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# Mathematics of Computation

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Groups .................................................... N. Esper

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Manuscripts should be typewritten double-spaced in the format used by the journal. For journal abbreviations, see the latest Mathematical Reviews volume index. An author should submit the original and one copy of the manuscript and retain one copy. The author may suggest an appropriate editor for his paper. It is recommended that the author acquaint himself with the pertinent material contained in “A Manual for Authors of Mathematical Papers,” which is available from the American Mathematical Society. All contributions intended for publication and all books for review should be addressed to James H. Bramble, Chairman, Editorial Committee, Mathematics of Computation, Center for Applied Mathematics, 275 Olin Hall, Cornell University, Ithaca, New York 14853. Institutions sponsoring research reported in the journal are assessed page and microfiche charges.

Each article submitted for publication must be accompanied by a brief and reasonably self-contained abstract, and by AMS (MOS) subject classification numbers. If a list of key words and phrases is included, it will be printed as a footnote on the first page. A list of the classification numbers may be found in the Index to Mathematical Reviews, Volume 39 (June 1970).
The five articles in this volume are expository in nature, and they all deal with various aspects of
the theory of bounded linear operators on Hilbert space. The volume is very timely, because in the
last year or two great progress has been made on hard problems in this field, and thus operator
theory today is a very exciting area of mathematical research. One particular problem on which
considerable progress has been made recently is the invariant subspace problem. This is the question
whether every bounded linear operator on a separable, infinite-dimensional, complex Hilbert space H
has a nontrivial invariant subspace. Even though this problem remains unresolved, there are some
operators T on H for which the structure of the lattice of all invariant subspaces of T is known, and
the first article in this volume, “Invariant subspaces”, by Donald Sarason, is devoted to a discussion
of such operators. One of the interesting features of this lucid presentation is the interplay between
operator theory and classical analysis.

The second article is entitled “Weighted shift operators and analytic function theory” and was
written by Allen Shields. He has taken essentially all of the information presently known about weighted
shift operators (with scalar weights) and incorporated it into this comprehensive article. A central
theme of the exposition is the interaction between weighted shift operators and analytic function
theory, and as an added bonus for the reader, the article contains a list of thirty-two interesting
research problems.

The third article in the volume is a treatise entitled “A version of multiplicity theory” by Arlen
Brown. The problem treated is how to decide when two normal operators are unitarily equivalent.
(Unitary equivalence is the analog for operators of the concept of isomorphism for groups, rings,
etc.) The unitary equivalence problem for arbitrary operators is exceedingly difficult, but the theory
of spectral multiplicity, which can be approached in several different ways, furnishes a reasonable
complete set of unitary invariants for normal operators. The author focuses attention on the concept
of a spectral measure, and his clear presentation of this circle of ideas should lead to a better
understanding of multiplicity theory by beginners and experts alike.

The fourth article in this volume, “Canonical models” by R. G. Douglas, is concerned with the
theory of canonical models for operators on Hilbert space. The central underlying idea is that if
T is any contraction operator on H (i.e., if the norm of T is at most 1), then there is a canonical
construction that associates with T an operator $M_T$ that is unitarily equivalent to T, called its
“canonical model”. One can therefore study T by studying $M_T$ instead, and this theory has made
significant progress in the past ten years. The author, who has contributed substantially to the
geometrization of this theory, exposes in his article various important components of the theory,
and thereby gives the reader much insight into its successes and failures.

The final article in this volume, “A survey of the Lomonosov technique in the theory of invariant
subspaces” by Carl Pearcy and Allen Shields, is a survey of some new invariant-subspace theorems
that resulted from the brilliant and elegant method of proof introduced by Victor Lomonosov early
in 1973. Further study and refinement of this technique should lead to additional progress on the
invariant subspace problem.
The Influence of Computing on Mathematical Research and Education,
Edited by Joseph P. LaSalle

This volume contains seven of the invited addresses and fourteen of the contributed papers that were presented at the joint American Mathematical Society and the Mathematical Association of America Conference on the Influence of Computing on Mathematical Research and Education held at the University of Montana, August 13—24, 1973.

The invited addresses were directed primarily to the influence of the computer on mathematical research and the applications of mathematics and secondarily on what this means for the teaching of mathematics and the education of mathematicians. The contributed papers describe more specifically some experiments in developing courses in mathematics with computing and algorithmic orientations and a few reports on computer influenced research.

The titles of the seven invited addresses and their authors follow:

The Influence of Computing on Research in Number Theory by D. H. Lehmer
The Influence of Computers on Algebra by Charles C. Sims
Computational Probability and Statistics by Ulf Grenander
An Introduction to Some Current Research in Numerical Computational Complexity by J. F. Traub
Applied Mathematics and Computing by Peter D. Lax
The Unexpected Impact of Computers on Science and Mathematics by Thomas E. Cheatham, Jr.

