REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the American Mathematical Society classification scheme. The 1980 Mathematics Subject Classification (1985 Revision) can be found in the December index volumes of Mathematical Reviews.

7[65–01].—CARL-ERIK FRÖBERG, *Numerical Mathematics—Theory and Computer Applications*, The Benjamin/Cummings Publ. Co., Menlo Park, Calif., 1985, xi + 436 pp., 24 cm. Price $37.95.

Originally published in Swedish, in 1962, two subsequent editions were published (in English) by Addison-Wesley under the title "Introduction to Numerical Analysis."

The new title and publisher are the result of a complete revision and restructuring in line with modern teaching trends. The current book covers a very wide range of topics in five distinct parts; Part 1 (Mathematical Introduction, 9 Chapters) Numerical Computation, Vectors and Matrices, Series Expansion, Orthogonal Functions, Linear Operators, Difference Equations, Special Functions, Laplace Transforms, Calculus of Variations; Part 2 (Equations, 4 Chapters) Systems of Linear Equations, Nonlinear Equations and Systems of Equations, Algebraic Eigenvalue Problems, Linear Programming; Part 3 (Approximation, 3 Chapters) Interpolation, Function Representation and Curve Fitting, The Monte Carlo Method; Part 4 (Integration and Summation, 2 Chapters) Numerical Integration, Summation; Part 5 (Differential and Integral Equations, 3 Chapters) Ordinary Differential Equations, Partial Differential Equations, Integral Equations.

Years of opportunity to test and refine this material have led to a presentation which is concise and clear as well as being mercifully free from typographic errors. There are many exercises which seem well thought out and at about the correct level for the text. Most of the sections also have at least one carefully worked out example which can be followed with pencil or pocket calculator. I found reading this text to be a distinctly pleasurable experience. Areas which I had lost track of were nicely refreshed, and in fields near and dear to me the coverage was classical, correct and showed experience in trying to get concepts across. The one single word to describe this book is "smooth." In particular, I liked the introductory chapters and think they will be excellent references for students.

Nevertheless, I must raise the question of who is the intended audience. It is unlikely to be computer science students, whose background and interest in mathematics is generally far too weak. Engineering and physical science students have the correct training. But because the author has elected to cover so many diverse areas, he must be very brief or ignore important subtopics; 2 pages each for Gaussian elimination, splines, overdetermined linear systems, nonlinear minimization, 1 for FFT, and less than 30 for partial differential equations (including parabolic, elliptic

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and eigenvalue problems). An earlier reviewer's objection to the lack of discussion of pivoting for linear systems has been corrected by the addition of one paragraph, but that chapter is obviously slanted toward large problems, with more than half the material on iterative and gradient methods, including selection of the optimum SOR parameter. The text reads so smoothly that I fear many students will either miss significant details or will find it very tough going.

Further, all issues of software are omitted. The author explains that "With the availability of large and modern program packages, there is hardly need for such programs in a modern textbook. In fact, this view is shared by many reviewers, and it seems safe to assume that such programs will be run only occasionally. Further, programs that are really good should soon become part of a suitable package." This reader agrees with these statements, but feels that it is an incorrect interpretation to ignore all software issues on the assumption that the students will find out about these on their own somehow. My experience has been that engineering and science curricula emphasize computing more each year, but that these students, and many of their faculty too, are woefully ignorant of the many well-established software packages. It seems sad to teach, for example, about Gaussian elimination and not mention LINPACK, about ODEs without any mention of excellent programs which are available from Hindmarsh or Shampine, about integrals without any mention of adaptive quadrature, or about eigenvalues without mentioning EISPACK.

I believe that the most suitable audience for this book will be professional scientists who would like an extremely well-written introduction or review of the mathematical-numerical techniques which were developed through the mid 1970's. As a course text, the book would benefit from an instructor with real computing experience and insight, who was also willing to expand on some of the overly brief presentations.

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8[65F05, 65F10, 65F50, 65N05, 65N10, 65N15, 65N30, 65N35].—GARRETT BIRKHOFF & ROBERT E. LYNCH, *Numerical Solution of Elliptic Problems*, SIAM Studies in Applied Mathematics, SIAM, Philadelphia, Pa., 1984, xi + 319 pp., $23\frac{1}{2}$ cm. Price $31.50.

As stated by the authors, "The aim of this monograph is twofold: first to describe a variety of powerful numerical techniques for computing approximate solutions of elliptic boundary value problems and eigenproblems on high speed computers, and second, to explain the reasons why these techniques are effective." An attempt is made "to provide a reasonably well-rounded and up-to-date survey of these methods." The authors succeed in striking a delicate balance between exposition of underlying theory and its application to numerical solution techniques, thereby providing an impressive addition to the literature, which could well become a classic reference on this subject. In a work of this breadth, the depth of treatment is limited. Nevertheless, the crucial theorems are given with reasonable outlines of basic arguments in their proofs and extensive reference to appropriate literature. Current

research is centered on vector and parallel computation, a field which is still in its infancy. The practitioner would do well to use this monograph as a springboard from which to launch on new methods for the incredible variety of emerging parallel and vector architectures. A summary of the nine chapters follows.

Chapter 1. "Typical Elliptic Problems," in which a description of a variety of physical problems which lead to elliptic systems motivates this treatise.

Chapter 2. "Classical Analysis," wherein a concise overview of the most essential and commonly used classical results are given which serve as a guide to formulation and solution of discrete approximations.

Chapter 3. "Difference Approximations" is a thorough treatment of several differencing techniques with analysis of approximation error and interrelationship of associated properties with solution techniques.

Chapter 4. "Direct and Iterative Methods" and

Chapter 5. "Accelerating Convergence" contain in-depth reviews of many of the most prevalent numerical techniques for solving large elliptic systems. Relative advantages of direct and iterative methods as a function of type of problem are discussed along with methods which utilize a combination of both approaches.

Chapter 6. "Direct Variational Methods" and

Chapter 7. "Finite Element Approximations" deal with variational principles characterizing solution of boundary-value problems, application via patchwork finite element approximation, and error estimation with the aid of classical polynomial approximation theory.

Chapter 8. "Integral Equation Methods," in which there is a concise description of Green's functions, boundary element methods, conformal mapping, capacitance matrix methods and other techniques for solving elliptic equations. Methods discussed here are "quasi-analytic" in that numerical approximations are made in conjunction with extensive analytic reduction.

