REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the revised indexing system printed in Volume 28, Number 128, October 1974, pages 1191–1194.

19 [2.05].– HELMUTH SPÄTH, *Spline-Algorithmen zur Konstruktion glatter Kurven und Flächen,* Oldenbourg Verlag, Munich, 1973, 134 pp., 24 cm. Price DM 40.

The algorithms and programs in this handy book deal almost exclusively with cubic splines.

A short introduction stresses the maximum "smoothness" of "natural" splines. The next chapter reviews the numerical solution of tridiagonal linear systems by Gauss elimination and of block tridiagonal linear systems by relaxation.

Chapter 3 offers programs for cubic spline interpolation with various (separated) boundary conditions. The next chapter deals with periodic splines and with the interpolation of curves by interpolation of their component functions with respect to some parametrization. Also, various "smoothing methods" are described, all of which turn out to be interpolation methods based on piecewise cubic Hermite interpolation with the slopes at the interpolation points estimated locally. The author's own construction of an area matching cubic spline approximation to histograms follows.

Quintic spline interpolation is the subject of Chapter 5, and block underrelaxation (!) is used for the solution of the appropriately ordered linear system for the determination of the interpolant. The interpolating quintic spline fares badly in the comparison with the cubic spline interpolant as given in five accompanying pictures.

The cubic smoothing spline of (Schoenberg and) Reinsch is introduced in the next chapter without any motivation at all as a cubic spline whose deviation at each data point (and knot) is proportional to the jump in its third derivative across the knot, with proportionality factor at each knot to be determined. Block underrelaxation is used in its construction, and the user is expected to choose the proportionality factors, perhaps interactively.

Chapter 7 gives a very nice summary of the author's efforts to obtain simple interpolating functions (and curves) which are less wiggly than the cubic spline is at times. Starting with Schweikert's spline in tension, the author discusses a general class of methods in which each cubic piece is replaced by a more flexible function, involving an exponential perhaps or a rational function, and offers programs for the construction of such "generalized cubic spline" interpolants. Again, much is left to interactive user efforts and the simple and attractive alternative of sticking to ordinary cubic splines and combatting extraneous inflection points by the suitable addition of knots instead is not mentioned at all.

The last chapter uses tensor products of the various univariate approximation schemes to produce interpolants or approximants to data on a rectangular grid.

An English translation, by W. D. Hoskins and H. W. Sager, with the title *Spline*

Algorithms for Curves and Surfaces has been published in 1974 by Utilitas Mathematica Publ. Inc., Winnipeg, Manitoba (viii + 198pp).

C. D.

20 [2.05].—SAMUEL KARLIN, CHARLES A. MICCHELLI, ALLAN PINKUS & I. J. SCHOENBERG, *Studies in Spline Functions and Approximation Theory,* Academic Press, New York, 1976, xii + 500 pp., 23.5cm. Price $19.50.

This book is a collection of 15 research papers written by the authors individually, and in various coauthorship combinations. The papers are related in content, and by the fact that each of the authors spent some time at the Weizmann Institute of Science, Rehovot, Israel, between September, 1970 and June, 1974. The papers are arranged into four categories, and we briefly describe each of them below.

Part I of the book deals with best approximations, optimal quadrature, and monosplines. The papers are:

(1) *On a class of best non-linear approximation problems and extended monosplines,* by S. Karlin. This paper deals with existence, uniqueness, and characterization of best (nonlinear) approximations to 0 from the class of extended monosplines with free knots. The results are applied to optimal quadrature.

(2) *A global improvement theorem for polynomial monosplines,* by S. Karlin. The main result is that given a polynomial monospline with odd multiple knots and with a maximum set of zeros, there exists another monospline with simple knots and the same set of zeros which has a smaller absolute value at all points.

(3) *Applications of representation theorems to problems of Chebyshev approximation with constraints,* by A. Pinkus. The author develops representation theorems for Chebyshev systems satisfying side constraints, and applies them to constrained best approximation problems.

(4) *Gaussian quadrature formulae with multiple knots,* by S. Karlin and A. Pinkus. It is shown that there is a Gaussian quadrature rule using derivatives to (odd) order $\mu_i - 1$ at τ_i, $i = 1, 2, \ldots, k$, which is exact for a given ECT-system of $n = k + \Sigma_1^k \mu_i$ functions.

