REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the revised indexing system printed in Volume 28, Number 128, October 1974, pages 1191–1194.

1 [2.30, 7.05].– RICHARD P. BRENT, *γ and e^γ to* 20700D *and Their Regular Continued Fractions to* 20000 *Partial Quotients,* Australian National University, 1976, 76 computer sheets deposited in the UMT file.

These are the four tables referred to in Brent's paper [1]. They give γ and e^γ to 20700D and their regular continued fractions

$$q_0 + \frac{1}{q_1} + \frac{1}{q_2} + \cdots \frac{1}{q_{20000}}$$

to 20000 partial quotients. For historical, computational and statistical details, see Brent's paper.

A paradoxical point in Brent's paper is this: He attributes to Euler the "suggestion that e^γ could be a more natural constant than γ". Euler should know and I might add that I am inclined to agree: certainly e^γ and $e^{-\gamma}$ occur more frequently in analysis. Now γ is known, by experience, to be harder to compute than π, and π is harder than e. Yet here Brent first computes γ by Sweeney's method, which is the most efficient known, and *then* he computes e^γ from γ. Why not reverse the order? So the question is: What occurrence of e^γ in analysis will lead to an efficient algorithm for its (direct) computation?

Originally, Brent computed γ to 10488D and when this was compared with the previous computation of γ by Beyer and Waterman a discrepancy between the two led to the discovery of an error in the value of Beyer and Waterman. A Table Errata in this issue gives details.

D. S.

1. RICHARD P. BRENT, "Computation of the regular continued fraction for Euler's constant," *Math. Comp.*, v. 31, 1977, pp. 771–777.

2 [3.00, 4.00].–A. C. BAJPAI, I. M. CALUS & J. A. FAIRLEY, *Numerical Methods for Engineers and Scientists,* Taylor & Francis Ltd., London, 1975, xii + 380 pp., 25 cm. Price £6.75.

This is a course book written for undergraduates and engineering students. It emphasizes the practical side of the subject and the more theoretical aspects have been largely omitted. The book is unconventional in that it is a programmed text book. It is divided into three units: Unit 1 – Equations and Matrices, Unit 2 – Finite Differences and their Applications, and Unit 3 – Differential Equations. Each unit is then subdivided into programs (between 3 and 5) and each program consists of a sequence of frames (on the average 3 frames per page). Miscellaneous examples with answers and hints are given at the end of each program.

The topics are presented through a sequence of carefully chosen examples mostly with some physical or technical background. The reader first works through one or several numerical examples and then arrives at a more mathematical presentation of the method. The book reflects the authors' interest in educational technology and their experience in teaching mathematics to engineering students, who often do not have a very high mathematical ability and are in need of motivation.

In an introductory course book in an applied subject like this, it is useful to have a good list of references for those students who later want, or indeed must, complement their knowledge. Unfortunately the list given in this book is inadequate, and in fact fails to list almost all of the more popular textbooks that exist. The typography of the book is not very appealing and lack of easy to find headlines makes it unnecessarily difficult to find the results you are looking for. Fortunately, however, there is a useful subject index giving both page and frame number.

The overall content of the book corresponds, in general, well to what should be included in an introductory course for engineering students. There are some notable exceptions where the treatment is not up to modern standards, and I list some of these here. On p. 91 it is stated that the better of the two approximate solutions to a system of linear equations $Ax = b$, is the one for which $\|b - Ax\|_2$ is less. This choice, however, does not necessarily make the error in x small, and the choice must, in general, depend on one's criterion of goodness. Also, in Gaussian elimination rounding errors are *always* correlated so as to give a small residual, which is not the case for, e.g. Gauss-Jordan elimination. On p. 111, one of the advantages of Gauss-Seidel's iteration method over elimination is said to be that round-off errors generally have less effect. This was a widely held belief in "pre-Wilkinson times", but in fact the opposite is more true. Doolittle's algorithm for computing the LR-factorization (called Choleski's method by the authors) is introduced without pointing out that Gaussian elimination will produce the same L and R. It should also have been stated that the symmetric factorization $A = LL'$ works only when A is positive definite. In the program on least squares the normal equations are used throughout, without hinting at the numerical difficulties that can result. Indeed the use of a false origin is described, but this is said only to be useful for reducing the amount of arithmetic in hand calculations. In the program on interpolation I was surprised to see that divided differences are introduced at great length, but then only used to derive Lagrange's interpolation formula.

The fact that one can find details to criticize should not detract from the basic fact that this is a practical textbook to use for the group of students at which it is aimed. Whether you as a teacher will like this book probably depends on your own attitude to programmed education. The approach as used here certainly has advantages. It stimulates the student to work actively, and this should be a good text for self-instruction. For the students in the group with good mathematical abilities, the numerical examples may sometimes obscure rather than simplify many derivations. Often theoretical knowledge can be very practical to have, and I am not sure the authors always have found the right balance.