Chapter 9. "ELLPACK" describes some of the capabilities of the Purdue ELLPACK software package for solving elliptic problems.

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9[65N99, 68-04].—JOHN R. RICE & RONALD F. BOISVERT, *Solving Elliptic Problems Using ELLPACK*, Springer-Verlag, New York, 1985, x + 497 pp., 24 cm. Price $46.50.

While I suspect most readers interested in this book will have some knowledge of the ELLPACK project and its origins, it seems advisable to sketch them because to some such background is necessary to understand this software. The solution of elliptic boundary value problems requires a sequence of operations, i.e., discretization of the domain, discretization of the equations and their boundary conditions, solution of the resulting linear equations, preparation of output. From the early 1970's, researchers in mathematical software have produced many high-quality program packages for individual steps in this sequence. One of the major motivations of the ELLPACK project was to standardize the large-scale testing of such

program 'parts' of the PDE solving process, e.g., so that three linear equation solvers could be easily compared using the same discretization, output stages. The problems confronting the designers of ELLPACK then were not ones of how to design software for PDE's, but how to design a testing framework into which modules could be plugged that were prepared and contributed by others. The result has been a successful software system, now distributed by IMSL Ltd., with capabilities:

(i) for use as a test bed;

(ii) for instructional use (as has been done by the reviewer);

(iii) for 'production' solution of PDE problems (although to the reviewer's knowledge, it has had limited acceptance for this purpose).

The ELLPACK system contains a large library of state-of-the-art mathematical software contributed from many sources to the project. It is used by preparing a 'program' in the ELLPACK language which is then translated by the ELLPACK processor into FORTRAN code with references to routines in the ELLPACK library. One of the features that distinguishes ELLPACK from a software system for solving PDE's is the flexibility to evolve that is built into its design. Software maintenance tools are provided for modifying the preprocessor so that new modules can be added to the library, or even modification of the grammar of the preprocessor can be made.

The book is an aggregate of documentation and literature for the project that has appeared previously in various levels of formality. To have this material edited into one volume provides a very useful reference for the ELLPACK user. The first roughly two thirds of the book deals with the use of ELLPACK to solve boundary value problems and the last third documents it as a language translator. The first two thirds provide an excellent tutorial introduction which moves from elementary to sophisticated uses, a description of the software parts in the library (i.e., a brief, standard format summary of each), and a description of the performance of these parts on a test suite of problems provided in an appendix. The latter third of the book describes how a 'user' can extend ELLPACK by adding code to the existing library or by modifying the ELLPACK language through generating a new preprocessor using a preprocessor generator distributed as part of ELLPACK. The ultimate test of software documentation is, of course, the experiences of a reader using it to accomplish some goals with the software. The reviewer has had happy experiences with the documentation in the first two thirds of the book, but has never tried modifying the ELLPACK as per the last third. Nevertheless, he found the material interesting and confidence-inspiring, although one would have to be a real enthusiast to try to regenerate a new ELLPACK preprocessor.

Two comments: The organization of the book leaves something to be desired in places, particularly in the systems documentation; be prepared for some page flipping and rereading due to repetition and inadequate cross-referencing. Secondly, the book requires a background in numerical methods and software for elliptic boundary value problems for the user documentation and a background in applications software systems and language translation for the systems documentation.

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10[65-01].—DOUGLAS QUINNEY, *An Introduction to the Numerical Solution of Differential Equations*, Research Studies Press Ltd., Wiley, New York, 1985, xi + 283 pp., $23\frac{1}{2}$cm. Price $34.95.

This book is an attempt to introduce the theory and techniques of numerical solution of differential equations to the reader who is not mathematically accomplished. Instead of analysis, the author often uses computed examples for simple model problems to exhibit the phenomena he discusses. This lends concreteness to the presentation, which should be reassuring to the scientist who needs to solve practical problems.

The exposition begins with a chapter on background material relating to recurrence relations and iterative methods, including techniques for extrapolation and acceleration of convergence. The following chapter, which is also the longest one, concerns numerical methods for initial value problems for ordinary differential equations, and covers general single-step methods, such as Runge-Kutta type methods, and linear multi-step methods. The basic concepts of consistency, convergence, and a variety of different stability concepts are defined and illustrated by examples. The subsequent chapter on two-point boundary value problems describes shooting methods and finite difference methods and ends with a short introduction to finite element type techniques, based on the collocation, Galerkin and Rayleigh-Ritz methods.

The remaining chapters deal with partial differential equations. First comes a chapter on parabolic equations, which starts with a discussion of the forward Euler method for the standard heat equation in one space dimension and a statement of the Lax equivalence theorem. Examples are given to illustrate von Neumann stability analysis, explicit and implicit schemes and alternating direction methods. The next chapter deals with the method of characteristics and the most common finite difference schemes for hyperbolic equations, particularly first-order equations and the standard second-order wave equation in one space dimension. The final chapter on elliptic equations begins with the five-point difference method for Laplace's equation in the unit square, with Dirichlet boundary conditions, and proceeds to discuss modifications required near curved boundaries and the case of Neumann boundary conditions. The closing few pages are devoted to the finite element method.

A first appendix exhibits the eigenvalues and eigenvectors of a tridiagonal symmetric Toeplitz matrix and proves Gerschgorin's theorem, and a second appendix discusses the classification of second-order differential equations into elliptic, hyperbolic, and parabolic equations.

The main emphasis is thus on ordinary differential equations, and here a readable account is provided of the various phenomena and dangers confronting the user. Conspicuously missing is a serious discussion of step-size control. The section on partial differential equations is more superficial and is essentially restricted to finite difference methods for the simplest model problems. Less than ten pages are afforded the finite element method, and here one can find the somewhat surprising statement that this method suffers from "the lack of a simple error estimate".

The author has avoided mathematics to a degree which could sometimes result in misunderstandings. For example, the Lax equivalence theorem is stated without a proper presentation of the framework within which it is valid and is then somewhat

too freely applied. In variable coefficient and nonlinear cases it is sometimes unclear what are established results and what are conjectures. In the elliptic chapter the author states incorrectly (p. 243) that the formally $O(h)$ Shortley-Weller approximation in the case of a curved boundary detracts from the overall $O(h^2)$ approximation of the five-point approximation. He further hints (p. 235) that higher-order approximation should be used in the case of nonsmooth solutions, although, in general, such methods require at least as much regularity to be competitive.

