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The research journals of the American Mathematical Society carry a page charge of $40.00 per page to help defray the cost of publication. This amount is charged to the institution or to a contract supporting the research reported in the published paper. The publication charge policy of the United States Federal Council for Science and Technology (FCST) is reported on page 112 of the February, 1975 issue of the Notices of the American Mathematical Society. In no case is the author personally responsible for paying the page charge, nor is acceptance of the author's paper for publication dependent upon payment of the page charge.
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R. Tyrrell Rockafellar, *Lagrange multipliers in optimization*

O. L. Mangasarian, *Unconstrained methods in nonlinear programming*

J. E. Dennis, Jr., *A brief survey of convergence results for quasi-Newton methods*

These Proceedings are based on lectures delivered at the symposium on Nonlinear Programming held March 23 and 24, 1975, as part of the American Mathematical Society's annual New York meeting. This event was the ninth in a series of Symposia in Applied Mathematics jointly sponsored by the Society for Industrial and Applied Mathematics and the American Mathematical Society with financial support from the Energy Research and Development Agency (formerly the Atomic Energy Commission) and the National Science Foundation.

The organizing committee for the symposium consisted of R. W. Cottle (chairman), C. E. Lemke, S. M. Robinson, and J. B. Rosen. The committee's intent was to help bring to the attention of a larger mathematical audience some of the history, theory, applications and vigorous research activity of the Nonlinear Programming field. The editors feel that the results included in these Proceedings can be recommended as well to the worker in the field as to the interested initiate.

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H. T. Kung and J. F. Traub, Computational complexity of one-point and multipoint iteration

During the last decade computational complexity has become one of the most active research areas within the mathematical theory of computation. Workers in computational complexity seek to derive efficient algorithms for computational problems of practical interest, to prove the optimality of particular algorithms relative to well-defined measures of computational efficiency, and to derive general lower bounds on the time or space intrinsically necessary for the performance of computational tasks. The specific problems considered are drawn from diverse areas, including numerical computation, symbolic algebraic computation, combinatorics, computational logic and the manipulation of data structures. The mathematical tools called upon are correspondingly diverse, ranging from algebraic geometry through computability theory. Nevertheless, some characteristic proof techniques and approaches to algorithm construction are emerging as complexity theory matures and strives for unification.

This volume is the proceedings of a symposium held in New York City on April 18 and 19, 1973, under the joint sponsorship of the American Mathematical Society and the Society for Industrial and applied Mathematics.

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The Influence of Computing on Mathematical Research and Education,
Edited by Joseph P. LaSalle

This volume contains seven of the invited addresses and fourteen of the contributed papers that were presented at the joint American Mathematical Society and the Mathematical Association of America Conference on the Influence of Computing on Mathematical Research and Education held at the University of Montana, August 13—24, 1973.

The invited addresses were directed primarily to the influence of the computer on mathematical research and the applications of mathematics and secondarily on what this means for the teaching of mathematics and the education of mathematicians. The contributed papers describe more specifically some experiments in developing courses in mathematics with computing and algorithmic orientations and a few reports on computer influenced research.

The titles of the seven invited addresses and their authors follow:
- The Influence of Computing on Research in Number Theory by D. H. Lehmer
- The Influence of Computers on Algebra by Charles C. Sims
- Computational Probability and Statistics by Ulf Grenander
- An Introduction to Some Current Research in Numerical Computational Complexity by J. F. Traub
- Applied Mathematics and Computing by Peter D. Lax
- The Unexpected Impact of Computers on Science and Mathematics by Thomas E. Cheatham, Jr.

The titles of the fourteen contributed papers and their authors follow:
- Computational Complex Analysis by Peter Henrici
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- Computer Supplemented Business Oriented Mathematics by Kenneth L. Hankerson and Gene A. Kemper

Only some college training in mathematics is needed to read most of the volume. It should be of some interest to high school teachers of mathematics.

INDEX TO MATHEMATICS OF COMPUTATION, 1943—1969

Classified and Edited by
Yudell L. Luke, Jet Wimp and Wyman Fair

462 + xviii pages; list price $19.95; institutional member price $14.96; individual member price $9.97
ISBN 0-8218-4000-2; to order, please specify MCOMIN/1

The INDEX TO MATHEMATICS OF COMPUTATION is a compilation, by author and by subject, of all material which has appeared in MATHEMATICS OF COMPUTATION and its predecessor, MATHEMATICAL TABLES AND OTHER AIDS TO COMPUTATION, during the years 1943—1969—twenty-three published volumes. The INDEX contains over 7,000 entries. This is an unusual compilation because of the unique character of the journal which not only publishes research papers, but also publishes reviews of material on mathematics of computation and a table errata section covering a number of other publications. In addition, an unpublished mathematical tables (UMT) file is maintained.

A new classification system, which was developed in 1969 by Yudell L. Luke, Jet Wimp and Wyman Fair is used in the subject classification index. In this section, all articles, tables, reviews, etc. are classified. The classification scheme is designed as an indexing system for retrieval of information in MATHEMATICS OF COMPUTATION, and the present index contains classification numbers for all entries beginning with 1943.

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The identifying code appearing in both indexes provides a cross-reference system that enables the reader to retrieve information rapidly. For example, if the following entry appears in the subject classification index

3.10 Linear Equations
Smith, A. B. 1PR, 3E

the reader knows immediately that under Smith’s name in the author index, there will be listed both a research article on the subject of linear equations, written by Smith, and a review of the article. In addition, an errata to a table which Smith published in another journal will be listed.

The preparation and publication of the subject classification index was supported in part by a grant from the National Science Foundation (GN-691).

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