(5) *An extremal property of multiple Gaussian nodes,* by S. Karlin and A. Pinkus. If $\{u_i\}_1^{n+1}$ is a Chebyshev system, it is shown that there is a unique $u = u_{n+1} + \Sigma_1^n a_i u_i$ with prescribed n zeros (each of odd multiplicity) which minimizes $\int_a^b u$. The connections with best quadrature are explored.

Part II of the book is devoted to cardinal splines and related matters. The papers are:

(6) *Oscillation matrices and cardinal spline interpolation,* by Ch. Micchelli. Interpolation of an infinite set of periodic data by spline functions with periodic knots is studied in terms of a generalized eigenvalue problem involving an oscillation matrix.

(7) *Cardinal L-splines,* by Ch. Micchelli. Interpolation of (bounded) data given

at the integers by splines which are piecewise in the null-space of a differential operator with constant coefficients is examined.

(8) *On Micchelli's theory of cardinal L-splines,* by I. J. Schoenberg, The author's methods for polynomial cardinal spline interpolation are used to provide a new development of cardinal L-spline interpolation of data of power growth.

(9) *On the remainders and the convergence of cardinal spline interpolation for almost periodic functions,* by I. J. Schoenberg. The convergence of cardinal spline interpolation at the integers to an almost periodic function with frequencies in $[-\pi, \pi]$ is examined with the help of an integral expression for the error.

Part III of the book is titled Interpolation with splines. It contains the following papers:

(10) *Interpolation by splines with mixed boundary conditions,* by S. Karlin and A. Pinkus. The question of when it is possible to find a unique spline interpolating given data and satisfying mixed boundary conditions (involving combinations of the derivatives at both ends) is discussed via determinants.

(11) *Divided differences and other non-linear existence problems at extremal points,* by S. Karlin and A. Pinkus. Given $1 \leqslant k \leqslant n-1$, and appropriately alternating data $\{e_i\}_k^n$, it is shown that there exists a polynomial of degree n and points $\{t_i\}_k^n$ so that $p^{(k)}(t_i) = 0$, $i = k, \ldots, n-1$, and $[t_{i-k}, \ldots, t_i]p = e_i$, $i = k, \ldots, n$.

(12) *Notes on spline function* VI. *Extremum problems of the Landau-type for the differential operators* $D^2 \neq 1$, by I. J. Schoenberg. Best bounds on $\|f'\|_\infty$ given bounds on $\|f'' + \alpha^2 f\|_\infty$ (or $\|f'' - {}^2 f\|_\infty$) and on $\|f\|_\infty$ are derived. Landau's inequality is recovered when $\alpha \longrightarrow 0$.

Part IV of the book deals with generalized Landau and Markov type inequalities and with generalized perfect splines. The papers are:

(13) *Oscillatory perfect splines and related extremal problems,* by S. Karlin. Certain perfect splines generalizing the Zolotareff polynomials are constructed, and their extremal properties discussed.

(14) *Generalized Markov Bernstein type inequalities for spline functions,* by S. Karlin. Using the perfect splines of the previous paper, Markov Bernstein type inequalities are established for certain cardinal L-splines.

(15) *Some one-sided numerical differentiation formulae and applications,* by S. Karlin. Differentiation formulae of the form $f'(0) = \Sigma_1^{n+1} c_i f(z_i) + \int_0^1 M(t) f^{(n)}(t)\, dt$ are examined.

LARRY L. SCHUMAKER

Courtesy of *Siam Review,* v. 20, 1978, p. 406.

Department of Mathematics
The University of Texas at Austin
Austin, Texas 78712

21 [3.10, 6.15].—A. N. TIKHONOV & V. Y. ARSENIN, *Solutions of Ill-Posed Problems,* Halsted Press, New York, 1977, xiii + 258 pp., 22½cm. Price $19.75.

A recent trend in applied mathematics can best be described as a disavowal of the statement: "The only problems which merit investigation are those which are properly posed." The study of improperly-posed problems has grown dramatically in the past 10 years. Unfortunately, as with most emerging research areas, an introduction to the subject is not easily acquired; the literature is spread throughout journal articles except for a few monographs [1], [2], [3], [4]*. However, these books deal almost exclusively with ill-posed problems involving partial differential equations. The resulting gap in the literature has been filled by the recent translation of *Solutions of Ill-Posed Problems* by A. N. Tikhonov and V. Y. Arsenin.