In conclusion, the reviewer feels that the book could be a useful introduction for the applied scientist with a weak mathematical background. It provides easy reading at the expense of generality, depth and precision. In its emphasis on finite differences, however, it does not properly account for the advances in computational techniques of the last few decades.

V. T.

11[65–01].—G. D. SMITH, *Numerical Solution of Partial Differential Equations, Finite Difference Methods*, 3rd ed., Clarendon Press, Oxford, 1985, xi + 337 pp., 22 cm. Price $19.95.

This book is the third, somewhat modified, edition of a text which first appeared in 1965. It presents the finite difference method in a manner which was standard at that time, with emphasis on formulation of finite difference equations, often motivated by manipulations with Taylor series. It describes the most basic explicit and implicit methods for model parabolic and hyperbolic equations in one space dimension and the usual five-point method for Poisson's equation, together with some common devices for increasing the accuracy. Special attention is paid to time discretization by means of Padé type schemes of equations which are already discretized in the space variable, and to the concept of stiff equations. Stability analysis is carried out for simple model problems in the time-dependent case, using von Neumann's approach, and Gerschgorin's matrix theorem plays a central role in the discussion of the matrix equations. In the elliptic part, the Jacobi, Gauss-Seidel, and SOR iterative procedures for solving the system of difference equations are also considered.

Although some additions have been made in the new edition, the exposition has been only slightly affected by the developments of the last three decades. Thus, the increasing impact of the theory of partial differential equations on the formulation and analysis of numerical methods is essentially ignored or inadequately represented. For instance, application of energy arguments and a discussion of the effect on the error of the regularity of the solution are missing. In the new section on stability for initial value problems the author is somewhat influenced by the Lax-Richtmyer theory, but does not present or apply it properly. Trying to illustrate the Lax equivalence theorem, he describes (p. 72) the existence part of the condition for correctness as: "A solution always exists for initial data that is arbitrarily close to initial data for which no solution exists." In the elliptic part he says (p. 248, unchanged from the first edition): "Although no useful general results concerning the magnitude of the discretization error as a function of the mesh lengths have yet been established, it seems reasonable to assume that this error will usually decrease as the mesh lengths are reduced." In spite of this statement he shows (in the new

edition) a basic error estimate for the five-point operator, which he describes as extremely useful. Referring to this estimate, he then comments incorrectly (p. 254): "It also proves that the discretization error is proportional to h^2 so Richardson's 'deferred approach to the limit' method can be used effectively to improve the accuracy of the solution of the difference equations." He remarks (p. 255), somewhat misleadingly, that the requirement of the fourth derivatives of the solution to be bounded "... is not satisfied if, for example, the boundary contains corners with internal angles in excess of 180°."

The list of references for supplementary reading also reveals the author's lack of familiarity with recent developments.

In spite of the dominance today of the finite element method, particularly as concerns elliptic and parabolic problems, the reviewer feels that the finite difference method is still of sufficient importance to justify the publication of a textbook. Unfortunately, however, the present one must be considered severely outmoded.

V. T.

12[65N99].—CARLOS A. BREBBIA (Editor), *Topics in Boundary Element Research*, Volume I: *Basic Principles and Applications*, Springer-Verlag, New York, Heidelberg, 1984, xiii + 253 pp., $24\frac{1}{2}$ cm. Price $49.50.

This is the first volume in yet another series of publications on the boundary element method edited by Carlos Brebbia. In the Introduction to the Series, it is stated that the series was launched to satisfy an unfilled need, namely that which "exists for a serial publication in which the most recent advances in the method are documented in a more complete form than is usually the case in papers presented at conferences or scientific gatherings". Whether such a need actually existed is debatable in view of the fact that another series "Developments in Boundary Element Methods" edited by P. K. Banerjee et al., with somewhat similar goals, had been started, the first two volumes [1], [2] having been published in 1979 and 1982. Since then two more volumes have appeared [3], [4]. Several of the contributors to the present volume have also contributed to Banerjee's series.

In this volume, there are eleven chapters (three of which are co-authored by the editor) covering such topics as time-dependent problems, fluid mechanics, hydraulics, geomechanics, and plate bending as well as mathematical aspects of the boundary element method. Three chapters are devoted to the latter, the first of which is Chapter 0, entitled "Boundary Integral Formulations" written by the editor and J. J. Connor. This chapter is aptly numbered because there is very little in it of relevance to boundary element methods. Material which is relevant is presented in a more understandable fashion by other authors in later chapters. Much of the discussion in Chapter 0 is devoted to one-dimensional problems, and any mention of boundary integral formulations is through the weighted residual scheme, an approach which does nothing to simplify the derivation of integral equation formulations of boundary value problems, but much to confuse it. Moreover, the weighted residual scheme is an *approximate* method whereas the standard derivation, in which the fundamental solution of the equation in question is combined with a reciprocal theorem, is simply a reformulation of the problem, not an approximation to it. There seems to be nothing that can be accomplished using the weighted residual scheme

that cannot be done in a more rigorous and straightforward manner using the standard approach. This chapter has only 10 references, all to publications (books and conference proceedings) by Brebbia.

Chapter 1, "A Review of the Theory" by M. A. Jaswon, is well-written, but it is rather mathematical in its approach and the book's intended audience of scientists and engineers will find it hard going. The third chapter devoted to mathematical aspects of the method is Chapter 10, "Trefftz Method" by I. Herrera. This chapter is exceedingly theoretical, and, in the reviewer's opinion, totally inappropriate for a publication of this nature, despite the editor's claim in the Preface that it "is written in a way the engineer can easily interpret". It appears to have little or no connection with the theory and application of boundary element methods described in the other chapters.

The remaining eight chapters are devoted to applications of the boundary element method. Transient problems in two space variables are discussed in Chapter 2, "Applications in Transient Heat Conduction" by H. L. G. Pina and J. L. M. Fernandes, and Chapter 3, "Fracture Mechanics Application in Thermoelastic States". In Chapter 2, the standard boundary integral formulation of the heat equation is derived in a rather confusing and long-winded fashion. Boundary element methods are described and two time-marching procedures are introduced and applied to a few test problems. Despite its title, much of Chapter 3 duplicates parts of Chapter 2. The notation is slightly different and, in contrast to Chapter 2, the correct fundamental solution is used, that of Chapter 2 having no fewer than two errors. When the discussion turns to thermoelastic problems it is very difficult to follow because some notation is not defined. Moreover, the reader unfamiliar with fracture mechanics will not be enlightened by the material presented in this chapter. Only 8 references are listed; one reference is omitted from the list and one appears in the list but not in the text.

