REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the indexing system printed in Volume 28, Number 128, October 1974, page 1191.

16 [2.00, 3, 4].—R. W. HAMMING, *Numerical Methods for Scientists and Engineers,* 2nd ed., McGraw-Hill Book Co., New York, 1973, ix + 721 pp., 24 cm. Price $14.95.

After eleven years, the first edition of Hamming's text had gradually vanished from the consciousness of numerical analysts. Thus, the encounter with this second edition will be a first encounter for many of the younger scientists in the computer field. Moreover, the text has been rewritten and expanded to such an extent that it reads like new, even to those who have known the previous version rather well. What has remained and is even more present than before is the author's original opinion on many subjects in numerical mathematics, an attitude which is based on a wealth of experience and a continuous striving to understand what happens in numerical computations. "The purpose of computing is insight, not numbers"; the author's motto for this book indeed describes his basic attitude well. (And his pun "the purpose of computing numbers is not yet in sight" characterizes a lot of the work that keeps our computers busy.)

Although most sections treat standard problems of numerical mathematics, there is hardly a page in the book which does not contain something of interest even to those who have handled these problems for a long time. Here it is a particularly intuitive approach to a well-known result, there a new illustrative example, here an ironic side-remark, there a statement which throws new light onto a whole line of development. The careful consideration of the interaction between mathematics and digital computing is ever present.

The material is arranged into three main and two supplementary chapters. The first one ("fundamentals and algorithms") begins with an exposition of the author's ideas on numerical methods. Then he considers machine numbers (including the frequency distribution of mantissas), evaluation of functions, and finding zeros (with an unusual attention to complex zeros). The treatment of linear equations is very short and yet displays a few original thoughts on what constitutes ill-conditioning. A section on (pseudo-) random numbers and their generators is followed by an introduction into the computationally relevant parts of the difference and the summation calculus. Numerical summation of infinite series is discussed and various techniques are illustrated; difference equations and recurrences are studied. The discussion of round-off and its estimation includes the statistical approach; interval analysis is dismissed perhaps too lightly.

The second chapter is named "polynomial approximation—classical theory" to be followed by chapter three on "Fourier approximation—modern theory". This confrontation emphasizes the author's opinion that the frequency approach to the finite approximation of analytic operations is more relevant than the classical truncation error analysis. Nevertheless, he devotes a considerable effort to an excellent introduction into the classical ideas. The derivation of Peano's theorem and the discussion of its applications are beautifully clear and elementary. Formulas using differences are given a fair attention while splines receive less emphasis than one would expect. The treatment of ordinary differential equations is introduced by a discussion of indefinite integrals where the stability problem is simpler; the author then describes predictor-

corrector and Runge-Kutta methods and points out special situations like stiff systems. The essentials of both least squares and Chebyshev approximation are presented and the importance of orthogonal functions and Chebyshev polynomials, resp., is well explained. The "classical" chapter ends with a section on approximation by rational functions.

Chapter 3 distinguishes this book from virtually any other in the field. Of course, numerical Fourier analysis does appear in most books, but only as one of many ways of approximation in the "time" domain whereas the "frequency" domain is hardly ever considered. The basis of Hamming's treatment is the "aliasing" effect of equidistant sampling which makes higher frequencies appear in the disguise of lower ones. This idea permits a new and often very natural appraisal of sampling distances in general when it is extended to nonperiodic functions by the consideration of the Fourier transform, and it also provides an interesting comparison with the polynomial error theory of Peano's theorem. It makes it natural to attempt a minimization of the error in the frequency domain which yields new types of formulae. While smoothing, filtering and similar subjects are treated at length, an explicit discussion of attenuation factors is strangely missing. Of course, the fast Fourier transform appears prominently. A section on the quantization of signals concludes this chapter.

Two short chapters follow. The one on exponential approximation discusses the characteristics and pitfalls of the approach; Prony's method is suggested for the determination of unknown exponents. The author then gives a short introduction to the Laplace transform; finally, he discusses some of the inherent problems of simulation, again emphasizing the frequency approach. The last chapter presents the author's view of some unrelated subjects: Numerical treatment of singularities, nonlinear optimization, and eigenvalues of Hermitian matrices. Two more "philosophic" sections, on linear independence and on general aspects of scientific computing, round up the heavy volume.

It may be difficult to use the book as a text for a course on numerical mathematics: Math majors may miss the formal rigor, science students may shun some of the mathematics, beginning computer adepts will not appreciate the wisdom of the author's remarks and more advanced ones may be deceived by the elementary looking treatment. Nevertheless, the book is a must for everybody teaching numerical mathematics at any level to any audience: There is hardly a subject in the field on which such a person will not find some new stimulation for his teaching job. And for those whom Hamming is addressing directly—scientists and engineers with numerical computing needs and some computing experience—the book is probably the best there is.

J. S.

17 [2.00, 3, 4].—THE OPEN UNIVERSITY, *Course Books,* Harper and Row, New York, 1973.

In 1969, The Open University (United Kingdom) received its charter to bring higher education into the private home. It uses radio, television, specially written correspondence material, cassettes and tapes, residential summer schools and local study centers. The readings and assignments are carefully coordinated with the BBC's broadcasts.

In Mathematics, there are four courses so far: Foundations (36 units), Linear Mathematics (33 units), Elementary Mathematics for Science and Technology (17 units), Mechanics and Applied Calculus (16 units). As an illustration, here are the topics in

the Linear Mathematics course: vector spaces, linear transforms, Hermite normal form, differential equations I, eigenvalues, recurrence relations, numerical solution of $Ax = b$, homogeneous differential equations, Jordan normal form, non-homogeneous differential equations, linear functionals and duality, bilinear and quadratic forms, affine geometry and convex cones, inner product spaces, linear programming, least-squares approximations, convergence, numerical solution of ODE's, Fourier series, wave equation, orthogonal and symmetric transformations, boundary value problems, Chebyshev approximation, theory of games, Laplace transforms, numerical solution of eigenvalue problems, differential equations II (resonance), heat conduction, existence and uniqueness theorems.

With each unit comes a 50-page paperback booklet whose style reflects the absence of teaching assistants, office hours, and discussion sections. There are plenty of exercises, lots of pictures, and, what is most striking, frugality in the exposition. This last comment is intended to be a compliment: often a text presents too many results and the student is given no perspective on them. In these booklets, the key facts are set down clearly, with proofs where appropriate. Frequent use is made of summaries and glossaries.

The units appear to be excellent for self-instruction, and sell for about $3.00 each. The prerequisite structure of units within each course is indicated pictorially on the cover. However, these units are not always self-contained. Two specified texts (from the U. S. A.) are needed for the Linear Mathematics course.

The unit on Chebyshev approximation, for example, has three sections: best polynomial approximations, Chebyshev polynomials and Chebyshev series. Subsections on the Remez algorithm and the computation of Chebyshev coefficients are optional. This unit is almost entirely independent of the two textbooks, whereas the Fourier series unit is essentially a commentary on one of the textbook's treatment of the subject.

There are films supplementing each unit: $125 for black and white, $275 for color. The cassettes were priced at $7.50 each.

Britain's experiment in higher education without a campus will probably be watched with apprehension by academics and with keen interest by state legislators.* The idea does seem particularly relevant to adult education and retraining.

B. P.

18 [2.00, 3, 4, 5, 6, 8, 12].— GERMUND DAHLQUIST & ÅKE BJÖRCK, translated by NED ANDERSON, *Numerical Methods,* Prentice-Hall, Inc., Englewood Cliffs, N. J., 1974, xviii + 573 pp., 24 cm. Price $15.95 (clothbound).

