Tables of Reductions of Symmetrized Inner Products ("Inner Plethysms") of Ordinary Irreducible Representations of Symmetric Groups

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Abstract. Decompositions of symmetrized inner products \([\alpha] \boxtimes [\beta]\) of ordinary irreducible representations \([\alpha]\) of symmetric groups \(S_n\) and \([\beta]\) of \(S_m\) were evaluated on a CDC 6400. Tables were obtained for \(2 \leq n \leq 10\) and \(2 \leq m \leq 5\) as well as for \(m = 6\) and \(2 \leq n \leq 7\).

In [3] R. C. King published tables of reductions of symmetrized inner products \([\alpha] \boxtimes [\beta]\) which he calls inner plethysms, of ordinary irreducible representations \([\alpha]\) of \(S_n\) and \([\beta]\) of \(S_m\), where \(n = 4\) and \(m \leq 5\), \(n = 5\) and \(m \leq 4\), \(n = 6\) and \(m \leq 3\).*

He obtained the decomposition by restricting certain representations \([\beta]\) of the general linear group \(GL_n\) to symmetric subgroups.

Such decompositions can be obtained directly by evaluating the character of \([\alpha] \boxtimes [\beta]\) which is

\[
\chi_{[\alpha] \boxtimes [\beta]}(g) = \frac{1}{|S_m|} \sum_{\pi \in S_m} \xi_{[\beta]}(\pi) \prod_{k=1}^n \xi_{[\alpha]}(g^k)^a_k(\pi),
\]

where \(g \in S_n\) and \(a_k(\pi)\) denotes the number of cyclic factors of length \(k\) in \(\pi \in S_m\), \(1 \leq k \leq m\).

For this formula see [1], [2], and [4, p. 74]. The evaluation was carried out with the aid of a computer (CDC 6400 RWTH Aachen) by using the program described in [1], in double-precision arithmetic. The characters of the products \([\alpha] \boxtimes [\beta]\) were then decomposed into their irreducible constituents via orthogonality relations by using the character table of \(S_n\).

Tables were thus obtained of the reductions of the symmetrized inner products of the ordinary irreducible representations of the symmetric groups \(S_2\) up to \(S_{10}\) with the ordinary irreducible representations of \(S_2\) up to \(S_5\) and of the characters of \(S_2\) up to \(S_7\) with those of \(S_6\). These tables appear on the microfiche card in this issue.

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*Dr. King wants me to point out that in the tables of [3] two printing errors occurred in the decomposition of \([3, 1] \boxtimes [2, 2]\) and \([3, 1^2] \boxtimes [2, 1^2]\). The correct values can be obtained from the microfiche.