Two chapters, Chapter 4, "Applications of Boundary Element Methods to Fluid Mechanics" by J. A. Liggett and P. L.-F. Liu, and Chapter 8, "Applications in Mining" by G. Beer and J. L. Meek, contain excellent reviews of their respective areas. In Chapter 4, the authors describe interesting applications of the method to porous media flow, free boundary and water wave problems, in addition to presenting a short history of the development and use of boundary methods in fluid mechanics. This chapter has a comprehensive list of 65 references. In Chapter 8, the authors give a very good description of the complexity of the problems arising in mining and discuss different formulations used in their analysis. Both direct and indirect boundary element methods are described, as well as a formulation known as the displacement discontinuity method which is of use when an excavation in a mine is considered to have negligible thickness. The combination of the boundary element method and the finite element method is discussed and several interesting examples of its use to solve some typical nonlinear mining problems are presented. This chapter has 69 references.

In Chapter 5, "Water Waves Analysis" by M. C. Au and the editor, the application of boundary element methods in the study of water waves diffraction and radiation problems involving fixed, free-floating or moored offshore structures is described. The main point made by the authors is that the boundary integral equation formulation of an exterior problem for Laplace's equation with a radiation

type boundary condition at infinity can be simplified by imposing this boundary condition on a finite boundary sufficiently far from the structure. This idea is not new but was discussed and improved upon by Harten et al., in the mid-seventies (see [6] and references cited therein). What is not obvious from the Preface, and the chapter's introduction and conclusions is that problems which involve the Helmholtz equation are also discussed in this chapter. Moreover, no mention is made of the important "fictitious eigen-frequency problem" which can arise in the solution of exterior boundary value problems involving the Helmholtz equation. A number of numerical examples are provided, some of which involve the solution of the Helmholtz equation and the remainder Laplace's equation.

Chapter 6, "Interelement Continuity in the Boundary Element Method" by C. Patterson and M. A. Sheikh, is not only very poorly written but almost unintelligible in parts. The description of the boundary element method is the worst that this reviewer has seen in print and ought not to have been published.

Chapter 7, "Applications in Geomechanics" by W. S. Venturini and the editor, deals with the solution of geomechanical problems modelling rock and soil behavior using the boundary element method. Several examples are presented to illustrate the efficacy of the boundary element formulation and, whenever possible, results obtained using this method are compared with known theoretical solutions or solutions obtained using the finite element method. This chapter seems to draw heavily on material already published by the editor in publications which he has edited.

In Chapter 9, "Finite Deflection of Plates" by N. Kamiya and Y. Sawaki, the application of the boundary element method to solve finite deflection of elastic plates, shallow shells, and sandwich plates and shells is presented. Integral equation formulations based on the von Karman theory are described, as well as the so-called Berger approximation. Some numerical examples in which the Berger method is used are presented, and it is noted that this method gives "fairly good results" under certain specified limitations.

The volume is rather lavishly produced, being typeset and printed on thick glossy paper, and consequently it is exceedingly expensive. However, it contains more than its fair share of typographical errors, inaccuracies, and grammatical mistakes. Moreover, in contrast to the volumes edited by Banerjee et al., it is very carelessly compiled, and it is clear that little editing was done. The chapters have no common thread and no attempt has been made to relate them. Most of the chapters, including those co-authored by the editor, begin as if the boundary element method were never mentioned elsewhere in the book. Few of the key references are to papers published in refereed journals; most are to works appearing in conference proceedings, usually edited by Brebbia.

The series by Banerjee et al. [1]–[4] is to be recommended over this volume, as is the book by Banerjee and Butterfield [5]. This volume contributes little to the literature on the boundary element method that is not presented better elsewhere. If the editor is not more discriminating and more selective in the preparation of subsequent volumes, *caveat emptor*.

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1. *Developments in Boundary Element Methods*, Vol. 1, Edited by P. K. Banerjee and R. Butterfield, Elsevier, London, 1979.

2. *Developments in Boundary Element Methods*, Vol. 2, Edited by P. K. Banerjee and R. P. Shaw, Elsevier, London, 1982.

3. *Developments in Boundary Element Methods*, Vol. 3, Edited by P. K. Banerjee and S. Mukherjee, Elsevier, London, 1984.

4. *Developments in Boundary Element Methods*, Vol. 4, Edited by P. K. Banerjee and J. O. Watson, Elsevier, London, 1986.

5. P. K. BANERJEE & R. BUTTERFIELD, *Boundary Element Methods in Engineering Science*, McGraw-Hill, London, 1981.

6. A. HARTEN & S. EFRONY, "A partition technique for the solution of potential flow problems by integral equation methods," *J. Comput. Phys.*, v. 27, 1978, pp. 71–87.

13[76–02, 76–08].—EARLL M. MURMAN & SAUL S. ABARBANEL (Editors), *Progress and Supercomputing in Computational Fluid Dynamics*, Progress in Scientific Computing, Vol. 6, Birkhäuser, Boston, 1985, ix + 403 pp., 23 cm. Price $44.95.

This volume constitutes the proceedings of the U.S.–Israel Workshop entitled "The Impact of Supercomputers on the Next Decade of Computational Fluid Dynamics" held in Jerusalem, Israel, during the week of December 16, 1984. From the editors' preface the intent was to "... present to the community a sort of 'State of the CFD-Nation' report consisting of two elements: technical papers by leading researchers and an attempt to assess where the field is going".

Taken as a whole, the papers provide an excellent overview of the present status of computational fluid dynamics with particular emphasis on problems relevant to aerodynamics. The Euler and compressible and incompressible Navier-Stokes equations are considered. A variety of numerical algorithms and their vectorization properties are discussed, including explicit and implicit methods, multigrid and spectral methods. Finally, the important topics of prediction of transition and turbulence are presented. Reflecting the present emphasis in the community, most of the papers utilize finite difference approximations rather than finite elements.

One paper surveys the present status of supercomputer hardware and projects capabilities into the immediate future. Unfortunately, there is not a companion paper to speculate on the kind of software systems that the new supercomputers will require in order to be effective for computational fluid dynamics, but many authors did comment from their own perspective on what they considered necessary and desirable. In addition, these issues were discussed during panel sessions, and a summary of those discussions is given in the introductory paper by the editors.