This is a remarkable textbook as well as a handbook for scientific computation. It is filled with well-written, succinct descriptions of methods and algorithms, together with mathematical analyses, practical observations, splendid exercises, and references to the literature for more detailed treatments. The scope of the topics (as indicated in the chapter headings that are listed at the end of this review) is greater than I would have thought to be feasible in a volume of this size. Nevertheless, the authors succeed admirably. The verbiage is kept to a minimum, so that it is easy to find clear explicit descriptions of the methods. This feature should make it possible to use the work as a handbook and as a text in an undergraduate numerical methods course, where the emphasis is on learning how to solve problems, rather than on the mathematical analysis

* No riots, no panty raids, no library.

of the procedures. On the other hand, for users with a good background in calculus, linear algebra, and ordinary differential equations, the book makes it possible to become expert in numerical analysis by reading the text and doing the excellent problem sets that follow each major section. Solutions for most of the exercises, appear in Chapter 12. The authors present a wise balance of small-scale and large-scale computing methods. An instructor would have to make a judicious selection of the material to be covered by his class, since the book contains such a wealth of material treated with varying levels of mathematical sophistication.

This book is a beautifully written and improved translation of the authors' Swedish work published in 1969. The translator, Ned Anderson, is credited by the authors with improving the presentation. The chapter headings are:

E. I.

19 [2.05].– G. G. LORENTZ, Editor, in cooperation with H. BERENS, E. W. CHENEY, L. L. SCHUMAKER, *Proceedings of an International Symposium Conducted by the University of Texas and the National Science Foundation, January* 22–24, 1973, Academic Press, Inc., New York, 1973, xiii + 525 pp., 24 cm. Price $17.00.

These proceedings contain six long articles and forty-nine shorter articles (eight typed pages maximum length). The shorter articles cover the entire field of approximation theory and include announcements of new results, summaries of previous work, and short research papers complete with proofs.

The long article by P. L. Butzer is a survey of the recent work by his colleagues in Aachen. There are eight books and 101 papers cited in the references, not counting works from outside Aachen. This seventy-page paper has nine distinct parts, the first four of which are the longest. These are

Basic approximation theory
Semigroup related results
Fourier analysis on R^n
Fourier analysis in Banach spaces
Approximation on compact manifolds

Best asymptotic constants
Kernels of finite oscillation
Spline approximation
Calculus for Walsh functions

The thirty-page paper by H. Berens and G. G. Lorentz considers Korovkin-type theorems for positive linear operators on Banach lattices. The paper primarily contains new results (extensions of previous results to Banach lattices). There are also many historical remarks, and thus the paper provides a thorough presentation of the results in this area.

The fifty-page paper by T. W. Gamelin outlines the ways in which the abstract theory of uniform algebras may be used to extend some classical approximation results to more general and/or abstract settings. The presentation revolves about three approximation problems: *Problem* I. Find conditions on a continuous complex-valued function $f(z)$ defined on K which guarantee that f can be approximated uniformly by functions analytic in a neighborhood of K. *Problem* II. What conditions on K guarantee that every continuous real-valued function on ∂K can be approximated uniformly by the real parts of functions which are analytic on a neighborhood of K. *Problem* III. Find conditions on a bounded analytic function $f(z)$ defined on K^0 which guarantee that f be a pointwise limit of a sequence of functions analytic in a neighborhood of K^0 and uniformly bounded on K^0.

The forty-eight-page paper by J. W. Jerome on multivariate approximation selects six topics from this field and summarizes the current state of approximation theory research for each of them. The six topics are

Generalized Peano kernel theorems
Spline approximations
Asymptotic estimates of widths
The finite element method
Rectangular grids in R^n
Convergence of Galerkin methods

The works of many people have been organized into a smooth and coordinated presentation.

The fourteen-page paper by D. J. Newman integrates the classical Jackson and Muntz theorems into a single setting. Consider approximation to continuous functions by $c_0 + \Sigma_{i=1}^n c_i x^{\lambda_i}$ where $0 < \lambda_1 < \lambda_2 < \cdots$. A typical result is that, if $\lambda_{i+1} - \lambda_i \geqslant 2$, then

$$\|f(x) - P_n(x)\| \leqslant K\omega_f[e^{-2\Sigma_{i=1}^n (1/\lambda_i)}],$$

where $P_n(x)$ is the best polynomial approximation and ω_f is the modulus of continuity.

The twenty-five-page paper by Daniel Wulbert considers the following problem: let $f(x)$ be continuous on the C^k-manifold M and assume $p(x) > 0$ on M. Is there a C^k function $g(x)$ defined on M so that $|f(x) - g(x)| < p(x)$ for all $x \in M$? The answer is "yes" if M is finite-dimensional and the extensions of this result to infinite-dimensional cases is the topic surveyed in this paper.

In summary, this is a valuable addition to the collection of books that a worker in approximation theory requires. The long (and some of the shorter) papers give an organized presentation of the current status of some important areas in approximation theory. The remaining papers give a broad cross section of the current activity in

approximation theory which furnishes valuable insight into 'who considers what worthwhile and interesting'.

J. R. R.

20 [2.05.2].—R. P. FEINERMAN & D. J. NEWMAN, *Polynomial Approximation*, The Williams & Wilkins Co., Baltimore, Md., 1974, viii + 148 pp., 24 cm. Price $13.00.

A descriptive title for this book is "Degree of convergence for polynomial and rational approximation on the real line". This is a thorough and compact presentation of most of the known theory on this topic, the primary exclusions being those results that involve complex functions, analyticity, etc. There is a short (ten pages) chapter on the existence, uniqueness and characterization of best Tchebycheff approximations; and, otherwise, there is very little that does not relate directly to degree of convergence questions. Thus the scope of the book is rather narrow and it is not suitable as a general reference or text on approximation theory (even polynomial approximation).

As a special topics book, it is well done. The authors have organized the material well and concisely. There is a natural progression from traditional results to current research (to which one of the authors is a principal contributor) which the specialist in approximations theory will find readable and interesting. There are only thirty-eight items in the bibliography. The book is done economically as far as design, copy-editing and production are concerned; and only one misprint was noted (reference [25]).

J. R. R.

21 [2.05, 7].— HERBERT E. SALZER, *Laplace Transforms of Osculatory Interpolation Coefficients*, ozalid copy of handwritten ms. of six sheets, 11″ × 16″, deposited in the UMT file.

The Laplace transforms of the n-point $(2n-1)$th-degree osculatory interpolation coefficients based on the integral points $i = 0(1)n-1$, namely,

$$A_i^{(n)}(p) = \int_0^\infty e^{-pt}\{[L_i^{(n)}(t)]^2\,[1 - 2L_i^{(n)\prime}(i)(t-i)]\}\,dt,$$

$$B_i^{(n)}(p) = \int_0^\infty e^{-pt}\{[L_i^{(n)}(t)]^2\,(t-i)\}\,dt,$$

where

$$L_i^{(n)}(t) = \prod_{j=0,j\neq i}^{n-1} (t-j) \Big/ \prod_{j=0,j\neq i}^{n-1} (i-j),$$

are expressed exactly as functions of p, for $n = 2(1)9$. Both $A_i^{(n)}(p)$ and $B_i^{(n)}(p)$ underwent three functional checks that were made on the exact fractional coefficients of p^{-r}, $r = 1(1)2n$, on the final manuscript. All computations were performed with a desk calculator before 1962, except for the recent completion of the final checks by hand.

Given $f(i)$ and $f'(i)$, $i = 0(1)n-1$, we have the approximation

$$\int_0^\infty e^{-pt} f(t)\,dt \approx \sum_{i=0}^{n-1} [A_i^{(n)}(p)f(i) + B_i^{(n)}(p)f'(i)].$$

AUTHOR'S SUMMARY

22 [2.25, 4, 7].—F. W. OLVER, *Asymptotics and Special Functions*, Academic Press, Inc., New York, 1974, xvi + 572 pp., 24 cm. Price $39.50.