The emphasis of the authors is on the development of methods for obtaining stable approximations to ill-posed problems and, thereby, rendering their numerical treatment possible.

We may cast the problem in an appropriate setting by considering the equation

$$Az = u, \tag{1}$$

where A is a continuous map from a metric space F with metric ρ_F to a metric space U with metric ρ_U. In this context, ill-posed problems arise when we try to invert A; i.e., given $u_T \in U$ find $z_T \in F$ such that

$$Az_T = u_T.$$

Specifically, suppose we have only approximate data u_δ such that $\rho_U(u_\delta, u_T) \leqslant \delta$. We wish to find an approximate solution z_δ corresponding to u_δ in some sense, so that as $\delta \longrightarrow 0$, $z_\delta \longrightarrow z_T$. We cannot in general set $z_\delta = A^{-1}u_\delta$ since

(i) u_δ need not lie in Range of A.

(ii) Even when A^{-1} is well defined it need not be continuous; i.e., for $u_\delta \in$ Range(A) and $z_\delta \in F$ such that $Az_\delta = u_\delta$ $\rho_U(u_\delta, u_T) \longrightarrow 0 \not\Rightarrow \rho_F(z_\delta, z_T) \longrightarrow 0$.

Thus, in our abstract setting, the study of ill-posed problems consists of trying to determine z_δ from u_δ in a stable manner when A^{-1} is not continuous (or even well defined). To stabilize (1) we must utilize additional a priori information about the solution z_T. This information can be of two forms: 1) quantitative information about the solution; or, 2) qualitative information about the solution. For instance, we may have an explicit bound on the size of the solution or we may know that the solution must satisfy certain regularity conditions.

The first part of the book is a development of stabilization methods, the emphasis being on the regularization method which uses information of type 2 to stabilize the problem.

The first chapter briefly surveys other methods of stabilization; in particular, the authors consider the selection method, the quasisolution method, and two techniques

*Contains an excellent bibliography.

of perturbing the equation. The selection method restricts consideration of possible solutions z of (1) to a subset $M \subset F$. If A is 1-1, M is compact, and $u_\delta \in M$, then we have stabilized the problem since $A^{-1}: AM \longrightarrow M$ is continuous.

Problems arise when the measured data $u_\delta \notin D(A^{-1})$. This difficulty motivates the notion of a quasisolution of (1) on M. A quasisolution is simply a $z_\delta \in M$ such that

$$\rho_U(Az_\delta, u_\delta) = \inf_{z \in M} \rho_U(Az, u_\delta).$$

Under certain restrictions on $N = AM$ the quasisolution is unique and depends continuously on the data.

The two perturbation methods discussed are quasireversibility and a technique of Lavrentiev. In both cases the equation is perturbed to a well-posed problem. In the latter (1) is replaced by

$$(A + \alpha I)z = u,$$

and the behavior is examined as $\alpha \longrightarrow 0$.

The quasireversibility method is a technique for dealing with unstable evolution equations such as the backward heat equation. The idea is to perturb the unstable differential operator to a stable operator by adding on a spatial operator depending on ϵ. As in Lavrentiev technique, the behavior is examined as $\epsilon \longrightarrow 0$.

The above methods are severely limited since they presuppose the class of possible solution can be a priori restricted to a compact set M. However, in many applied problems this is impossible. To remove this restrictive assumption the authors introduce the regularization method. Specifically, let u_T denote the true data; and suppose we have measured data u_δ such that $\rho_U(u_\delta, u_T) \leqslant \delta$. We shall introduce regularizing operators to define z_δ's corresponding to u_δ's such that $z_\delta \longrightarrow z_T$ as $u_\delta \longrightarrow u_T$. To be precise, a regularizing operator $R(u, \delta)$ of (1) in a neighborhood of u_T is an operator with the properties

1) $\exists \delta_i > 0$ such that $R(u, \delta)$ is defined $\forall \delta\ 0 \leqslant \delta \leqslant \delta_1$ and $\forall u\, \rho_U(u, u_T) \leqslant \delta$ and

2) $\forall \epsilon > 0\ \exists \delta_0 = \delta_0(\epsilon, u_T) \leqslant \delta_1$ such that

$$\rho_U(u_\delta, u_T) \leqslant \delta \leqslant \delta_0 \Rightarrow \rho_F(z_\delta, z_T) \leqslant \epsilon,$$

where $z_\delta = R(u_\delta, \delta)$.