The titles of the fourteen contributed papers and their authors follow:

Computational Complex Analysis by Peter Henrici
Combinatorial Games with an Annihilation rule by Aviezri S. Fraenkel
The Integration of Computing and Mathematics at the Open University by F. B. Lovis and R. V. M. Zahar
Real Time Computer Graphics Techniques in Geometry by Thomas Banchoff and Charles Strauss
Visual Geometry, Computer Graphics and Theorems of Perceived Type by Philip J. Davis
The Design and Use of an Undergraduate Numerical Analysis Laboratory by Myron Ginsberg
Statistical and Numerical Analysis: A Computer Oriented Approach by Andre R. Brousseau
Some Problems in Computational Probability by Marcel F. Neuts
The Influence of Computing on Generalized Inverse Applications in Statistical Analysis by Cecil R. Hallum
On Using the Electronic Analog Computer to Illustrate Mathematical Concepts by Tyre A. Newton
An Inexpensive Computer Assist in Teaching Large Enrollment Mathematics Courses by Edward L. Spitznagel, Jr.
A new Computer Oriented (Algorithmic) Linear Algebra Course—Preliminary Report by Robert Ducharme
Computer Supplemented Business Oriented Mathematics by Kenneth L. Hankerson and Gene A. Kemper

Only some college training in mathematics is needed to read most of the volume. It should be of some interest to high school teachers of mathematics.

INDEX TO MATHEMATICS OF COMPUTATION, 1943—1969

Edited by
Yudell L. Luke, Jet Wimp and Wyman Fair

462 + xviii pages; list price $19.95; institutional member price $14.96; individual member price $9.97
ISBN 0-8218-4000-2; to order, please specify MCOMIN/1

The INDEX TO MATHEMATICS OF COMPUTATION is a compilation, by author and by subject, of all material which has appeared in MATHEMATICS OF COMPUTATION and its predecessor, MATHEMATICAL TABLES AND OTHER AIDS TO COMPUTATION, during the years 1943—1969—twenty-three published volumes. The INDEX contains over 7,000 entries. This is an unusual compilation because of the unique character of the journal which not only publishes research papers, but also publishes reviews of material on mathematics of computation and a table errata section covering a number of other publications. In addition, an unpublished mathematical tables (UMT) file is maintained.

A new classification system, which was developed in 1969 by a committee chaired by Yudell Luke at the Midwest Research Institute, is used in the subject classification index. In this section, all articles, tables, reviews, etc. are classified. The classification scheme is designed as an indexing system for retrieval of information in MATHEMATICS OF COMPUTATION, and the present index contains classification numbers for all entries beginning with 1943.

The author index has been set up so that it gives bibliographical information on all of the items published in the journal. Information in this index includes title of article; translated title of books not in English, French, German, or Italian; title of book which is a collection of articles written by a number of authors; volume, year, and number of pages; publisher of a book; journal title of periodicals, information concerning translations; MATHEMATICAL REVIEWS numbers; subject classification. Items are listed alphabetically by author and chronologically under each author's entry. Each article is identified by both an ordering numeral and an identifying code in order that the reader may tell quickly whether the information listed concerns a primary research publication, a review of the work, errata to a table, or any of the other types of information covered by the journal. The following codes are used:

P = research paper
R = review of article or book
T = table in UMT (unpublished mathematical table) file
E = table errata
Q = queries and replies
M = microfiche

These identifying codes appear in the subject classification index, also, to refer the reader to the author index.

The identifying code appearing in both indexes provides a cross-reference system that enables the reader to retrieve information rapidly. For example, if the following entry appears in the subject classification index

3.10 Linear Equations
Smith, A. B. 1 PR, 3E

the reader knows immediately that under Smith's name in the author index, there will be listed both a research article on the subject of linear equations, written by Smith, and a review of the article. In addition, an errata to a table which Smith published in another journal will be listed.

The preparation and publication of the subject classification index was supported in part by a grant from the National Science Foundation (GN-691).

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Edited by Gilbert Baumslag
- as printed in MATHEMATICAL REVIEWS, volumes 1—40
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- classified under 24 major and 264 minor headings
- 2 parts, 1,062 pages (1974)

REVSIEWS OF PAPERS ON FINITE GROUPS
Edited by Daniel Gorenstein
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- classified under 21 major headings
- 3,200 reviews
- 1 volume, 736 pages (1974)

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The editorial committee would welcome readers' comments about this microfiche
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Hall, Cornell University, Ithaca, New York 14853.
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