This is a very satisfactory book, which combines sound mathematical analysis with

a pervading sense of realism and practicality that will make it an extremely useful volume for applications of mathematics involving second-order linear ordinary differential equations and the classical special functions. The author has been a well-known contributor to the asymptotic theory of such equations for over twenty years. He has worked on the computational as well as on the theoretical aspects of these problems. In his own research, as in this book, he emphasizes results that can be used to compute, be it with pencil and paper or on electronic machines.

So much is known on the asymptotic approximations to solutions of ordinary linear differential equations that no single book can do justice to this whole body of knowledge. The author has wisely limited himself to differential equations of order two and has omitted all theories that do not imply computational results. A very distinctive feature of this book—and also of the author's own work—is the emphasis on error estimates. Usable, realistic inequalities for the remainder in asymptotic expansions are rarely found in the literature. The author has developed a practical scheme for the derivation of such bounds and he applies it throughout the volume.

The mathematical prerequisites are kept simple: Undergraduate level courses in advanced calculus and complex variable theory, and a first course in ordinary differential equations should suffice. It is true, on the other hand, that the presentation becomes more condensed as the book progresses, and some analytic proofs are described so briefly that the reader has to put in quite a bit of thinking to supply the details. To the serious student of the subject, the many examples and the over 500 exercises will be welcome. The variety and interesting nature of the exercises is impressive.

The first seven chapters contain the essentials of the subject: The classical special functions, the basic properties of second-order linear differential equations, the nature of asymptotic series and the various techniques for obtaining them from integral representations as well as from formal expansions. Chapter 6 is probably the most distinctive section of the book. It describes the author's version of what is frequently called the WKB method, a name he sensibly avoids in favor of the historically more accurate one of Liouville-Green Approximation. As developed by the author, it becomes a very flexible asymptotic tool complete with a general formula for a bound on the remainder. The technique is presented in such a way that it applies to asymptotic problems with a large parameter as well as to large independent variables, to unbounded domains as well as to turning point problems. One price that has to be paid for this generality is some lack of motivation at the beginning. It is not clear to the uninformed why certain terms are treated as small with respect to others. However, as the technique is applied in chapter after chapter, eventually the motivation becomes quite transparent.

Most of the material in the later chapters is of a more specialized nature. It includes, among other things, the Euler-Maclaurin formula, refinements of the saddlepoint method, turning and other transition points, and asymptotic connection problems.

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23 [2.25, 4, 7].—F. W. OLVER, *Introduction to Asymptotics and Special Functions*, Student Edition, Academic Press, Inc., New York, 1974, xii + 297 pp., 24 cm. Price $10.00.

The first seven chapters of the above reviewed volume are well suited to form the

basis of a one-semester course. In recognition of this fact, that part of the book has been made available separately as a paperback volume. In view of the high cost of books these days, the author and the publisher are to be commended for this service to the public.

WOLFGANG WASOW

24 [2.35].–W. MURRAY, Editor, *Numerical Methods for Unconstrained Optimization,* Academic Press, New York, 1972, xi + 144 pp., 24 cm. Price $8.95.

During the last fifteen years the field of unconstrained optimization has experienced a phenomenal rate of growth. As in other fields that have grown at such a fast rate, it is rare to find a book that provides a coherent overview of the subject and also clearly describes the latest important research results. This is such a book, and it is a very welcome addition to numerical analysis, and in particular, optimization literature.

The book provides an excellent survey of unconstrained optimization methods, successfully presenting both theoretical results and practical matters such as computer implementation. The material covered is up-to-date and includes results obtained subsequent to the joint IMA/NPL conference in January 1971 at which the papers were originally presented. Considering the very active roles that all of the contributors to this book have played in extending the frontiers of optimization, this is not surprising. Moreover, this reviewer shares the view of the editor that "most" of the material presented will not become obsolete during the next few years.

As in any book containing the contributions of several authors, the style of the chapters varies considerably. On the whole, however, the book is extremely readable. A description of each chapter follows.

Chapter 1 – *Fundamentals,* authored by W. Murray, outlines some basic theory upon which subsequent chapters rely. This includes: definitions, necessary and sufficient conditions for a minimum, properties of quadratic and convex functions, and methods for minimizing functions of a single variable.

Chapter 2 – *Direct Search Methods,* authored by W. H. Swann, surveys methods which depend only upon values of the objective function; i.e., methods which do not use derivative information. Discussed here are the well-known "pattern search" method of Hooke and Jeeves, Rosenbrock's method, the Davies, Swann and Campey method, the simplex methods of Spendley, Hext, and Himsworth, and of Nelder and Mead, (not to be confused with the simplex method for linear programming), generalized Fibonacci search, and modifications of these methods. Random search, Box's technique of evolutionary operation and the technique of minimizing with respect to each independent variable in turn are also very briefly described. Just after publication of the book, Powell showed that the latter method can fail on differentiable functions.

Chapter 3 – *Problems Related to Unconstrained Optimization,* authored by M. J. D. Powell, is concerned with the solution of two types of problems via unconstrained optimization: nonlinear least-squares and constrained optimization problems. There is a clear and informative discussion of the Gauss-Newton and Marquardt methods and of modified versions of these for dealing with least-squares problems. There is also an excellent discussion of algorithms requiring only function values based upon the generalized secant method and the quasi-Newton approach. For constrained problems, transformation of variables, penalty function methods and Lagrangian methods are discussed. Practitioners who have such problems to solve should take special note of the section

on Lagrangian methods, as recent work has shown this approach to be more promising than the standard penalty function or barrier type of approaches.

Chapter 4 – *Second Derivative Methods,* authored by W. Murray, is exclusively concerned with Newton's method and modifications of it. After briefly describing methods proposed by several other authors, Murray devotes the rest of the chapter to a numerically stable method of his own based upon Cholesky factorization. This appears to be an excellent method if the time required for computing the matrix of second derivatives is not excessive.

Chapter 5 – *Conjugate Direction Methods,* authored by R. Fletcher is, in this reviewer's opinion, the best introductory discussion of these methods in print. Methods described include those developed by Powell, Smith, Fletcher and Reeves, and Zoutendijk and the Partan method.

Chapter 6 – *Quasi-Newton Methods,* authored by C. G. Broyden surveys all of the well-known quasi-Newton, (variable metric), updating methods and families of updating formulas. Theoretical properties of these methods are discussed, with the principal emphasis on convergence results. For some methods, statements are made about computational experience.

Chapter 7 – *Failure, the Causes and Cures,* authored by W. Murray, attempts to provide some helpful hints to the practical optimizer. Besides some general remarks on rounding errors and numerical stability, there is a good discussion of these aspects with regard to quasi-Newton algorithms. Here, recent work on implementing these algorithms using the Cholesky factorization of an approximate Hessian, rather than the inverse of that matrix, is described. There are also some remarks on computer input and interpretation of output.

Chapter 8 – *A Survey of Algorithms for Unconstrained Optimization,* authored by R. Fletcher, documents several fully available Fortran IV and ALGOL 60 codes which implement some of the better optimization methods. The information given should be of considerable interest to problem solvers.

Finally, there is an appendix which sets forth several definitions and results in linear algebra without proof with which the reader needs to be familiar. The common mistake of referring to the Sherman and Morrison modification rule as Householder's rule is made here.

Some obvious typographical errors that were noticed are: first line below (3.6.4): the first "infinity" should be "minus infinity"; p. 69, equation for g and (6.2.5): f omitted; (4.12.1): T_3 inside both sets of parentheses should be T_2; second line above (7.3.2): (1.6.1) should be (1.5.1); p. 133, line 4: (8) should be (7); p. 136, third reference: pollution should be solution.

D. G.

25 [3, 7, 8, 10].– JÜRG NIEVERGELT, J. CRAIG FARRAR & EDWARD M. REINGOLD, *Computer Approaches to Mathematical Problems*, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1974, xiii + 257 pp., 24 cm. Price $8.95 (clothbound).