Regularizing operators, by definition, stabilize (1). Thus, the problem of finding approximate solutions of (1) is reduced to the construction of regularizing operators.

The construction can be accomplished by a variational principle. First, we say $\Omega[z]: F_1 \longrightarrow \overline{\mathbf{R}}^+$, where $\overline{F}_1 = F$ is a stabilizing functional if

a) $z_T \in D(\Omega)$,

b) $\forall d > 0\ \{z \in F_1 \ni \Omega[z] \leqslant d\}$ is compact in F_1.

Thus, for $Q_\delta = \{z \ni \rho_U(Az, u_\delta) \leqslant \delta\}$, if $\widetilde{R}(u_\delta, \delta)$ is defined as a z minimizing $\Omega[z]$ over $Q_\delta \cap F_1$, then $\widetilde{R}$ is a regularizing operator. Under certain conditions this mini-

mization is equivalent to choosing z_δ to minimize the functional

$$M^\alpha[z, u_\delta] = \rho_U^2(Az, u_\delta) + \alpha\Omega[z].$$

(α is called the regularization parameter. It depends on δ and u_δ.)

Having thus developed the regularization method, the remainder of the book details its application to a wide variety of ill-posed problems. Problems discussed include:

1) Singular and ill-conditioned systems of linear algebraic equations;
2) Fredholm integral equations of the first kind (with emphasis on kernels of convolution type);
3) Stable methods (in the space of continuous functions) for summing Fourier series with approximate coefficients in l_2; and
4) Problems in optimal control and mathematical programming.

In each case, the authors construct the regularization operator for the problem in detail. Methods for determining the optimal regularization parameter α under assumptions on the distribution of noise are also discussed.

The book also has an unexpected but particularly welcome feature: an extensive bibliography of the Russian literature. The reference list will be extremely valuable both to the active researcher and the student surveying the ill-posed problems literature.

In conclusion, it should be noted that although the book presents results not commonly included in an applied mathematics education, the techniques are accessible to graduate students and the engineering community. Thus, the book will serve as an excellent reference to anyone whose research leads him into the realm of ill-posed problems.

JOHN B. BELL

Department of Mathematics
Cornell University
Ithaca, New York 14853

1. M. M. LAVRENTIEV, *Some Improperly Posed Problems of Mathematical Physics*, Springer-Verlag, New York, 1967.

2. M. M. LAVRENTIEV, V. G. ROMANOV & V. G. VASILIEV, *Multi-dimensional Inverse Problems for Differential Equations*, Lecture Notes in Math., vol. 167, Springer-Verlag, Berlin, 1967.

3. R. LATTES & J. L. LIONS, *Methods of Quasireversibility: Applications to Partial Differential Equations*, American Elsevier, New York, 1969.

4. L. E. PAYNE, *Improperly Posed Problems in Partial Differential Equations*, Regional Conference Series in Applied Mathematics, No. 22, SIAM, Philadelphia, 1975.

22 [4.00, 12.00].—G. HALL & J. M. WATT, Editors, *Modern Numerical Methods for Ordinary Differential Equations*, Clarendon Press, Oxford, 1976, ix + 336 pp, 24cm. Price $21.50.

This volume is intended as an up-to-date account of theoretical questions and practical questions and methods in the numerical solution of ordinary differential equations. It consists of twenty-one chapters (eight on general initial value problems, six on stiff problems, five on boundary value problems, and two on functional differential equations); these chapters are written by thirteen scholars from England, New Zealand, and Scotland.

The authors and editors succeed admirably in achieving the goal mentioned. Naturally, the theoretical discussion is mainly limited to results and glimpses of proofs, but adequate references are always given. Frequently, the discussion is very elucidating. Hints as to "best algorithms" are often given.