With the expected explosive growth in raw computational power, the participants look forward to an exciting decade of discovery in computational fluid dynamics. However, the following statement from the editors' introduction indicates there is a clear understanding that such power alone is not sufficient: "It is important to understand that the powerful new supercomputers will only yield useful results if the mathematical and numerical analysis formulation is carefully done."

There follows a list of papers and authors included in the volume:

The Impact of Supercomputers on the Next Decade of Computational Fluid Dynamics
Earll M. Murman and Saul S. Abarbanel

Current Status of Supercomputers and What is Yet to Come
Sidney Fernbach

Experience with a Personal Size Supercomputer—Implications for Algorithm Development
W. T. Thompkins

Remarks on the Development of a Multiblock Three-Dimensional Euler Code for Out of Core and Multiprocessor Calculations
Antony Jameson, Stefan Leicher and Jef Dawson

Developments in the Simulation of Compressible Inviscid and Viscous Flow on Supercomputers
Joseph L. Steger and Peter G. Buning

High Resolution Solutions of the Euler Equations for Vortex Flows
Earll M. Murman, Arthur Rizzi and Kenneth G. Powell

An Efficient Iteration Strategy for the Solution of the Euler Equations
Robert W. Walters and Douglas L. Dwoyer

Numerical Methods for the Navier-Stokes Equations
Robert W. McCormack

Algorithms for the Euler and Navier-Stokes Equations for Supercomputing
Eli Turkel

Viscous Flow Simulation by Finite Element Methods and Related Numerical Techniques
R. Glowinski

Marching Iterative Methods for the Parabolized and Thin Layer Navier-Stokes Equations
Moshe Israeli

Multigrid Solutions to Quasi-Elliptic Schemes
Achi Brandt and Shlomo Ta'asan

Secondary Instability of Free Shear Flows
Marc E. Brachet, Ralph W. Metcalfe, Steven A. Orszag and James J. Riley

Turbulent Flow Simulation—Future Needs
Joel H. Ferziger

Numerical Calculation of the Reynolds Stress and Turbulent Heat Fluxes
Micha Wolfshtein

Numerical Investigation of Analyticity Properties of Hydrodynamic Equations using Spectral Methods
P. L. Sulem

Order and Disorder in the Kupamoto-Sivashinsky Equation
Daniel Michelson

Information Content in Spectral Calculations
Saul Abarbanel and David Gottlieb

Recovering Pointwise Values of Discontinuous Data within Spectral Accuracy
David Gottlieb and Eitan Tadmor

Numerical Problems Connected with Weather Prediction
C. Browning and Heinz-Otto Kreiss

Order of Dissipation Near Rarefaction Centers
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14[34–02, 34C15, 34C25, 34C29, 34E05].—E. A. GREBENIKOV & YU. A. RYABOV, *Constructive Methods in the Analysis of Nonlinear Systems* (Translated from the Russian by Ram S. Wadhwa), "Mir", Moscow, 1983, 328 pp., 22 cm. Price $9.95.

These authors represent a very strong Soviet specialty, asymptotic analysis of nonlinear oscillatory systems. They have published extensively in Russian, especially concerning the detailed development of asymptotic methods for problems of celestial mechanics. This translation of a 1979 book makes no attempt to relate its material to Western research or textbooks. This is a pity, because minor changes in terminology and references to standard English presentations could increase its usefulness substantially. The writing is quite satisfactory, though neither colloquial nor easy-reading mathematics. C. L. Siegel is a victim of double translation, having become K. L. Zigel. There is, indeed, much to be learned, by amateurs and experts, from reading this monograph. *Mathematics of Computation* readers will, however, find it quite analytical (rather than computational).

The first half of this book deals with the method of averaging. This method dates back to Lagrange and Laplace, but was largely developed and applied by Soviet mathematicians beginning about fifty years ago. Grebenikov and Ryabov first emphasize that coordinate transformations are basic to this Krylov-Bogoliubov theory for dealing with "multifrequency" systems of ordinary differential equations with slow and fast variables. The simplest averaging possibility relates to approximating solutions to

$$\frac{dz}{dt} = \mu Z(z, t, \mu),$$

for μ small, by solutions to

$$\frac{d\bar{z}}{dt} = \mu \lim_{T\to\infty} \frac{1}{T}\int_0^T Z(\bar{z}, t, 0)\, dt.$$

Such averaging theories and many generalizations to higher-order methods are presented in detail. For complete proofs, however, the reader must refer to the Russian literature. Resonance enters, once one considers systems of van der Pol equations

$$\frac{d^2x_k}{dt^2} + \omega_k^2 x_k = \mu f_k(x_1, \ldots, x_s, \dot{x}_1, \ldots, \dot{x}_s)$$

where the frequencies $\omega_1, \ldots, \omega_s$ become rationally commensurable. Illuminating examples are given, and the small-divisor problem is encountered through Fourier analysis. Among several applications, a very readable discussion of the bounded three-body problem is included.

Although we generally associate the names of Cauchy and Picard with iteration, these authors give Lyapunov major credit for introducing majorants. Many uncommon and useful techniques for solving operator equations are presented, including use of the trigonometric norm, estimates of the domain of existence and uniqueness, and the relation to (non)contraction. The authors apply such methods to their special interest of obtaining periodic solutions. They treat differential equations involving regular and singular perturbations in both resonant and nonresonant situations. Unlike most authors, they worry and obtain convergent expansions, in contrast to asymptotic approximations (which could provide a starting guess for a numerical algorithm). Special attention is given to obtaining the resonance curve for

Duffing's equation and to calculating eigenvalues for Mathieu's equation. New iterative methods with quadratic convergence properties are then used as successive approximation schemes to effectively obtain periodic solutions to differential equations.

The authors finally present their approach to numerical-analytic solutions, which emphasizes the use of a computer for intermediate steps in obtaining analytical solutions, as in using iterative methods to solve algebraic equations. Much of such necessary algebraic details might also benefit from symbolic computation. I cannot report that this monograph is preferable to all others available in English (such as Arnold [1], Guckenheimer and Holmes [2], and Sanders and Verhulst [3]). It does, however, present valuable material and a unique perspective on an important, though specialized, class of problems.

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1. V. I. ARNOLD, *Geometrical Methods in the Theory of Ordinary Differential Equations*, Springer-Verlag, New York, 1983.