This is a delightful book. It introduces the student to a medley of techniques, algorithms, and facts, which would be known by a well-educated computer scientist with a mathematical bent. Yet he (or she) probably acquired this material haphazardly over the years from courses, colloquia, and technical conversations.

Is the book simply an introduction to formal languages, combinatorics, and graph theory? By no means, and yet all three subjects are introduced along with random number generation, game playing, and the computation of mathematical constants.

Part of the charm of the book stems from a lively style, but its character comes from the desire of the authors to show how problems are attacked with the aid of a computer. The mathematical maturity demanded is that of a junior or senior mathematics major. Consequently, it could serve as a text for a valuable course for first year graduate students in Computer Science, although such a course will be opposed on the grounds that it broadens rather than deepens.

Each of the six chapters is self-contained and ends with an annotated list of references and exercises of varying difficulty. For example, (i) find a method for generating random permutations from random numbers so that each permutation should have an equal probability of occurrence, (ii) estimate (by simulation) the probability that three points chosen at random in the plane form an obtuse triangle. Lewis Carroll posed the latter problem which has a nice theoretical solution. Historical comments are woven into the text. This book should appeal to many mathematicians who admit to very little interest in Computer Science, because the intellectual difficulties in the problems addressed are so clearly brought out.

B. P.

26 [4,5].—J. ALBRECHT & L. COLLATZ, Editors, *Numerische Methoden bei Differentialgleichungen und mit Funktionalanalytischen Hilfsmitteln,* Birkhäuser Verlag, Basel, Switzerland, 1974, 231 pp., 25 cm. Price sfr. 59.—.

This volume contains papers presented at two meetings organized by Y. Albrecht and L. Collatz. The first meeting took place at the Technical University at Clausthal-Zellerfeld, Germany, from May 31–June 2, 1972, the second meeting was held at the Mathematical Research Institute at Oberwolfach, Germany, from June 9–10, 1972.

J. B. & V. T.

27 [5].—ROGER TEMAN, *Numerical Analysis.* Reidel Publishing Co., Doordrecht, Holland, and Boston, Mass., 1973, viii + 167 pp., 19 cm. Price $17.50.

This book is an updated translation of a French text which appeared in 1970. Despite its title, it concentrates on the analysis of numerical procedures for elliptic problems. The main emphasis in this study is on the use of functional analysis. The book thus contains discussions of the Lax-Milgram theorem, the Galerkin method, approximation theory, etc., and applications of these tools to finite difference and finite element methods applied to a few linear and nonlinear model problems.

In this way, the author provides an accessible, fairly elementary introduction to some of the work on theoretical numerical analysis in France during the last ten years. What the book lacks, in the reviewer's opinion, is material on the more practical aspects of elliptic equation solving. The author does describe the fractional step approach, a method which however is rarely used in real life applications. Apart from this discussion, only a few sentences are spent on the very important and interesting problems of how to handle the large systems of linear and nonlinear equations which arise in these applications.

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28 [7].—IRWIN ROMAN, *Tables of* $N^{3/2}$, ms. of 68 pp., $8\frac{1}{2}'' \times 11''$, deposited in the UMT file.

In *MTAC*, v. 1, 1945, p. 407, QR **14** Dr. Roman briefly described his manuscript tables of $N^{3/2}$. Subsequently he enlarged them, so that the copy deposited in the UMT file now consists of 10S values corresponding to $N = 0(1)9999$. He has included a single page of 9D values for $N = 1.0001(.0001)1.0099$, which appeared in his earlier version.

A successful comparison of this table has been made by this reviewer with the similar, less extensive 10D tables of Davis & Fisher [1].

J. W. W.

1. H. T. DAVIS & V.J. FISHER, *Tables of the Mathematical Functions,* Vol. III, The Principia Press of Trinity University, San Antonio, Tex., 1962, pp. 506–507. MR 26 #364. (See *Math. Comp.*, v. 17, 1963, pp. 459–461, RMT 68.)

29 [7].— E. ORAN BRIGHAM, *The Fast Fourier Transform,* Prentice-Hall, Inc., Englewood Cliffs, N.J., 1974, xiii + 252 pp., 24 cm. Price $19.95.

When the fast Fourier transform (FFT) algorithm was rediscovered and published by Cooley and Tukey, it came to the attention of a number of people who had an urgent need for it. Subsequently, a great many papers on applications and extensions of the basic idea were published. This fast algorithm led to increased applications of Fourier theory to digital processes. Consequently, the theory of the discrete Fourier transform (DFT) which had received relatively little attention in the literature, became a subject of great interest to applied mathematicians and engineers.

The present book gives a detailed description of the DFT and its relation to the Fourier theory of continuous functions in a form suitable for engineering students. It starts at a very elementary level, discussing basic Fourier transform theory and the DFT in great detail, with many graphs and diagrams, in a not too rigorous heuristic manner.

The essentials of the FFT algorithm can be given in a few pages. However, for pedagogical reasons, 49 pages are taken up with long algebraic derivations describing various forms of the algorithm along with system flowgraphs for each. This does achieve its purpose, i.e., completeness. However, it may take some perseverance to go through the algebraic manipulations. The elementary level of this treatment may be appealing to an engineer first encountering the subject. However, such a person is probably not ready to concern himself with the details of the FFT algorithm. On the other hand, the working scientist or engineer who has a need for the increased speeds available from the FFT algorithm, and who wants to make a detailed study of it, will find this too elementary. The part of the book dealing with the FFT algorithm takes only one-third of the book and is introduced by treating the 4-point form of the algorithm algebraically and with system flowgraphs. This simple treatment suffices to introduce the reader to flowcharts and programs and will be appreciated by the engineering student. Special algorithms for two real transforms via one complex transform and for one real transform are derived. Then the book gives derivations of the more general algorithms in several different forms and for the mixed-radix case.

Some essential parts of the explanation of the FFT algorithm are somewhat inadequate. For example, the explanation of the reasons for the economy of the method uses the case $N = 4$, showing that the FFT takes fewer operations. However, this is due to the fact that some powers of W_N are equal to one, which does not affect the

asymptotic behavior of the operations count. In fact, the discussion does not lead to the general asymptotic formulas given. In the more general FFT algorithms, the algebraic manipulations obscure the basic simplicity of the FFT algorithm, particularly for students at the level for which the rest of the book is designed.

The chapter on convolution calculations is effectively given at the same level as the first part of the book. It is strange that no mention is made of applications of the methods to engineering problems.

Aside from the fact mentioned above, that it is hard to define the type of student for whom the book is really designed, it can be a good text book for engineering students at a fairly elementary level. However, the teacher would have to condense and abstract the essential parts of the algorithm from the chapters on the FFT algorithm, since an elementary student would not really have much need or use for the detail given here.

For the working scientist or engineer, however, who merely wants to learn to use the FFT algorithm, it may be more efficient to consult a few of the references on the subject.

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30 [7].—K. A. STROUD, *Laplace Transforms, Programmes and Solutions,* John Wiley and Sons, New York, 1973, x + 275 pp., 23 cm. Price $5.75 (paperbound).

The reviewer can do no better than to quote from the author's preface. "The purpose of this book is to provide a sound introductory course in the use of Laplace transforms in the solution of differential equations and in their application to technological situations. The course requires no previous experience of the subject, but some knowledge of the solution of simple differential equations by the classical methods is assumed. The book forms a topic module. It approaches the subject in a practical way and has been devised specifically for courses leading to (i) B. Sc. Degree in engineering and science subjects, (ii) Higher National Diploma and Higher National Certificate in technological subjects and courses of a comparable standard. The module is self-contained and can therefore be introduced into any appropriate year of such courses and by its nature is equally applicable for individual or class use. The text has been based on self-learning methods developed and extensively tested over the past ten years. In controlled post-tests, each of the programmes has consistently attained a success rating in excess of 80/80, i.e., after working through each programme at least 80 per cent of the students scored at least 80 per cent of the possible marks. The individual nature of the method, the ability of a student to progress at his own rate, the immediate assessment of responses and, above all, the complete involvement of the student, all result in high motivation and contribute significantly to effective learning."