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3 [3.15].—G. GOOS & J. HARTMANIS, Editors, *Matrix Eigensystem Routines—EISPACK Guide,* 2nd ed., Lecture Notes in Comput. Sci., Vol. 6, Springer-Verlag, Berlin, 1976, vii + 551 pp. Price $17.30 paperback.

In 1971 J. H. Wilkinson and C. Reinsch published a set of Algol procedures for eigenvalue and eigenvector computations. From the mathematician's point of view, these algorithms settle the question of how these computations should be done (unless, of course, you disagree with the particular choice of algorithms). However, it is now widely recognized that it requires considerable effort and talent to turn good Algol procedures into programs that can be easily and widely used. Obviously, an Algol to Fortran translation must be made. In addition, to get top efficiency and reliability,

the Fortran programs must be carefully tailored for the particular computing environment where they are to be used.

The first project of the National Activity to Test Software (NATS) was to produce these programs. One may view this effort as one of technology transfer. A knowledgeable person starting with the Wilkinson-Reinsch procedures can expect to spend 30 days to produce a high quality program for a particular eigensystem calculation. The same person starting with EISPACK can expect to spend 30 minutes to obtain a similar program, one which is probably better than his own product. Even more significant is that a naive user can start with EISPACK and a simple interface routine and obtain the same high quality programs. Thus NATS has placed the eigensystem computation expertise of Wilkinson and Reinsch at the finger tips of the whole scientific community. One should note that NATS also made various improvements in the algorithms as well as corrected a few errors.

People who have not seen a full cost accounting of top quality software are amazed at its cost. The EISPACK project cost something less than a million dollars to produce about 12,000 lines of Fortran code. This cost of $80 per Fortran line is unusually high for several reasons, but $25 per Fortran statement is to be expected for top quality, reliable and documented programs.

This book has three distinct audiences: the naive user (he knows what eigenvalues are, but has little idea of how to compute them), the sophisticated user (he has a broad knowledge of modern methods for eigensystem computations and some familiarity with the EISPACK terminology) and the eigensystem software expert. There are about 50 pages of basics on "How to use EISPACK" which will challenge the naive user. He would be better off with 5 pages describing how to use a simple interface. The second release of EISPACK recognized the naive user's problem and provides an interface for IBM users which is described among these 50 pages.

The 120 pages of "How to use EISPACK" (50 pages of basics plus 70 pages of special considerations) are exactly aimed at the sophisticated user. Once one is familiar with the EISPACK terminology and organization, one can readily obtain the right programs for almost any eigensystem calculation.

The remaining 430 pages are for the eigensystem software expert. About 350 pages give the actual Fortran programs and their documentation. The validation and certification of EISPACK are described only briefly (5 pages) and references are given to other publications. It is unfortunate that these topics are slighted since they are crucial to the scientific merit of EISPACK. Another 20 pages of text would have been well spent to substantiate the claims of quality and reliability for EISPACK. There are 57 pages of tables which substantiate the efficiency claims for EISPACK. Somewhat random comparisons with other programs (not reported in this book) indicate that other programs are usually significantly less efficient or less reliable and often both.

This book is a must for the library of a mathematical software expert or a numerical analyst who does eigensystem calculations. People who occasionally do straightforward eigensystem calculations will prefer a short and simple description of how to use EISPACK.

J. R. R.

4 [5, 6.30].—R. GLOWINSKI, J. L. LIONS, & R. TRÉMOLIÈRES, *Analyse Numérique de Inéquations Variationnelles,* Vol. 1; *Applications aux Phénomènes Stationnaires et d'Évolution,* Vol. 2, Dunod, Paris, 1976, xiv + 290 pp., 25 cm. Price: Vol. 1 $36.00, Vol. 2 $42.00.

This two volume set is the long-awaited work dealing with the numerical analysis

of problems formulated in *Les inéquations en mécanique et en physique* by G. Duvaut and J. L. Lions.

The first volume deals with the general theory of stationary variational inequalities (Chapter 1), discusses algorithms for solving the finite-dimensional optimization problems which result from the approximation schemes (Chapter 2), and then considers in detail the specific model problem of elasto-plastic torsion of a cylindrical bar (Chapter 3).