The different chapters are well integrated towards a whole (only occasionally are forward references found), but they can also be read independently by a reader with a modest background.

A good bibliography and a subject index add to the value of this book.

LARS B. WAHLBIN

Department of Mathematics
Cornell University
Ithaca, New York 14853

23 [5.00].—W. E. FITZGIBBON & H. F. WALKER, Editors, *Nonlinear Diffusion,* Pitman Research Notes in Mathematics, Pitman Publishing Ltd., London, 1977. Price £7.50.

These notes constitute the lectures of the participants of the NSF–CBMS Regional Conference on Nonlinear Diffusion held at the University of Houston in June, 1976. The lectures of the principal speaker, D. G. Aronson, are to be published by SIAM in the CBMS Regional Conference Series in Applied Mathematics.

Over the last twenty years, there has been an ever increasing recognition on the part of physical scientists of the inadequacy of classical linear diffusion theory as a tool for predicting experimental observations.

Thus, it has become necessary to include previously neglected (or small) nonlinear terms in the mathematical modeling of many observed phenomena. As is usually the case, the physical problems have again provided mathematicians with a rich variety of research questions.

The present set of notes includes lectures of interest both to mathematicians and to applied scientists. For want of a better way to provide an overview of these notes, we have divided the articles into two rough classes: those of primarily a theoretical nature and those that would be of more interest to applied scientists. Naturally, this classification is based on the reviewer's own personal prejudices and he offers, in advance, his apologies to any of the authors who might feel that their work has been placed in the wrong class. Since this is a loose classification and since people have varying interests, some theoreticians will find the applied articles of interest and conversely, some experimentalists will find the theoretical articles relevant to their needs.

Among those articles of a theoretical nature are the following:

1. D. G. Aronson, The Asymptotic Speed of Propagation of a Simple Epidemic.
2. E. D. Conway and J. A. Smoller, Diffusion and Classical Ecological Interactions: Asymptotics.
3. P. C. Fife, Stationary Patterns for Reaction Diffusion Equations.
4. D. Henry, Gradient Flows Defined by Parabolic Equations.
5. R. M. Miura, A Nonlinear WKB Method and Slowly Modulated Oscillation in Nonlinear Diffusion Processes.
6. P. Nelson, Subcriticality for Submultiplying Steady State Neutron Diffusion.

7. M. E. Schonbek, Some Results on the Fitzhugh-Nagumo Equation.

The articles of a more applied nature, including the numerical aspects of diffusion theory are:

1. J. R. Cannon and R. E. Ewing, Galerkin Procedures for Systems of Parabolic Partial Differential Equations Related to the Transmission of Nerve Impulses.
2. J. W. Evans, Transition Behavior at the Slow and Fast Impulses.
3. N. J. Kopell, Waves, Shocks, and Target Patterns in an Oscillating Chemical Reagent.
4. J. Rinzel, Repetitive Nerve Impulse Propagation: Numerical Results and Methods.
5. A. D. Snider and D. L. Akins, Calculations of Transients for some Nonlinear Diffusion Phenomena.

HOWARD A. LEVINE

Department of Mathematics
College of Arts and Sciences
University of Rhode Island
Kingston, Rhode Island 02881

24 [7.30].—RALPH HELLER, 25D *Table of the First One Hundred Values of* $j_{0,s}$, $J_1(j_{0,s})$, $j_{1,s}$, $J_0(j_{1,s}) = J_0(j'_{0,s+1})$, $j'_{1,s}$, $J_1(j'_{1,s})$, Department of Physics, Worcester Polytechnic Institute, Worcester, Massachusetts, 1976. Ms. of six pages deposited in the UMT file.

This is an attractively arranged, definitive table of the first 100 zeros of the Bessel functions $J_0(x)$, $J_1(x)$ and of their first derivatives, $J'_0(x)$ and $J'_1(x)$, together with the associated turning values of $J_0(x)$ and $J_1(x)$ and the values of $J_1(j_{0,s})$, all to 25 decimal places.

It supersedes, particularly in precision, all previous related tables, such as those of Meissel (as reproduced in Gray, Mathews and MacRobert [1]), the British Association for the Advancement of Science [2], and Gerber [3].