The volume is divided into eight programs. Programs 1–4 develop use of the transforms to solve various types of differential equations. Programs 5–8 deal with the Heaviside unit step function, periodic functions and the impulse function. A set of worked examples provides an introduction to the application of transforms to engineering problems, and a concluding section includes a table of transforms and inverse transforms.

It appears that the volume is well suited for self-study. But the very nature of

the approach inevitably results in its taking more space to present the material than a typical textbook designed for classroom instruction. I feel that more material should have been covered—for example, application of transforms to solve partial differential equations and the integral representation for the inverse transform.

Y. L. L.

31 [8].—WILLARD H. CLATWORTHY, JOSEPH M. CAMERON & JANACE A. SPECKMAN, *Tables of Two-Associate-Class Partially Balanced Designs,* NBS Applied Mathematics Series 63, U.S. Government Printing Office, Washington, D.C., 327 pp., 26 cm. Price $3.45 (paperbound).

An extremely important problem in the design of experiments is the development of the capability of evaluating the significance of large numbers of variables and of estimating their effects, while maintaining control of the experimental error. To accomplish these purposes two broad classes of designs have been developed, known respectively as the Balanced Incomplete Block (BIB) and lattice designs. A major drawback to these designs consists of the constraints placed on the design configuration in terms of variables, blocks, and replicates. Owing to the resulting limitation on the number of ready-made designs of the BIB and lattice types, interest has developed in a broader class of designs that would remove these rather severe restrictions, to a large extent, while retaining most of the desirable features of the earlier designs. The underlying theory of the more general class of designs was developed by Bose & Nair [1]. These designs are called partially balanced incomplete block designs with m associate classes, and are designated as PBIB(m) designs.

The tables under review are new PBIBD(2) designs, representing the culmination of intensive research into combinatorial problems associated with experimental-design configurations studied since the publication of PBIBD(2) designs by Bose, Clatworthy and Shrikhande [2]. The present tables include more than 800 experimental designs of type $D(2)$ for which $2 \leqslant k \leqslant 10$ and $2 \leqslant r \leqslant 10$, where k is the number of experimental units in a block and r is the number of blocks in which each treatment appears.

Detailed explanations are given of the various means of accessing experimental designs to fit experimental situations. In Chapter III the reader is made fully aware of the models underlying the designs in the PBIBD(2) class. This chapter includes details of the computation required to develop the analysis of variance summaries, even to identifying the most appropriate schemes for the type of computer available.

However, this reviewer would have liked to see some sections directed at those not versed in the theory of statistical design, outlining in nontechnical terms the need for the new designs in preference to the customary ones.

This publication can be recommended as a necessary addition to the library of anyone fully qualified in the design of experiments.

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1. R. C. BOSE & K. R. NAIR, "Partially balanced incomplete block designs," *Sankhyā*, v. 4, 1939, pp. 337–372.

2. R. C. BOSE, W. H. CLATWORTHY & S. S. SHRIKHANDE, *Tables of Partially Balanced Designs with Two Associate Classes,* North Carolina Agricultural Experiment Station Technical Bulletin No. 107, Raleigh, North Carolina, 1954.

32 [8].—THE INSTITUTE OF MATHEMATICAL STATISTICS, Editors, and H. L. HARTER & D. B. OWEN, Coeditors, *Selected Tables in Mathematical Statistics,* Volume I, American Mathematical Society, Providence, R. I., second printing with revisions, 1973, 403 pp., 26 cm. Price $8.60.

This is the first of a series of specialized tables prepared and edited by the Committee on Mathematical Tables of the Institute of Mathematical Statistics and published by the American Mathematical Society under a joint agreement.

The present volume contains five sets of tables; namely, "Tables of the Cumulative Noncentral Chi-Square Distribution", by G. E. Haynam, Z. Govindarajula and F. C. Leone, "Tables of the Exact Sampling Distribution of the Two-Sample Kolmogorov-Smirnov Criterion D_{mn}, $(m \leqslant n)$", by P. J. Kim and R. I. Jennrich, "Critical Values and Probability Levels for the Wilcoxon Rank Sum Test and the Wilcoxon Signed Rank Test," by Frank Wilcoxon, S. K. Katti and Roberta A. Wilcox, "The Null Distribution of the First Three Product-Moment Statistics for Exponential, Half-Gamma, and Normal Scores," by P. A. W. Lewis and A. S. Goodman, and "Tables to Facilitate the Use of Orthogonal Polynomials for Two Types of Error Structures," by Kirkland B. Stewart.

Each set of tables is prefaced by an introduction, a description of the mathematical algorithms used in their preparation, a discussion of tabular accuracy and interpolation, examples of their application, and references to the relevant literature.

A possible criticism of this collection of tables is that it is too specific and selective; however, this reviewer believes that this selection reflects the fact that most statistical texts do not adequately address the problems to which these tables apply. For example, this reviewer has many times been confronted with problems in chemical and mechanical engineering where a cumulative-error model beyond that of the first degree would have been appropriate, but necessary guidance was not to be found in the available literature. The tables of Stewart would have been extremely useful in that connection, and it is to be hoped that these tables will inspire similar research with other types of error structures.

Similarly, the tables of Lewis and Goodman address certain reliability problems involving failure clustering patterns that do not conform to typical textbook problems.

Especially useful is the presentation by Haynam, Govindarajula and Leone of two types of tables displaying different aspects of the power of the chi-square distribution as illustrated by well chosen examples.

The importance of the tables of Kim and Jennrich and also of those of Wilcoxon, Katti and Wilcox cannot be overemphasized for those researchers who depend upon distribution-free statistics for the solution of many of their problems.

In conclusion, this reviewer endorses this approach adopted by the Institute of Mathematical Statistics of soliciting meritorious material for mathematical statistical tables. This procedure should lead to a broad representation of those difficult statistical problems that continue to challenge researchers, and it should provide relevant tables not hitherto accessible in the literature.

HARRY FEINGOLD

33 [9].—I. O. ANGELL, *Table of Complex Cubic Fields,* Royal Holloway College, University of London, Surrey, England, 1972, 53 computer output sheets deposited in the UMT file.

There are listed here the 3169 nonconjugate cubic fields $Q(x)$ having discriminants $-D$ between 0 and -200000. For each $Q(x)$ there is given: D; a generating equation

$$x^3 - Ax^2 + Bx - C = 0$$

of discriminant $-N^2D$ and index N; the fundamental unit $\epsilon_0 = (Ix^2 + Jx + K)/L$ where $0 < \epsilon_0 < 1$ and L is a divisor of N; the class number H and an ideal norm bound P used in its calculation. The H and ϵ_0 are computed by Voronoi's method. This table should be useful and informative for all students of algebraic number theory.

