TABLE ERRATA

523.—MILTON ABRAMOWITZ & IRENE A. STEGUN, *Handbook of Mathematical Functions With Formulas, Graphs, and Mathematical Tables,* National Bereau of Standards, Applied Mathematics Series, no. 55, U. S. Government Printing Office, Washington, D. C., 1964, and all known reprints.

On pp. 610–611, in Table 17.2, the 15D values of the complete elliptic integrals $K(\alpha)$ and $E(\alpha)$ have been reproduced from the tables of G. W. & R. M. Spenceley [1], as acknowledged in a footnote, and accordingly contain the 23 final-digit errors discovered therein by DiDonato [2]. The corresponding corrections have been confirmed by a new, carefully checked calculation, extending to 20D. Only two of the errors exceed the tolerance specified on p. ix of the *Handbook*; namely, the terminal tabular digits of $K(\alpha)$ should read 86 and 30 in place of 89 and 25, respectively, when $\alpha = 79°$ and $87°$.

PAUL NININ

Lycée Victor Louis
Talence-Bordeaux, France

1. G. W. SPENCELEY & R. M. SPENCELEY, *Smithsonian Elliptic Functions Tables,* The Smithsonian Institution, Washington, D. C., 1947. (See MTAC, v. 3, 1948–1949, pp. 89–92, RMT 485.)

2. A. R. DIDONATO, MTE 318, *Math. Comp.,* v. 16, 1962, pp. 511–512.

524.—A. ERDÉLYI, W. MAGNUS, F. OBERHETTINGER & F. G. TRICOMI, *Higher Transcendental Functions,* McGraw-Hill Book Co., New York, 1953.

In Volume I the following corrections are required.,

On p. 225, the right sides of formulas (20) and (21) should read, respectively,

$$\sum \frac{(\alpha)_{m+n}(\beta)_m}{(\gamma)_{m+n} m! n!} x^m y^n, \qquad |x| < 1.$$

and

$$\sum \frac{(\beta)_m (\beta')_n}{(\gamma)_{m+n} m! n!} x^m y^n.$$

On p. 241, the right side of formula (10) should read

$$\sum \frac{\Gamma(\gamma)\Gamma(\rho - \lambda)\Gamma(\sigma - \mu)}{\Gamma(\rho)\Gamma(\sigma)\Gamma(\gamma - \lambda - \mu)} (-x)^{-\lambda}(-y)^{-\mu}$$

$$\times F_2(\lambda + \mu + 1 - \gamma, \lambda, \mu, \lambda + 1 - \rho, \mu + 1 - \sigma; 1/x, 1/y).$$

W. P. WOOD

Department of Mathematics
University of Newcastle
Newcastle, New South Wales 2308, Australia

In Volume II, on p. 186 the left side of formula (30) should read

$$2 \sum_{n=1}^{\infty} n^{-1} T_n(x) z^n.$$

NEVILLE I. ROBINSON

Department of Civil Engeering
University of Waterloo
Waterloo, Ontario, Canada

EDITORIAL NOTE: For Notices of additional errata in these volumes see *Math. Comp.*, v. 29, 1975, p. 670, MTE 516 and the editorial footnote thereto; also v. 17, 1963, p. 485, MTE 338; v. 18, 1964, p. 687, MTE 360; v. 24, 1970, p. 504, MTE 460; v. 25, 1971, p. 199, MTE 471, ibid., p. 635, MTE 481; v. 26, 1972, p. 598, MTE 490.

525.—F. HORNER, "A table of a function used in radio-propagation theory", *Proc. Inst. Elec. Engineers,* Part C, v. 102, 1955, pp. 134–137.

The function tabulated to 3D is

$$F(\omega) = 1 + 2j\sqrt{\omega}e^{-\omega}\int_{-j\sqrt{\omega}}^{\infty} e^{-x^2}\,dx,$$

where j denotes the imaginary unit, when $0 \leqslant \operatorname{Re}\omega \leqslant 10$, $-10 \leqslant \operatorname{Im}\omega \leqslant 0$. The function can be written as

$$F(\omega) = 1 + j\sqrt{\pi\omega}e^{-\omega} + \sum_{n=0}^{\infty} \frac{(-2\omega)^{n+1}}{(2n+1)!!},$$

where $(2n + 1)!! = 1 \cdot 3 \cdot \ldots \cdot (2n + 1)$. The table shows that $\operatorname{Im} F(\omega) \leqslant 0$ when $\operatorname{Im}\omega = 0$, from which it can be inferred that the table uses the branch of $\sqrt{\omega}$ for which $\operatorname{Re}\sqrt{\omega} \leqslant 0$, $\operatorname{Im}\sqrt{\omega} \geqslant 0$ for the values of ω under consideration.