J. W. W.

1. A. GRAY, G. B. MATHEWS & T. M. MACROBERT, *A Treatise on Bessel Functions*, 2nd ed., Macmillan, London, 1922.

2. BRITISH ASSOCIATION FOR THE ADVANCEMENT OF SCIENCE, *Mathematical Tables*, v. 6: *Bessel Functions, Part I*, Cambridge University Press, Cambridge, 1950.

3. H. GERBER, "First one hundred zeros of $J_0(x)$ accurate to 19 significant figures," *Math. Comp.*, v. 18, 1964, pp. 319–322.

25 [10].—JACOB T. B. BEARD, JR. & KAREN I. WEST, *Factorization Tables for Binomials over* GF(q), The University of Texas at Arlington, Arlington, Texas and Mobil Research & Development Corporation, Dallas, Texas, ms. of 42 pp. 8½″ × 14″ + 7pp. 8½″ × 11″, deposited in the UMT file.

The thirteen tables herein give the complete factorization over the Galois field GF(q) of each monic binomial $B(x)$ of degree n, $2 \leqslant n \leqslant d$ as below, such that $x \nmid B(x)$, together with the generalized Euler Φ-function whenever $B(x)$ is not prime and $\Phi[B(x)] < 10^8$. Furthermore, the numerical exponent and the q-polynomial are given for each $B(x)$ whenever $2 \leqslant n \leqslant d_1$. The numerical exponent assigned to a nonprime

binomial in these tables is the multiplicative order of the companion matrix of $B(x)$.

The tables correspond, respectively, to the following sets of values of q, d, and d_1:

$q = 2^2, d = 16, d_1 = 15$	$q = 5, d = 21, d_1 = 11$
$q = 2^3, d = 8$	$q = 5^2, d = 10$
$q = 2^4, d = 6$	$q = 7, d = 10 = d_1$
$q = 2^5, d = 4$	$q = 11, d = 10, d_1 = 8$
$q = 3, d = 26, d_1 = 15$	$q = 13, d = 10$
$q = 3^2, d = 9$	$q = 17, d = 10$
	$q = 19, d = 10.$

The representation for $GF(p^\alpha)$, $\alpha \geqslant 1$, is that discussed in [1] and used previously in [2], [3], and [4]. In the introduction to the present tables the authors prove that a prime binomial of degree $n \geqslant 2$ is not primitive of the first, second, or third kind [1].

J. W. W.

1. J. T. B. BEARD, JR., "Computing in GF(q)," *Math. Comp.*, v. 28, 1974, pp. 1159–1166.

2. J. T. B. BEARD, JR. & K. I. WEST, "Some primitive polynomials of the third kind," *Math. Comp.*, v, 28, 1974, pp. 1166–1167.

3. J. T. B. BEARD, JR. & K. I. WEST, "Factorization tables for $x^n - 1$ over GF(q)," *Math. Comp.*, v. 28, 1974, pp. 1167–1168.

4. J. T. B. BEARD, JR. & K. I. WEST, "Factorization tables for trinomials over GF(q)," *Math. Comp.*, v. 30, 1976, pp. 179–183.

26 [2.05, 2.10, 3.00, 4.00, 5.00, 6.15].—D. A. H. JACOBS, Editor, *The State of the Art in Numerical Analysis,* Academic Press, London, 1977, xix + 978 pp., 23 cm. Price $39.00.

This volume is based on material presented at a conference held at the University of York in the spring of 1976. The topics surveyed are: linear algebra, error analysis, optimization and non-linear systems, ordinary differential equations and quadrature, approximation theory, parabolic and hyperbolic problems, elliptic problems, and integral equations. In all there are twenty-three authors each contributing a section of one of the above-mentioned chapters.

J. B.

27 [2.00].—J. DESCLOUX & J. MARTI, Editors, *Numerical Analysis,* Proceedings of the Colloquium on Numerical Analysis, International Series of Numerical Mathematics, Birkhäuser Verlag, Basel, Switzerland, 1977, 248 pp., 24 cm. Price approximately $22.00.

This volume contains papers presented at a meeting organized by the editors. This meeting took place at Lausanne, Switzerland, October 11–13, 1976.