To illustrate the type of problems dealt with in the books, the elasto-plastic torsion problem can be formulated as the variational inequality:

Find $u \in K$ such that

$$\int_\Omega \nabla u \cdot \nabla(v-u)\,dx \geqslant C\int_\Omega (v-u)\,dx \;\forall v \in K,$$

$$\text{where } K = \{v \in H_0^1(\Omega) : |\text{grad } v| \leqslant 1 \text{ a.e. in } \Omega\},$$

Ω is the cross section of the bar, and u represents the stress potential. It also has the equivalent formulation:

Find $u \in K$ minimizing over K the functional

$$J(v) = \frac{1}{2}\int_\Omega |\nabla v|^2\,dx - C\int_\Omega v\,dx.$$

Basically, one obtains an approximation scheme by replacing the infinite-dimensional minimization problem by a finite-dimensional one through the use of finite differences or finite elements.

As pointed out in the introduction to Volume 1, this model contains many of the essential difficulties of the general theory (e.g., how should the convex set K be approximated). Hence, by studying this concrete problem in detail, the authors are able to lend a great deal of insight into the special difficulties encountered in the approximation of variational inequalities. The discussion of this problem in Chapter 3 includes the formulation of various approximation schemes, the proof of convergence of the methods, a discussion of algorithms for solving the resulting finite-dimensional optimization problems, and a presentation of numerical computations.

Other applications that can be formulated as stationary variational inequalities appear in Volume 2. In particular, the problem of a nondifferentiable cost functional is studied in the context of problems of temperature control and of diffusion of fluids through semipermeable walls in Chapter 4 and the stationary flow of a Bingham fluid in a cylindrical pipe in Chapter 5. As an illustration, the cost functional for the last problem is similar to the $J(v)$ described previously with the addition of the nondifferentiable term $\int_\Omega |\text{grad } v|\,dx$. Finally, Chapter 6 contains a discussion of some general approximation schemes for variational inequalities of evolution type. Once again the theory is illustrated by several examples and the results of numerical computations are given.

As evidenced by the book of Duvaut and Lions, many problems in mechanics and physics can be formulated in the context of variational inequalities. The work of Glowinski, Lions, and Trémolières should provide a valuable reference for the numerical solution of such problems, especially for the reader already acquainted with finite-difference and finite-element approximations for unconstrained problems.

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5 [9].—A. O. L. ATKIN, *The Number of Subgroups of the Classical Modular Group of Index N,* University of Illinois, Chicago Circle, 1976, 61 sheets of computer output deposited in the UMT file.

Newman's earlier table of this same function $M(N)$ was reviewed in [1]. It had the range $N = 1(1)255$ and was computed to check C. R. Johnson's conjecture (see [1]) which asserted that $M(N)$ is odd iff $N = 2^n - 3$ or $2(2^n - 3)$ for $n = 2, 3, \ldots$. It did check the conjecture in that range and so preprints of [1] were sent to several investigators in this field. There were two responses.

The table deposited here by Atkin goes further: $N = 1(1)1024$. It agrees with Newman's table at $N = 255$ and verifies that $M(N)$ is odd in the continuation only for $N = 506, 509, 1018$ and 1021. Subsequently, Atkin proved the conjecture. In the meantime, a proof was sent by W. Wilson Stothers of the University of Glasgow [2]. It uses results in his dissertation [3]. The whole episode is a nice example of the interplay of table computation, thoughtful examination of tables, conjectures, and new theory.

These large numbers $M(N)$ are printed here in blocks of five decimals and, as is so common in number-theoretic tables, the high-order digits in each block are suppressed if they equal zero. I have been arguing for years, cf. [4], against this easily eliminated inelegancy, but to little avail.

The value of $M(1024)$ here is

$$21655\ {}^*(449)^*\ 38688$$

where $^*(k)^*$ means that k digits are not shown. Newman's asymptotic formula [5, Theorem 4] is

$$M(N) \sim K \exp\left(\frac{N \log N - N}{6} + N^{1/2} + N^{1/3} + \frac{\log N}{2}\right) \tag{1}$$

where $K = (12\pi e^{1/2})^{-1/2}$. For $N = 1024$ the right side of (1) gives 3.3229×10^{458}, an error of 53%. Of course, $\log M(N)$ is much more accurate, then the error is only 0.023%. Perhaps an interested reader may wish to determine the second-order term for (1).

D. S.

1. DANIEL SHANKS, "Review of Newman's Table," UMT 6, *Math. Comp.*, v. 31, 1977, p. 612.

2. W. W. STOTHERS, Letter to M. Newman, Sept. 9, 1976.

3. W. W. STOTHERS, *On Some Discrete Triangle Groups,* Dissertation, Cambridge, 1971.

4. DANIEL SHANKS, "Review of Kortum and McNiel's Table," UMT 30, *Math. Comp.*, v. 16, 1962, pp. 377–379.

5. MORRIS NEWMAN, "Asymptotic formulas related to free products of cyclic groups," *Math. Comp.*, v. 30, 1976, pp. 838–846.