An ALGOL procedure [1] computes the function

$$G(p) = 1 + i\sqrt{\pi p}e^{-p}\operatorname{erfc}(-i\sqrt{p}), \quad \operatorname{erfc}(q) = \frac{2}{\sqrt{\pi}}\int_q^{\infty} e^{-x^2}\,dx,$$

where $\operatorname{Re} p \geqslant 0$, $\operatorname{Im} p \geqslant 0$, and the branch of $\sqrt{p}$ used is such that $\operatorname{Re}\sqrt{p} \geqslant 0$, $\operatorname{Im}\sqrt{p} \geqslant 0$. Thereby, the relation between the two functions is

$$F(\omega) = \overline{G(p)}, \quad p = \overline{\omega}.$$

By means of the procedure [1], which has an accuracy better than 3D, the values for $\operatorname{Re}\omega = 0(0.5)10$ and $\operatorname{Im}\omega = -10(0.5)0$ have been computed and compared with Table 1. This disclosed 87 errors of a unit in the last decimal and two errors of two units:

For $\operatorname{Re} F(8 - j) = -0.076$, read -0.078,

for $\operatorname{Im} F(10 - 5j) = -0.025$, read -0.027.

The function $G(p)$ is closely related to the function $w(z) = e^{-z^2}\operatorname{erfc}(-iz)$, which has been tabulated [2] to 6D, in that

$$G(p) = 1 + i\sqrt{\pi p}\,w(\sqrt{p}).$$

Computations based upon the table of $w(z)$ confirm the two corrections given above:

$$G(8 + i) \doteq -0.0779 + 0.0146i, \quad G(10 + 5i) \doteq -0.0435 + 0.0266i.$$

The table under consideration has been differenced with respect to one of the independent real variables in the vicinity of the two above-mentioned entries: for $\operatorname{Im}\omega = -1$ near $\operatorname{Re}\omega = 8$, and for $\operatorname{Re}\omega = 10$ near $\operatorname{Im}\omega = -5$. This revealed a maximum absolute sixth-order difference equal to 34 and 39, respectively, which shows that the accuracy of the table leaves something to be desired ([3, §II − *N*], [4, §1.8]), but after the correction of the errors of both one unit and two units the differences decreased to 19 and 16.

SØREN CHRISTIANSEN

Laboratory of Applied Mathematical Physics
The Technical University of Denmark
DK-2800 Lyngby/Copenhagen, Denmark

1. S. CHRISTIANSEN, "Computation of Sommerfeld's attenuation function, Sommerfeld cox, Algol procedure," *Apl. Mat.*, v. 18, 1973, pp. 379–384.

2. V. N. FADDEYEVA & N. M. TERENT'EV, *Tables of Values of the Function* $w(z) = e^{-z^2}(1 + (2i/\sqrt{\pi}) \int_0^z e^{t^2}\, dt)$ *for Complex Argument*, Pergamon Press, Oxford, 1961.

3. Z. KOPAL, *Numerical Analysis*, 2nd ed., Chapman & Hall, London, 1961.

4. R. A. BUCKINGHAM, *Numerical Methods*, Pitman Press, London, 1962.

526.—E. MEISSEL, *Ueber die Besselschen Funktionen* $J_0(x)$ *und* $J_1(x)$, Jahresbericht ueber die Oberrealschule in Kiel (Programm No. 284), 1890.

On p. 4 are tabulated to 16D the zeros, $j_{1,s}$, of $J_1(x)$ and the corresponding values $J_0(j_{1,s})$ for $s = 1(1)50$.

The only error in the tabulated zeros occurs in the final digit of $j_{1,4}$: *for* 2231, *read* 2230.