J. B.

28 [10.35].—DAN ZWILLINGER, *Magic Labellings,* Massachusetts Institute of Technology, Cambridge, Massachusetts, 1977, iii + 81 pages of computer output filed in stiff covers and deposited in the UMT file.

These are not the β-valuations of Rosa [4] (graceful numberings of Golomb [1]) nor the magic configurations of Murty [3]. They are closer to, but not identical with, the magic labellings of Stanley [5], [6], [7].

Page i defines a magic labelling as an assignment of integers to the edges of a tree, so that the sum of the labels on edges incident with a given node is at most ["is equal to" in Stanley] N. The number of such labellings is a polynomial in N. Associated with each polynomial is the generating function

$$g(x) = (1 - x)^{1+\deg[f]} \sum_{J=0}^{\infty} f(J)x^J.$$

Pages ii and iii are unacknowledged copies of [2].

Pages 1–81 list all trees on at most 10 points, with the polynomial and generating function for each. The computations were done by MACSYMA at M.I.T.

RICHARD K. GUY

University of Calgary,
Calgary, Alberta, Canada, T2N 1N4

1. S. W. GOLOMB, "How to number a graph," in R. C. Read, *Graph Theory and Computing*, Academic Press, New York, 1972, pp. 23–37.

2. FRANK HARARY, *Graph Theory*, Addison-Wesley, Reading, Mass., 1969, pp. 233–234.

3. U. S. R. MURTY, "How many magic configurations are there?," *Amer. Math. Monthly*, v. 78, 1971, pp. 1000–1002.

4. A. ROSA, "On certain valuations of the vertices of a graph," *Théorie des Graphes*, Dunod, Paris, 1967, pp. 349–355.

5. RICHARD P. STANLEY, "Ordered structures and partitions," *Mem. Amer. Math. Soc.*, no. 119, 1972.

6. RICHARD P. STANLEY, "Linear homogeneous diophantine equations and magic labellings of graphs," *Duke Math. J.*, v. 40, 1973, pp. 607–632.

7. RICHARD P. STANLEY, "Magic labellings of graphs, symmetric magic squares, systems of parameters and Cohen-Macaulay rings," *Duke Math. J.*, v. 43 1976, pp. 511–531.

29 [9].—ROBERT BAILLIE, *Solutions of $\varphi(n) = \varphi(n + 1)$ for Euler's Function*, University of Illinois, Urbana, Illinois, 1978, eleven computer output sheets deposited in the UMT file.

This is an extension of Baillie's earlier table [1] of the 306 solutions of

$$\varphi(n) = \varphi(n + 1) \tag{1}$$

that have $n \leqslant 10^8$. Here he gives all 85 additional solutions that satisfy $10^8 < n \leqslant 2 \cdot 10^8$. See [1] for more detail. There is now sufficient data here to encourage practitioners of heuristic to attempt a conjecture for the asymptotic number of solutions of (1), having $n \leqslant N$, as $N \longrightarrow \infty$.

No additional example of $\varphi(n) = \varphi(n + 1) = \varphi(n + 2)$ was found. Only one of these 85 solutions has the property that multiplication (mod n) is isomorphic to multiplication (mod $n + 1$) for the $\varphi(n) = \varphi(n + 1)$ residue classes prime to the modulus. This occurs for $n = 184611375$ where both Abelian groups equal $C(2) \times C(2) \times C(60) \times C(378300)$. Such isomorphic multiplication is becoming increasingly rare; frequently, even the 2-ranks of the two groups are unequal. There are only 24 examples for $n \leqslant 2 \cdot 10^8$, (see [1]).

D. S.

1. ROBERT BAILLIE, UMT 6, *Math. Comp.*, v. 30, 1976, pp. 189–190.

30 [10.35].—L. COLLATZ, G. MEINARDUS & W. WETTERLING, Editors, *Numerische Methoden bei Optimierungsaufgaben,* Band 3, *Optimierung bei graphentheoretischen und ganzzahligen Problemen,* International Series of Numerical Mathematics, Birkhäuser Verlag, Basel, Switzerland, 1977, 216 pp., 24 cm. Price approximately $22.00.

This volume contains papers presented at a meeting organized by the editors. This meeting took place at the Mathematical Research Institute at Oberwolfach, Germany, from February 22–28, 1976.

J. B.