A Goldbach Conjecture Using Twin Primes

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Abstract. The numbers $2N = 2(2)1000000$ are checked to determine if they can be written as the sum of two twin primes. Thirty-three numbers are found that cannot be so represented; they are all less than 5000. The largest number in the range $2N = 2(2)500000$ that can be written as the sum of two twin primes in only one way is $2N = 24098$.

A natural extension of the Goldbach conjecture is to use only a restricted set of primes instead of all the primes. The primes used could be of a special form, or have special properties. This note describes the case where the allowed primes are twin primes ($3, 5, 7, 11, 13, 17, \ldots$).

Define $H(N)$ to be the number of decompositions of $N$ into two twin primes. If a Goldbach type conjecture were to be true about twin primes, then the $H(N)$ function would have no zeros. Unfortunately, in the range $N = 2(2)500000$, $H(N)$ is equal to zero for the following values of $N$:

\begin{align*}
94 & 96 & 98 & 400 & 402 & 404 \\
514 & 516 & 518 & 784 & 786 & 788 \\
904 & 906 & 908 & 1114 & 1116 & 1118 \\
1144 & 1146 & 1148 & 1264 & 1266 & 1268 \\
1354 & 1356 & 1358 & 3244 & 3246 & 3248 \\
4204 & 4206 & 4208 & & & 
\end{align*}

A further computation found no additional zeros of $H(N)$ for $N$ in the range $500000(2)1000000$. It is easy to show that if $H(6N) = 0$ then $H(6N - 2) = H(6N + 2) = 0$. This explains, somewhat, why the zeros of $H(N)$ come in threes.

Some interesting numbers concerning the $H(N)$ function: the smallest $N$ for which $H(N) = 1000$ is $N = 30240$, the largest $N$ such that $H(N) = 1$ is $N = 24098$.

This work was carried out on CCNY's computer system in early 1974.

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Received February 23, 1978.


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0025-5718/79/0000-0115/$01.25

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