In addition to six errors reported by Russon and Blair [1], there are two additional errors in the 16D turning values of $J_0(x)$; namely, in $J_0(j_{1,1})$, *for* 5547, *read* 5530, and in $J_0(j_{1,3})$, *for* 8259, *read* 8432.

Meissel's table of $j_{1,s}$ and $J_0(j_{1,s})$ was reprinted in its entirety in Gray and Mathews [2] and subsequently in Gray, Mathews and MacRobert [3], and it was abridged to 10D in Davis and Kirkham [4]. It is well known [5] that Meissel's correct value for $j_{1,9}$ was copied incorrectly in these references, the tenth decimal figure having been printed erroneously as 0 instead of 9.

RALPH HELLER

Department of Physics
Worchester Polytechnic Institute
Worchester, Massachusetts 01609

1. A. E. RUSSON & J. M. BLAIR, MTE **452**, *Math. Comp.*, v. 24, 1970, p. 240.

2. A. GRAY & G. B. MATHEWS, *A Treatise on Bessel Functions and Their Applications to Physics*, Macmillan, London, 1895 (Table III, p. 280).

3. A. GRAY, G. B. MATHEWS & T. M. MACROBERT, *A Treatise on Bessel Functions and Their Applications to Physics*, 2nd ed., Macmillan, London, 1922; reprinted by Dover, New York, 1966 (Table IV, p. 301)

4. H. T. DAVIS & W. J. KIRKHAM, "A new table of the zeros of the Bessel functions $J_0(x)$ and $J_1(x)$ with corresponding values of $J_1(x)$ and $J_0(x)$," *Bull. Amer. Math. Soc.*, v. 33, 1927, pp. 760–772 (Table II, p. 769).

5. A. FLETCHER, J. C. P. MILLER, L. ROSENHEAD & L. J. COMRIE, *An Index to Mathematical Tables*, 2nd ed., 1962, Addison-Wesley Publishing Co., Reading, Mass. (Article 17.7311, pp. 405–406).

527.—L. J. SLATER, *Confluent Hypergeometric Functions*, Cambridge Univ. Press, New York, 1960; W. MAGNUS, F. OBERHETTINGER & R. P. SONI; *Formulas and Theorems for the Speical Functions of Mathematical Physics*, Springer-Verlag, New York, 1966.

Equations 3.7.4 to 3.7.6 and 3.7.14 to 3.7.16 of Slater are incorrect; as are the first, second and fourth equations on p. 280 and the first equation to p. 322 of Magnus et al. The conditions given do not guarantee convergence of the integrals in question. Further, even when the integrals do converge, they are generally not equal to the functions listed, as can be seen by noting that both sides of each equation are meromorphic functions of their parameters but that the integrals are analytic at points where the right-hand sides have poles.

The conditions under which the integrals converge can be found by inspection of the asymptotic forms of the integrands. In the case

$$\operatorname{Re} \alpha > 0, \quad \operatorname{Re} k < 0, \quad \operatorname{Re} z > 0 \quad \text{and} \quad \operatorname{Re}(a + c' - a' - \alpha) > 0$$

the fourth integral on p. 280 of Magnus et al. should read

$$\begin{aligned} &\int_0^\infty t^{\alpha-1}\,{}_1F_1(a; c; kt)\,{}_1F_1(a'; c'; zt)e^{-zt}\,dt \\ &\qquad = \Gamma(\alpha)z^{-\alpha}F_2(\alpha; a, a'; c, c'; k/z, 1) \end{aligned} \tag{1}$$

which can be obtained by continuation from Eq. 14 on p. 216 of [1]. The integral in Eq. 3.7.4 of Slater is this same integral (in a slightly different notation). All other integrals mentioned above are either restrictions of this result to special values of the parameters, or a restatement in terms of Whittaker functions.

Other conditions for which the integral (1) has been evaluated are given in reference [2].

MARK D. SEMON
JOHN R. TAYLOR

Department of Physics and Astrophysics
University of Colorado
Boulder, Colorado 80309

1. A. ERDÉLYI, Editor, *Tables of Integral Transforms*, Vol. I, McGraw-Hill, New York, 1954.

2. K. ADLER et al., *Rev. Modern Phys.*, v. 28, 1956, p. 432.