2. J. GUCKENHEIMER & P. HOLMES, *Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields*, Springer-Verlag, New York, 1983.

3. J. A. SANDERS & F. VERHULST, *Averaging Methods in Nonlinear Dynamical Systems*, Springer-Verlag, New York, 1985.

15[45D05, 65R20].—PETER LINZ, *Analytical and Numerical Methods for Volterra Equations*, SIAM Studies in Appl. Math., SIAM, Philadelphia, Pa., 1985, xiii + 227 pp., $23\frac{1}{2}$ cm. Price $32.50.

This book contains an elementary but thorough and self-contained introduction to the theory and the numerical solution of Volterra integral equations; as stated in the preface, "The audience for which this book is intended is a practical one with an immediate need to solve real-world problems." Thus, the chosen mathematical setting is that of (continuous) real-valued functions of one or several real variables, and proofs are often either just sketched or omitted entirely (the reader is then directed to an appropriate reference).

The first part of the book (six chapters, covering some 90 pages) deals with the classical quantitative theory of linear and nonlinear Volterra equations. It includes a brief chapter on some typical applications of Volterra integral and integro-differential equations, and it introduces some elementary results on the asymptotic behavior of solutions to certain second-kind integral equations. These chapters are particularly valuable, as most books on integral equations focus on Fredholm equations and treat Volterra equations only in a passing manner.

Numerical methods are discussed in the second part of the book, comprising about 110 pages. Chapter 7 covers direct quadrature methods (including a convergence analysis, asymptotic error estimates, and numerical stability), block-by-block methods, and explicit Runge-Kutta methods for second-kind equations with bounded kernels. Various product integration methods for second-kind equations with unbounded (or otherwise poorly behaved) kernels form the contents of Chapter 8. The

next two chapters are devoted to the solution of integral equations of the first kind having either differentiable or Abel-type kernels; here, the discussion focuses on the midpoint method and the trapezoidal method (and their product analogues), as well as on certain block-by-block methods. Chapter 11 is on numerical methods for first-order integro-differential equations; in addition to a description of linear multistep methods and block-by-block methods, we also find brief remarks on numerical stability and on more general Volterra functional equations.

Motivated partly by the lack of Volterra subroutines in software libraries, the author gives, in Chapter 12, listings (in Pascal) of a number of simple programs for (systems of) first-kind and second-kind integral equations. These algorithms are based on the midpoint rule and the trapezoidal rule and employ a fixed step size. Finally, Chapter 13 contains three case studies, involving the problem of error estimation, a nonstandard system of integral equations arising in polymer rheology, and the solution of a first-kind integral equation with nonexact data (this complements remarks, made in Chapters 9 and 10, on the ill-posed nature of first-kind equations). An extensive bibliography (some 280 references) concludes the book.

The book is well written and contains numerous examples which serve to illuminate the general discussion. The only slight shortcoming is that, in Chapter 8, the convergence orders are derived under the assumption that the solution of the second-kind Volterra integral equation with weakly singular factor $(t - s)^{-1/2}$ in its kernel have continuous derivatives of sufficiently high order (the order of, e.g., the block-by-block method based on quadratic interpolation is then $p = 7/2$). This is somewhat misleading since, typically, the solution of such an equation has derivatives which are unbounded at the left endpoint of the interval of integration (thus reducing the order of convergence on uniform meshes to $p = 1/2$). However, this minor criticism is greatly outweighed by the overall quality of this book, which is a most welcome addition to the literature on integral equations and their numerical solution.

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16[65–02, 65–04, 65F15, 65F20].—JANE K. CULLUM & RALPH A. WILLOUGHBY, *Lanczos Algorithms for Large Symmetric Eigenvalue Computations*, Vol. I: *Theory*, Progress in Scientific Computing, Vol. 3, Birkhäuser, Boston, 1985, xiv + 268 pp., 23 cm. Price $29.95. Vol. II: *Programs*, Progress in Scientific Computing, Vol. 4, Birkhäuser, Boston, 1985, vii + 496 pp., 23 cm. Price $49.95.

Cornelius Lanczos (another brilliant Hungarian) was a student of Albert Einstein. During World War II he put aside his studies in General Relativity and turned his abundant energy to the struggle against Nazi Germany. This led him into problems of engineering and scientific computation, and he never lost his interest in them, even after he was settled in Dublin in the Institute for Theoretical Physics.

He contributed a number of seminal ideas for applying classical analysis to the standard mathematical tasks where computation is vital. It is a pleasure to see a

book with the word Lanczos in the title, and it is fitting that a work of this length is needed to elaborate a clever idea he presented in a couple of papers in 1950.

Lanczos showed how to reduce any symmetric matrix A to a similar tridiagonal matrix T without applying any explicit similarity transformations to A. However, our way of looking at this scheme has undergone a number of significant transformations since 1950. An ambitious implementation of the method is a far harder task than an outsider might suspect.

The book under review is an excellent account of the present state of affairs. In fact, it is a lot more than that. The authors, both with IBM Research Center at Yorktown Heights, have been in the forefront of research on how best to implement, on a computer, the famous three-term recurrence presented in the original papers of Lanczos. They address the more general problem of how to compute some or all of A's eigenvalues, either with or without the corresponding eigenvectors, in the important case when A has many rows and many zero elements in them.

The exposition is exceptionally clear, in part because the authors wish to reach a broadly based readership: physicists, engineers, chemists, as well as applied mathematicians. Moreover, great care has been taken with references, with descriptions of work by others in this field, and with notation (see Chapter 0). Chapter 1 presents necessary background material and could serve as a model for textbook writers, particularly the list of applications. Chapter 2 presents what is now known about the behavior of the Lanczos algorithm both in exact and in finite precision arithmetic. New material appears here when the authors undertake a thorough review of all relevant research known to them. This is a daunting task because there is a certain amount of rivalry: different groups have espoused different strategies. Chapter 3 will tell you more than you ever wanted to know about tridiagonal matrices but, it turns out, this is the key to good implementations and to theoretical analysis of what is called 'convergence'.

The heart of the book is Chapter 4 wherein the authors develop and justify their implementation of the Lanczos recursion. The remaining chapters are concerned with extensions (such as block algorithms) and further applications (singular values and complex matrices). Volume II contains programs (in FORTRAN) and further detailed documentation. The references are impeccable and bear witness to the high scholarship practiced by Cullum and Willoughby. An example to us all.

