

## CORRIGENDA

D. M. GAY, "Modifying singular values: Existence of solutions to systems of non-linear equations having a possibly singular Jacobian matrix," *Math. Comp.*, v. 31, 1977, pp. 962-973.

This note corrects an error pointed out by K. Tanabe [1978]. Theorem (5) of this paper should have been stated as:

(5) THEOREM. *If  $F: \mathbf{R}^n \rightarrow \mathbf{R}^n$  is continuous, then for each  $x \in \mathbf{R}^n$  and  $t_0 \in \mathbf{R}$  there exist  $a \in [-\infty, t_0)$ ,  $b \in (t_0, +\infty]$ , and a continuously differentiable function  $x: (a, b) \rightarrow \mathbf{R}^n$  such that*

$$(6a) \quad x(t_0) = x_0 \quad \text{and}$$

$$(6b) \quad x'(t) = F(x(t)) \quad \text{for all } t \in (a, b).$$

*If  $\|F(x)\| \leq c$  for  $\|x - x_0\| \leq d$ , then  $a < t_0 - d/c$  and  $b > t_0 + d/c$ . Moreover, if  $F$  is locally Lipschitz continuous, then the solution  $x(t)$  of (6) is unique.*

In [Gay, 1977] it was erroneously asserted that  $a = -\infty$  and  $b = +\infty$ . This has no effect on the rest of the paper, except that the proof of Theorem (23) must be revised to show that  $b = +\infty$  for the  $F$  of interest. The revised proof may be stated as follows:

*Proof.* Fix  $x_0$ . As already remarked, the existence of  $x(t)$  on some interval  $[0, b)$  follows easily from Theorems (13) and (5). We first show for  $s, t \in [0, b)$  that

$$(24.1) \quad \|f(x(t))\| \leq \|f(x_0)\| e^{-\theta t} \quad \text{and}$$

$$(24.2) \quad \|x(s) - x(t)\| \leq [\|f(x_0)\|/(\theta\epsilon)] |e^{-\theta s} - e^{-\theta t}|.$$

Indeed, let  $\phi(t) = \|f(x(t))\|^2$ . Then  $\phi'(t) = -2f^T J \hat{J}^+ f$ , so (22) implies  $\phi'(t) \leq -2\theta \|f(x(t))\|^2 = -2\theta\phi(t)$ . Hence,  $\psi(t) \equiv \ln \phi(t)$  has  $\psi'(t) \leq -2\theta$ , so (for  $t \geq 0$ )

$$\psi(t) = \psi(0) + \int_0^t \psi'(\tau) d\tau \leq \psi(0) - 2\theta t$$

and

$$\|f(x(t))\|^2 = \phi(t) = e^{\psi(t)} \leq \|f(x_0)\|^2 e^{-2\theta t},$$

which establishes (24.1). Because of (9a), we have

$$\|x'(t)\| = \|\hat{J}^+ f(x(t))\| \leq \|f(x(t))\|/\epsilon \leq (\|f(x_0)\|/\epsilon) e^{-\theta t},$$

whence

$$\|x(s) - x(t)\| = \left\| \int_s^t x'(\tau) d\tau \right\| \leq \left| \int_s^t \|x'(\tau)\| d\tau \right| \leq \frac{\|f(x_0)\|}{\epsilon} \left| \int_s^t e^{-\theta \tau} d\tau \right|,$$

which gives (24.2).

Now let  $d = \|f(x_0)\|/(\theta\epsilon)$ ,  $c = \max\{\|f(x)\|: x \in \bar{B}(x_0, d)\}/\epsilon$ , and  $b_0 = 0$ . By (9a), (24.2), Theorem (5), and induction on  $k$  we find:

$$b > b_k,$$

$$\|x(b_k) - x_0\| \leq [1 - \exp(-\theta b_k)]d,$$

$$\|\hat{V}^+ f(x)\| \leq c \quad \text{for } x \in \bar{B}(x(b_k), \exp(-\theta b_k)d),$$

$$b > b_{k+1} \equiv b_k + \exp(-\theta b_k)d/c = \frac{d}{c}(1 + e^{-\theta b_1} + e^{-\theta b_2} + \dots + e^{-\theta b_k}).$$

From this it follows that  $b = +\infty$ , for if  $b$  were finite, then we would have  $b > d(1 + ke^{-\theta b})/c$  for all  $k$ , which is impossible.

From (24.2) it follows that the sequence  $x(t_1), x(t_2), x(t_3), \dots$  is a Cauchy sequence for any choice of  $t_1, t_2, \dots$  with  $\lim_{t \rightarrow \infty} t_i = +\infty$ , whence  $x^* = \lim_{t \rightarrow \infty} x(t)$  exists. By the continuity of  $f$  and (24.1),  $f(x^*) = \lim_{t \rightarrow \infty} f(x(t)) = 0$ .  $\square$

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K. TANABE, (1978), "Global analysis of continuous analogues of the Levenberg-Marquardt and Newton-Raphson methods for solving nonlinear equations." (Preprint.)

I. S. GRADSHTEYN & I. M. RYZHIK, *Table of Integrals, Series, and Products*, 4th ed., Academic Press, New York, 1965.

On p. 906 of MTE 428 (*Math. Comp.*, v. 22, 1968, pp. 903–907) listing corrections in this set of tables there appears a typographical error in the correction of Formula 8.521(4). The emended correction should read

$$-\frac{1}{\sqrt{(2k\pi - z)^2 + x^2 + y^2}}.$$

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REIJO ERNVALL & TAUNO METSÄNKYLÄ, "Cyclotomic invariants and  $E$ -irregular primes," *Math. Comp.*, v. 32, 1978, pp. 617–629.

On p. 619, the three first lines of the first table should read as follows:

$x$	$\pi_B$	$\pi_E$	$\pi_{BE}$	$\pi_B/\pi$	$\pi_E/\pi$	$\pi_{BE}/\pi$
2000	121	113	56	0.399	0.373	0.18
4000	218	213	91	0.396	0.387	0.17

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