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Mathematics of Computation

Volume 29 · Number 129

Special Issue ~ Dedicated to

DERRICK HENRY LEHMER

January 1975

Published by
American Mathematical Society
Providence · Rhode Island
Derrick Henry Lehmer

THE EDITORS of Mathematics of Computation are pleased to dedicate this issue to Derrick Henry Lehmer on the occasion of his seventieth birthday, February 23, 1975.

Lehmer played a role in the founding of MTAC and was an active member of the executive committee that produced the first issue in January, 1943, under the editorship of R. C. Archibald. In the 1946—1949 volumes, Lehmer joined Archibald as an editor, while in the 1950—1954 volumes, Lehmer was the chairman of an enlarged group of editors.

In an effort to keep this dedication a surprise and to keep the issue modest in size, the editors did not inform all of the people who would have liked to contribute articles to celebrate Lehmer's birthday. In fact, the editors have postponed the publication of a paper jointly authored by the dedicatee. To those who do not appear here, we apologize.

We must make mention of a special one, who was not informed about this issue—Emma Lehmer. She has also been associated with this journal from its beginning and continues to assist us to this day. We are in her debt too!

The editors are also pleased to acknowledge that this issue was made possible through the generous support that was received from friends of the Lehmers.

Eugene Isaacson
for the editors
# Mathematics of Computation

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INDEX TO MATHEMATICS OF COMPUTATION, 1943—1969

Classification System by
Yudell L. Luke, Jet Wimp and Wyman Fair

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The INDEX TO MATHEMATICS OF COMPUTATION is a compilation, by author and by subject, of all material which has appeared in MATHEMATICS OF COMPUTATION and its predecessor, MATHEMATICAL TABLES AND OTHER AIDS TO COMPUTATION, during the years 1943—1969—twenty-three published volumes. The INDEX contains over 7,000 entries. This is an unusual compilation because of the unique character of the journal which not only publishes research papers, but also publishes reviews of material on mathematics of computation and a table errata section covering a number of other publications. In addition, an unpublished mathematical tables (UMT) file is maintained.

A new classification system, which was developed in 1969 by a committee chaired by Yudell Luke at the Midwest Research Institute, is used in the subject classification index. In this section, all articles, tables, reviews, etc. are classified. The classification scheme is designed as an indexing system for retrieval of information in MATHEMATICS OF COMPUTATION, and the present index contains classification numbers for all entries beginning with 1943.

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Smith, A. B. 1PR, 3E

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The preparation and publication of the subject classification index was supported in part by a grant from the National Science Foundation (GN-691).

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