For the only mildly interested reader let me try to sketch the reason why all this activity is necessary. After all, we do not really want a book associated with every nontrivial program.

Roundoff error wrecks the very pretty theory but it does not wreck the method. The explanations of this fact was the valuable contribution of C. C. Paige in 1971/72. In practice, the method becomes iterative whereas in exact arithmetic it terminates in at most n steps, where n is the order of A. We have as yet no theory which puts a precise bound on the number of steps needed so that the associated tridiagonal matrix yields every eigenvalue of A. The authors' response is to establish, in finite precision, the connection between the Lanczos recursion and the conjugate gradient (CG) algorithm for solving the system $Ax = v$, where v is the starting vector for the Lanczos scheme. Then they use the monotone decline in a measure of the error in the CG algorithm to give credence to the eventual appearance of each eigenvalue of A among the eigenvalues of T. It must be said that the clear, careful

pursuit of this approach does not make for easy reading. The wood gets lost in the trees. Here is an example of how hard it is to keep a global perspective. The authors make use of Paige's error bounds, and these bounds are based on the model that the multiplication $w = Av$ is what it seems to be, that is, $w_k = \sum a_{kj} v_j$ where the sum is over nonzero elements in row k of A. Consequently, the results do not apply to some important applications where Av is merely shorthand for the solution of a set of equations of the form $(K - \sigma M)w = Mv$ and so $A = (K - \sigma M)^{-1}M$. This example is not to be construed as a blemish in the book but as an illustration of the quandary faced by the designers of an algorithm that succeeds but cannot be formally proved to succeed. What is to be done?

Apart from 'Convergence' theory, an important contribution made by the authors is the test they have gradually developed and refined for discriminating between desirable and undesirable eigenvalues of the tridiagonal matrix T. This is a fascinating topic but too technical for description here. After all, if T's order is three times that of A then some selection has to be made.

In their careful discussion of other ways of implementing the Lanczos algorithm, Cullum and Willoughby clearly believe that there is a "best", or preferred, way to do the job. Certain turns of phrase suggest that there may have been intellectual skirmishes between rival research groups. It is all rather tantalizing. To me, it seems far more likely that efficient execution of different tasks will require different implementations of the Lanczos recursion. Sophisticated structural engineers may well stick to their own nearly orthogonal set of Lanczos vectors. On the other hand, those scientists who need half or all of the spectrum of a conventional sparse symmetric matrix A will find the Cullum/Willoughby algorithm very hard to beat. And this book's description of it is a lesson to us all.

B. P.

17[30-02, 65E05, 42-XX, 30E20, 30C30, 30C50].—PETER HENRICI, *Applied and Computational Complex Analysis*, Vol. 3: *Discrete Fourier Analysis—Cauchy Integrals—Construction of Conformal Maps—Univalent Functions*, Wiley, New York, 1968, xiii + 637 pp., $23\frac{1}{2}$ cm. Price $59.95.

This text is another excellent, long awaited, important and welcome addition to the previous two volumes written by the same author. The titles and chapter headings of the now available three volumes are:

Vol. 1: Title, "Power Series—Integration—Conformal Mapping—Location of Zeros"; Chapters, 1. Formal Power Series, 2. Functions Analytic at a Point, 3. Analytic Continuation, 4. Complex Integration, 5. Conformal Mapping, 6. Polynomials, 7. Partial Fractions.

Vol. 2: Title, "Special Functions—Integral Transforms—Asymptotics—Continued Fractions"; Chapters, 8. Infinite Products, 9. Ordinary Differential Equations, 10. Integral Transforms, 11. Asymptotic Methods, 12. Continued Fractions.

Vol. 3: Title, given above; Chapters, 13. Discrete Fourier Analysis, 14. Cauchy Integrals, 15. Potential Theory in the Plane, 16. Construction of Conformal Maps: Simply Connected Regions, 17. Construction of Conformal Maps for Multiply

Connected Regions, 18. Polynomial Expansions and Conformal Maps, 19. Univalent Functions.

The study of applied mathematics and computation using tools of complex analysis provides an indispensable insight and understanding of not only when, why, and how well the methods of these subjects work, but also for deriving new methods. Unfortunately, an examination of the curriculum of most schools, or an examination of textbooks on complex variables, shows that the importance of this understanding is sadly underemphasized. Henrici's three volumes provide this understanding, and thus they fill a dire need in our educational system. The books are excellent reference books for applied mathematicians, engineers and physicists, and they may be used as textbooks for good students. Exercises, problems, and seminar topics appear at the end of chapters.

Henrici's Volumes 1 and 2 have been reviewed in *Math. Comp.*, v. 31, 1977, pp. 325–326, MR **51** #8378, and MR **56** #12235. The present reviewer wholeheartedly agrees with the praiseworthy remarks in these reviews.

Volume 3 shares features of Volumes 1 and 2, in that it contains exciting presentations which can be found in no other textbooks. Chapter 13 is a new, self-contained and well-written presentation of the DFT method. Here one finds an excellent exposition of the discrete Fourier transform method for Fourier, power series and integrals, numerical harmonic analysis for Fourier series and integrals, evaluation of coefficients of Laurent series, residues and zeros, the conjugate periodic functions, convolutions, and multivariate DFT. The material of Chapter 14, which was previously available only in technical monograph translations from Russian, provides powerful methods for solving problems of potential theory. Included in this chapter are methods of solution of the Riemann and Privalov problems for both closed and open arcs, as well as the recently developed Burniston-Siewert method for solving transcendental equations. Chapter 15 discusses a variety of methods of solution of Dirichlet and Neumann problems for planar potentials. Chapters 16 through 18 discuss conformal mapping, as well as the very exciting and newly developed area of the construction of conformal maps. One finds in these chapters a discussion of the osculation methods for numerical conformal mapping, the integral equations of Symm and Berrut, the mapping methods of Timman, Fornberg and Wegmann, and a formal theory of Faber polynomials and Faber functions. Finally, Chapter 19 presents an elementary and self-contained version of the recently discovered proof of the Bieberbach conjecture in the theory of univalent functions.

Congratulations to Peter Henrici for completing such a monumental, important and exciting work!

F. S.

18[41A50].—G. G. LORENTZ, *Approximation of Functions*, 2nd ed., Chelsea, New York, 1986, ix + 188 pp., $23\frac{1}{2}$ cm. Price $14.95.