There were two discrepancies between Angell's very brief paper [1] and the original table deposited in the Royal Society UMT. The paper states that $C(2) \times C(4)$ is the class group for $D = 16871$ while the table correctly had $H = 4$ since the group is really $C(2) \times C(2)$. The original table listed 3168 fields since one line was inadvertently omitted. It is included in the present version and is

D	N	A	B	C	I	J	K	L	P	H
5359	3	14	61	39	−17	236	−171	3	1	1

There follows a detailed critique of the conventions adopted in this table and then some further commentary and additions going beyond the table. In the equation selected for generating x, the coefficients A, B, C are all positive and such that the single real root satisfies $0 < x < 1$. That can always be accomplished simply by a translation $x = y + a$ or a reflection $x = a - y$. While this standardization certainly has merit it also has various minor faults: the coefficients are sometimes unduly large, and in further calculations it is frequently preferable to have the inflection point or any minimum of the cubic polynomial closer to $x = 0$. To illustrate, one of the 13 fields for $D = 63199139$, (far beyond this table), is generated by $y^3 - 183y^2 + 119y - 22 = 0$. By $y = 183 - x$, this becomes $f(x) = x^3 - 366x^2 + 33608x - 21755 = 0$ in Angell's convention. It has a minimum with $f(x) = \ldots 508, 85, 22, 325, 1000, \ldots$ far out at $x = 183$ while $f(x)$ has five or six decimals near $x = 0$. A more serious objection concerns the index N. It never exceeds 5 here but is not always minimized, not even when this can easily be done. For example, for the $D = 5359$ above, the $N = L = 3$ there can be eliminated since $x = (y + 1)/(y + 4)$ gives $y^3 - 59y - 175 = 0$ with $N = L = 1$. A minimal N is certainly preferable, both for practical computation and for theoretical studies concerning monogenic rings of integers, and, if and when N can be easily reduced, it seems desirable to do so.

The first five $N > 1$ here are for $D = 356, 424, 431, 440$, and 503, all being listed as $N = 2$. But, while 431 and 503 cannot be reduced to $N = 1$ since the prime 2 splits in these fields, the other three D can easily be reduced to $N = 1$. The next $N = 2$ is $D = 516$, but here I am uncertain whether it can be reduced or not. The first $N = 3$ here is for $D = 972$ (for $Q(\sqrt[3]{12})$) and this can be easily made $N = 1$. While $D = 2028$, for $Q(\sqrt[3]{26})$, can be easily reduced from $N = 3$ to $N = 2$, I am uncertain if it can be further reduced to $N = 1$. The first $N = 4$ and 5 here can be reduced to $N = 2$, and so forth.

The convention $0 < \epsilon_0 < 1$ also has a mixed assessment. Its reciprocal $\epsilon = \epsilon_0^{-1} > 1$ generally has much larger coefficients but ϵ can be used to easily compute the regulator $R = |\log \epsilon_0|$. In contrast, ϵ_0 may be exceedingly small and one has catastrophic loss of significance due to cancellation in its numerical evaluation unless one first inverts it *algebraically*. The programmer could circumvent this difficulty by printing ϵ_0 and evaluating and printing R in addition. (And why not? R is just as significant as H is.)

Davenport and Heilbronn have proven [2] that the asymptotic density of nonconjugate complex cubic fields is

$$[4\zeta(3)]^{-1} = 0.20798.$$

In this table one has an average density of only 3169/20000 = 0.15845. The ratio of the number of fields up to $D = 1000n$ divided by $1000n$ for $n = 1(1)20$ is shown in Table 1.

TABLE 1

n	*ratio*	n	*ratio*	n	*ratio*	n	*ratio*	n	*ratio*
1	0.1270	5	0.1458	9	0.1513	13	0.1544	17	0.1569
2	0.1350	6	0.1480	10	0.1520	14	0.1557	18	0.1572
3	0.1397	7	0.1514	11	0.1539	15	0.1561	19	0.1577
4	0.1435	8	0.1501	12	0.1540	16	0.1562	20	0.1584

The observed convergence is slowly from below and surprisingly smooth—except for a fluctuation at $D \approx 7000$–8000.

The growth here is associated mostly with those D for which m (> 1) nonconjugate fields exist. There are 58 D here with $m = 3$ and 22 with $m = 4$. $m > 4$ does not occur here. However, for larger D, there will be cases of $D = 27S^2$ where S is a square-free product of many primes. The multiplicity m increases exponentially with the number of prime factors of S. For fundamental discriminants $-D$, $m = (3^r - 1)/2$ where r is the 3-rank of $Q(\sqrt{-D})$. The 22 cases of $m = 4$ here are all of this type with $r = 2$. The maximum r known at present [3] is $r = 4$ and so its $D = 87386945207$ will have $m = 40$.

Most of the $m = 4$ discriminants here were already well-known, such as $D = 3299$, 4027, etc. Here are three known algebraic series [4], [5] that have $r \geqslant 2$ and therefore $m \geqslant 4$. These are the fundamental discriminants $-D$ where D equals

$$3\Delta(a, b) = 3(a^6 + 4b^6), \qquad b \equiv 0 \pmod 3,$$

$$D_6(z) = 108z^4 - 148z^3 + 84z^2 - 24z + 3, \qquad z(\neq 1) \equiv 1 \pmod 3,$$

$$4D_3(y) = 108y^4 - 296y^3 + 336y^2 - 192y + 48, \qquad y \equiv -1 \pmod 6.$$

For these D, N, A, B, C can be given a priori. Since I have not published this elsewhere, I include these formulas in Table 2. Note that the C in the first fields for $D_6(z)$ and $4D_3(y)$ are integral even when $z \equiv -1 \pmod 3$ and $y \equiv +1 \pmod 6$. In these cases

TABLE 2

D	N	A	B	C
$3\Delta(a, b)$	3	0	$3ab$	$2b^3 - a^3$
$3\Delta(a, b)$	3	0	$-3ab$	$2b^3 + a^3$
$3\Delta(a, b)$	3	0	$3b^2$	a^3
$3\Delta(a, b)$	6	0	$3a^2$	$4b^3$
$D_6(z)$	1	1	$1 - z$	$z(1 - 2z)$
$D_6(z)$	1	0	$-z$	$(6z^2 - 4z + 1)/3$
$D_6(z)$	1	0	$z(2 - 3z)$	$(6z^3 - 6z^2 + 4z - 1)/3$
$D_6(z)$	8	0	$8z - 3$	$(48z^2 - 40z + 10)/3$
$4D_3(y)$	2	1	$3 - 2y$	$4y^2 - 6y + 3$
$4D_3(y)$	2	0	$-2y$	$4y^2 - 8(2y - 1)/3$
$4D_3(y)$	2	0	$y(4 - 3y)$	$2y^3 - 4y^2 + 8(2y - 1)/3$
$4D_3(y)$	2	0	$4y - 3$	$4y^2 - 10(2y - 1)/3$

one only knows that $r \geqslant 1$ and $m \geqslant 1$, and these are valid cubic fields. Note also that the A, B, C in Table 2 do not follow Angell's convention.

There are known cases of $r = 3, 4$ in these series, such as the $D_6(28) = 63199139$ above, but they are far beyond Angell's table. Recently, F. Diaz y Diaz [6] sent me smaller D with $r = 3$. Two of these are

$$Q(\sqrt{-3321607}) \quad \text{with} \quad C(3) \times C(3) \times C(63),$$
$$Q(\sqrt{-3640387}) \quad \text{with} \quad C(3) \times C(3) \times C(18).$$

For these D an algebraic evaluation of the A, B, C for the 13 cubic fields is not possible and one must use numerical methods. By a delightfully sophisticated combination of the infrastructure [7] of the real fields $Q(\sqrt{3D})$ and unimodular and Tschirnhausen transformations, I computed the 13 cubic equations for these two D.

Table 3 shows the 13 fields for $D = 3321607$ with the A, B, C following Angell's convention. The splitting primes 2, 13, 19, 29, 41 and 43 split only in those four of the 13 fields marked S. This shows that the 13 fields are distinct and that I managed to make $N = 1$ except where 2 splits. In evaluating these cubic polynomials for $x = \pm 1$, ± 2, etc., one is struck with the large number of functional values equal to perfect cubes. These occur because these *cubic* fields have a 3-rank = 2 according to the Gras-Callahan theorem, cf. [8, p. 185].

