equivalent to the previous pair.

The values of C_1 and C_2 are required for two-sided tests of normal dispersion. An example of a problem that can be brought to a two-sided test of normal dispersion by a transformation is that of testing whether the counts of two or more organisms in soil samples of given volume have independent Poisson distributions.

The method used in calculating the tables, together with an account of previous tables of this type, is presented in a paper [1] by the author.

AUTHOR'S SUMMARY

1. S. JOHN, "Critical values for inference about normal dispersion," *Austral. J. Statist.*, v. 15, 1973, pp. 71–79.

28 [12].—GRANNINO A. KORN, *Minicomputers for Engineers and Scientists*, McGraw-Hill Book Co., 1973, New York, xiv + 303 pp., 24 cm. Price $17.75.

The book is a fair to good compilation of currently available minicomputer handbook literature. The book is well formated and reasonably well written.

The text has tutorial value, but could be misleading as a design reference, if the criterion of novelty is applied. That is, the book's content is extracted mostly from manufacturers' manuals that are recurrently updated to keep abreast of the latest products. Thus, the book's usefulness and longevity as a classroom text are doubtful, since the transience and innovativeness of the minicomputer doom the book to prompt obsolescence.

This reviewer finds it difficult to justify the cost of the book, $17.75, since the same information can be obtained gratuitously from manufacturer's handbooks.

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