TABLE 3, $D = 3321607$

N	A	B	C	2	13	19	29	41	43
8	45	664	404	S		S			
8	41	616	512	S	S				
8	59	960	656	S				S	S
6	37	498	288	S			S		
1	68	1179	755		S		S	S	
1	80	1601	27		S				
1	129	4174	883		S	S			S
1	2	95	27						S
1	17	144	125					S	
1	45	526	357			S	S		
1	9	112	103						
1	78	1555	1303			S		S	
1	126	4027	3637				S		S

For $D = 3640387$, no prime < 13 splits and $Q(\sqrt{-D})$ has $L(1, \chi) = 0.26674$. This is sufficiently close to the lower bound allowed by the Riemann Hypothesis [9] that it is unlikely that a much smaller quadratic class number than its $h = 27 \cdot 6$ can occur with $r \geqslant 3$. For this D, I found ten fields with $N = 1$, two with $N = 8$ and one with $N = 7$. I leave it as an exercise for the reader to reproduce these equations and to verify that 13 of the splitting primes and the 13 fields form the incomplete balanced block design [10] in Table 4. Any two fields intersect in only one of these splitting primes and any two primes both split in only one of these fields. Also, show that 149 splits in all 13 fields and 421 ramifies.

TABLE 4, $D = 3640387$

	13	31	43	53	73	109	173	193	227	239	281	337	617
I		*S*					*S*	*S*				*S*	
II			*S*	*S*	*S*							*S*	
III										*S*	*S*	*S*	*S*
IV	*S*					*S*			*S*			*S*	
V	*S*		*S*				*S*				*S*		
VI					*S*	*S*	*S*			*S*			
VII				*S*			*S*		*S*				*S*
VIII		*S*		*S*		*S*					*S*		
IX	*S*	*S*			*S*								*S*
X		*S*	*S*						*S*	*S*			
XI			*S*			*S*		*S*					*S*
XII					*S*			*S*	*S*		*S*		
XIII	*S*			*S*				*S*		*S*			

Since the infrastructure-Tschirnhausen method is quite efficient, and does not require much trial-and-error, one does not need a high-speed computer for D of this size, and I worked out these equations on a *nonprogrammable* HP-45 hand computer. One principal feature of the method is that as each cubic equation comes forth there is no need to show that it gives a field different than the others. That is automatic. I may publish this method elsewhere.

D. S.

1. I. O. ANGELL, "A table of complex cubic fields," *Bull. London Math. Soc.*, v. 5, 1973, pp. 37–38.

2. H. DAVENPORT & H. HEILBRONN, "On the density of discriminants of cubic fields. II," *Proc. Roy. Soc. London Ser. A*, v. 322, 1971, pp. 405–420.

3. DANIEL SHANKS & RICHARD SERAFIN, "Quadratic fields with four invariants divisible by 3," *Math. Comp.*, v. 27, 1973, pp. 183–187; "Corrigenda," *ibid.*, p. 1012.

4. DANIEL SHANKS & PETER WEINBERGER, "A quadratic field of prime discriminant requiring three generators for its class group, and related theory," *Acta Arith.*, v. 21, 1972, pp. 71–87. MR 46 #9003.

5. DANIEL SHANKS, "New types of quadratic fields having three invariants divisible by 3," *J. Number Theory*, v. 4, 1972, pp. 537–556. MR 47 #1775.

6. F. DIAZ Y DIAZ, "Sur les corps quadratiques imaginaires dont le 3-rang du groupe des classes est supérieur à 1." (To appear.)

7. DANIEL SHANKS, "The infrastructure of a real quadratic field and its applications," *Proceedings of the* 1972 *Number Theory Conference*, (Univ. of Colorado, Boulder, 1972), pp. 217–224.

8. T. CALLAHAN, "The 3-class groups of non-Galois cubic fields. II," *Mathematika*, v. 21, 1974, pp. 168–188.

9. DANIEL SHANKS, "Systematic examination of Littlewood's bounds on $L(1, \chi)$," *Proc. Sympos. Pure Math.*, vol. 24, Amer. Math. Soc., Providence, R.I., 1973, pp. 267–283.

10. MARSHALL HALL, JR., *Combinatorial Theory*, Blaisdell, Waltham, Mass., 1967, Chapter 10. MR 37 #80.

34 [9].–E. D. TABAKOVA, *A Numerical Investigation of Kummer Cubic Sums* (in Russian), Institute of Applied Mathematics of the USSR Academy of Sciences, Moscow, preprint No. 98, 1973 (22 pages).

We use the notation of Shanks's review [1] of Fröberg's recent table. The author has evaluated S_p for the first 21100 primes $p \equiv 1 \pmod 6$, that is for $p < 509757$. (Fröberg, at about the same time, had gone to $p < 200000$.) The number of S_p in the intervals I_1, I_2, I_3 are 5748, 6933, 8419 giving proportions 27.2%, 32.9% and 39.9%, respectively. Since $27.2\% > 4/15$, this goes to support Shanks's scepticism about Fröberg's conjecture that the asymptotic proportions should be 4 to 5 to 6.

There are two tables. The first gives the number of primes p in each consecutive 100 for which S_p is in I_1, I_2, I_3 together with the cumulative totals and their proportions (to 6 significant figures). Although there are minor fluctuations the proportion of S_p in I_1 has a rising upward trend and the proportion in I_3 has a decreasing trend. The second table lists for $n = 100000(100000)500000$ and for $x = -1(0.05) + 1$ the proportion of the primes $p < n$ for which $S_p < (2p^{1/2})x$ together with the value $1/2 + (\arcsin x)/\pi$ to which it would tend under the hypothesis of equidistribution. To the naked eye there is quite a good fit but perhaps this is not a severe test.

In the introduction the author briefly discusses the method of computation and how it was organized to minimize computer time. The residue r_x of x^3 modulo p is computed by the recurrence relation

$$r_{x+1} \equiv 2r_x - r_{x-1} + 6x \pmod p, \qquad 0 \leqslant r_{x+1} < p.$$

The value of $\cos(2\pi r_x/p)$ is then computed by interpolation between the values $\cos(2\pi j/1024)$ with integral j. This requires $O(p)$ calculations for each p, but so, as the author points out, does the reviewer's method [2]. As a check she has considered the intervals investigated by the reviewer and points to a discrepancy of one unit in one of his tables. The computations were done on a BESM-6 and in all required machine time "of the order of 24 hours".

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1. C.-E. FRÖBERG, "Kummer's Förmodan," *Math. Comp.*, v. 29, 1975, p. 331. UMT 5.
2. J. W. S. CASSELS, "On the determination of generalized Gauss sums," Arch. Math. (Brno), v. 5, 1969, pp. 79–84.

35 [12].– RICHARD V. ANDRÉE, JOSEPHINE P. ANDRÉE & DAVID D. ANDREE, *Computer Programming: Techniques, Analysis and Mathematics*, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1973, xvii + 549 pp., 24 cm. Price $12.95.

Despite the title, this text is largely an elementary introduction to FORTRAN programming. The first 220 pages provide a rather leisurely introduction to basic FORTRAN: integer and real variables, arrays, DO loops, and the elementary statement types. The approach is to work largely from example problems, motivating the need for each language feature and, at the same time, studying in detail the various difficulties that arise in problem formulation and analysis prior to coding. The remainder of the book consists of chapters on simulation, random number generation and Monte Carlo techniques (60 pages), errors in numerical computations (50 pages), FORTRAN subprograms (40 pages), and assembly language programming on the IBM 1130 (45 pages), followed by 100 pages of answers to problems.

The strength of the text lies in an interesting collection of problems (each chapter is based on a set of example problems and also ends with an extensive problem set). Also important is the early and continued emphasis on careful problem analysis before

coding begins, a stage that beginning programmers too often bypass in their rush to get something "on the machine".

The major weakness of the text (a weakness that unfortunately overshadows the strengths mentioned above) is the extremely narrow and rather dated view of computer programming. In this respect, the text might well have been written ten years ago. Subprograms are mentioned only briefly toward the end of the book, and almost no hint of the central role of subprograms in programming is given. The sort of problem analysis and program design suggested to precede coding is entirely concerned with questions of *run-time efficiency*—no emphasis is given to good program structure, readability, documentation (in spite of statements to the contrary in the text), or modifiability. The text is sprinkled with suggestions for writing programs, but most are of debatable value and in conflict with current ideas, e.g., "use the faster forms whenever feasible: ... $C*C*C*C$ is faster than $C**4$ which in turn is much faster than $C**4.0$" (p. 422), "make a simple case run first, then make it fancy" (p. 184). The beginning programmer studying this book is far too likely to get the impression that "efficiency" is not just the primary, but almost the only, criterion by which program design is to be judged.

In addition, *assembly language* is presented as the way to get more power and versatility, should FORTRAN prove too restricted; there is only the merest mention of other high-level languages. The book has a number of other oddities: A six-page section on "The Computer in our Society" included in a chapter on "Subscripts and DO Loops", 45 pages devoted to the details of assembly language programming on the IBM 1130 (a small pre-IBM 360 machine), and the mixing of FORTRAN language details, job control language statements, particular hardware restrictions (six-digit accuracy is taken as standard), and other trivia.

In sum, an instructor might glean some interesting problems and examples from this text, but it is not recommended as a primary text for student use. Basically, the contents reflect practices, languages and computer systems of 1964 rather than 1974. A beginning student would be better served by a text incorporating more recent practice in programming.

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36 [12].—FREDERICK W. WEINGARTEN, *Translation of Computer Languages*, Holden-Day, Inc., San Francisco, Calif., 1973, xi + 180 pp., 24 cm. Price $9.95.

The title of this book might lead one to infer that it treats compiler construction rather generally. The title is deceptive, however, in that the book is devoted almost exclusively to an expository presentation of different parsing methods and to the essential aspects of the theory underlying them. Such matters as run-time organization are not even mentioned, and code generation is given only the most cursory treatment. Thus, the book is not really a satisfactory text for a compiler course, although that would seem to be its intent.

After some preliminary introduction to relevant mathematical notation, the book discusses some of the early methods used for translating arithmetic expressions. It then goes on to a more general discussion of general formal grammars, context-free grammars, and the structure of translation trees. The author has chosen to represent such

trees in a purely binary form, claiming that this approach makes parsing easier. However, I found that, as a result, his algorithms were complicated and not intuitively convincing. He then discusses different parsing methods, treating, in sequence, the top-down parse, the bottom-up parse, the general left-to-right parse (using Earley's nodal span method), and parsing based on restricted grammars, specifically, $LR(k)$ grammars, bounded-context grammars, and precedence grammars. Unfortunately, the interesting special case of operator precedence is not treated.

The author has a clear and engaging expository style; but he is unfortunately fighting an uphill battle against a poor choice of algorithms and data representations. For instance, the discussion of the top-down parse occupies two chapters and requires about five pages of flow-charts; with proper choice of representation and algorithm, the top-down parse becomes exceedingly simple—in fact, the simplest of all methods. Furthermore, there are numerous minor errors which make the discussion hard to follow. After several hours of attempting to understand the discussion of the nodal span parse, I realized that my difficulties were due to a number of different misplaced or missing arrows in one diagram (figure 8.6)! Among other errors that I noticed were the use of $\bar{y}$ rather than $\bar{x}$ on line 10, page 47, the use of $\underline{m}$ rather than $\underline{i}$ on the last line of page 114, and the omission of $\bar{z}$ following ξ in the statement of the first part of the closure rule on the same page.

The advantages of this book are its excellent organization and fine expository style. I found these outweighed, however, by the disadvantages of cumbersome algorithms and confusing errors. The restriction of the subject matter to parsing is a mistake, from my point of view; but that is a matter of taste.

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37 [13, 25].—H. MELVIN LIEBERSTEIN, *Mathematical Physiology, Blood Flow and Electrically Active Cells,* American Elsevier Publishing Co., Inc., 1973, New York, xiv + 377 pp., 24 cm. Price $19.50

This book is a collection of the specific contributions of its author to the growing field of mathematical physiology, and the reader would be well advised not to form an overall impression of this field on the basis of this book.

The section on blood flow is based on a power series expansion for the velocity profile in pulsatile blood flow. The first term of this series is the parabolic profile of steady flow, and the subsequent terms contain successively higher time derivatives of the driving force. One would expect this series to converge rapidly only in the smaller arteries where the flow is, in fact, quasi-steady.

The section on electrophysiology is based on a modification of the Hodgkin-Huxley equations, which is referred to by the author as a "reformulation." The equations for nerve conduction, as stated by Hodgkin and Huxley [1], have the form:

$$ri = -v_x, \tag{1}$$

$$cv_t + I = -i_x, \tag{2}$$

where $I = I(v, s_1 \cdots s_N)$ is the ionic current through the membrane, and where the membrane parameters s_k obey ordinary differential equations of the form

$$ds_k/dt = f_k(s_k, v).$$

Traveling wave solutions of this system with velocity θ obey the ordinary differential equation

$$cv_t + I = (1/r\theta^2)v_{tt} \tag{3}$$

with I defined as above.

Unless the parameter θ is chosen correctly, the solutions to (3) are unbounded, and this fact can be used to determine θ.

Lieberstein attempts to avoid the difficulty of unbounded solutions as follows. First, he introduces in equation (1) the term li_t which arises from line inductance. This in itself cannot be wrong, since there is always some line inductance, and the Hodgkin-Huxley equations can be regarded as a limit as $l \longrightarrow 0$. Thus, Lieberstein obtains the equations

$$li_t + ri = -v_x, \tag{1$'$}$$

$$cv_t + I = -i_x, \tag{2$'$}$$

with I as above. The signal velocity for this hyperbolic system is given by $\theta^2 = 1/lc$, and Lieberstein determines the value of θ from the experimental propagation speed of a nerve impulse. The fact that equations of the Hodgkin-Huxley type with $l = 0$ exhibit stable traveling wave solutions with finite velocity (see for example [2]) shows that this procedure is not justified. Under Lieberstein's assumption, the ordinary differential equation for traveling waves is

$$cv_t + I = (1/rc\theta^2)I_t. \tag{3$'$}$$

Equations (3) and (3)$'$ are different, though of course they coincide in the case $\theta \longrightarrow \infty$ which corresponds to a "membrane" or "space-clamped" action potential [1].

Lieberstein's assumption that the nerve impulse travels at the velocity given by $\theta^2 = 1/lc$ is almost certainly incorrect. First, it is doubtful whether one could produce a field theory for transmission line conduction in nerves which would give such a low signal velocity as the observed propagation rate in nerves (20 m/sec in squid giant axon). Second, $\theta^2 = 1/lc$ yields a velocity which is uninfluenced by the active properties of the membrane and should therefore be independent of temperature, for example, but this is not the case. Finally, it is well known that electrical stimulation of a nerve at one point yields a stimulus artifact at distant points which arrives essentially instantaneously, long before the nerve impulse. This observation strongly suggests that the nerve impulse itself travels at a velocity far lower than the maximum possible signal velocity of the nerve cable.

If the term li_t is included in the formulation, and one looks for solutions which are traveling waves with velocity $\theta^2 < 1/lc$, then the ordinary differential equation to be satisfied by these waves is second order in time, like (3), and tends to (3) in the limit $l \longrightarrow 0$, θ fixed. This would appear to be the correct procedure.

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1. A. L. HODGKIN & A. F. HUXLEY, "A quantitative description of membrane current and its application to conduction and excitation in nerve," *J. Physiology (London)*, v. 117, 1952, pp. 500–544.

2. J. RINZEL & J. B. KELLER, "Travelling wave solutions of a nerve conduction equation," *Biophysical J.*, v. 13, 1973, pp. 1313–1337.