Approximation Theory is a well-established part of analysis which, after more than 100 years of activity, remains a vital research area with numerous applications. This branch of mathematics deals with the problem of approximating a complicated

(or possibly unknown) function by a relatively simple function (such as a polynomial). There are especially close ties to three other mathematical fields: 1) Fourier Analysis, 2) Numerical Analysis, and 3) Statistical data analysis. Its applications include data fitting, computer evaluation of special functions, numerical solution of differential and integral equations, and computer representation of curves and surfaces (for computer-aided design and manufacture). In addition it remains (sometimes under the alias Constructive Function Theory) an important tool in other branches of mathematics.

The importance of the subject can be judged by the fact that 1) papers on it can be found in numerous journals, including several devoted exclusively to Approximation Theory, 2) there continue to be numerous conferences in a variety of countries on the subject, and 3) over the years a large number of books on the subject have been written, and continue to be written.

The subject of this review is the second edition of a book which first appeared in 1966. Except for some minor corrections (and the insertion of one new result on rational approximation), this second edition is identical with the first, and thus those familiar with the original classic need read no further.

The author's stated aim in writing this book was to provide an accessible but reasonably complete treatment of several topics in Approximation Theory. The book is aimed at the graduate or advanced undergraduate level, and includes a considerable number of problems (some of which are rather more challenging than others). As everyone familiar with the book knows, the author has been very successful at meeting his goals, and the book is indeed a very readable introduction to the subject and makes an excellent textbook.

The heart of the book is contained in Chapters 1–5, which deal with best approximation from a finite-dimensional subspace of a normed linear space. Most of the results concern real functions, and in fact most of the action takes place in $C(A)$, where A is a compact set. In these chapters one may find the classical approximation results on existence of best approximants, uniqueness, characterization, and degree of approximation. Along the way one is treated to a discussion of positive linear operators, moduli of smoothness, classes of smooth functions, Bernstein and Markov inequalities, and a wealth of other tools and techniques. Special attention is devoted to both direct and inverse theorems for trigonometric and algebraic polynomials.

Nonlinear approximation is dealt with only briefly in Chapter 6 where rational approximation is considered. Except for an interesting chapter on Hilbert's Thirteenth Problem of approximating a function of several variables by superpositions of functions of fewer variables, the book deals almost exclusively with univariate functions. A special feature is the inclusion of chapters on n-widths (a subject which has attracted considerable recent attention) and on entropy and capacity. This material is hard to find in book form, especially at this level.

I am happy to recommend this book to anyone who wants an introduction to Approximation Theory. The reader should be aware, however, that in addition to a considerable body of classical work which was not treated here, a tremendous amount of new work has been done in the past twenty years (especially in numerical methods, spline functions, and multivariate approximation). While the reader will have to look elsewhere for these more recent results, I am sure that he will find the

present book an excellent foundation to build upon. We all owe a debt of gratitude to Chelsea for resurrecting this delightful little monograph.

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19[42–01, 42–04].—RONALD N. BRACEWELL, *The Hartley Transform*, Oxford Engineering Science Series, Vol. 19, Oxford Univ. Press, New York, 1986, vii + 160 pp., 24 cm. Price $24.95.

This book's stated purpose is to introduce and discuss applications of the Hartley transform and to compare it with the related Fourier transform.

After a brief introduction in Chapter 1, Chapter 2 defines the Hartley transform and shows how the Hartley transform coefficients relate to those of the Fourier transform. A number of examples are given pertaining to the calculation of the Hartley transform for a variety of problems. The power spectrum and the phase are shown to be readily expressible in terms of Hartley transform coefficients. Hartley and Fourier transform theorems are presented in Chapter 3, as well as relations between domains, while Chapter 4 presents a good discussion of discrete versus continuous transforms.

Much of the remainder of the book is devoted to the practical application of these transforms. Chapter 5 discusses digital filtering by convolution. Cyclic convolutions are discussed and contrasted with ordinary convolutions, and comparisons are made to Fourier convolutions. Examples are given involving low pass filtering and edge enhancement. The chapter ends with convolutions expressed in terms of matrix multiplications.

In Chapter 6, two-dimensional Hartley transforms are discussed; in Chapter 7, a description of a factorization technique is given and the transform is shown to be expressible as a matrix operation with the bulk of the chapter devoted to factorization of this matrix and the perturbation operations involved. Chapter 8 discusses details of a fast transform algorithm and various schemes to speed up all aspects of the calculation including rapid computation of trigonometric functions and methods for fast permutation. The concluding short Chapter 9 discusses optical Hartley transforms. A series of problems follows each chapter.

Appendix I gives a series of programs in BASIC for discrete Hartley transforms, while Appendix II presents an atlas of Hartley transforms.

This book presents a quite thorough discussion of Hartley transforms with special emphasis on the discrete transform. It is well written and gives the reader a number of applications of these transforms and detailed cases where computation via the Hartley transform has advantages over that done by more standard Fourier transform techniques.

There are a few minor typographical errors such as in the expression for the even part of the transform on page 11 and in the expression for the phase on page 18. In general, though, I found no errors of any consequence.

Anyone interested in Fourier and related transforms should find this book a valuable reference volume to have in his library.

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20[62Q05, 62E15, 62E99, 62G15, 62J99].—R. E. ODEH & J. M. DAVENPORT (Coeditors), N. S. PEARSON (Managing Editor), *Selected Tables in Mathematical Statistics*, Vol. 10, Amer. Math. Soc., Providence, R. I., 1986, xi + 347 pp., 26 cm. Price $39.00.

The volume contains two tables whose entries can be quite useful for statisticians in diverse applications. The opening table gives percentiles, P, of the distribution of positive definite quadratic forms in normal variables, $\Sigma_{i=1}^{k} \lambda_i x_i^2$ where x_i is $N(0, 1)$ and $\lambda_i > 0$, $\Sigma_{i=1}^{k} \lambda_i = 1$. There are percentile entries corresponding to $P = 0.001$, 0.005, 0.01 (0.01)0.19, 0.20 (0.05) 0.80, 0.81 (0.01)0.99, 0.995, 0.999; for $k = 2(1)10$, and λ's in multiples of 0.05.

The second table lists confidence limits on the correlation coefficient ρ, associated with a bivariate normal distribution. Let r_0 be the observed value of ρ, and n the sample size; then confidence limits are given for values of $r_0 = -0.98(0.02)0.98$; $n = 3(1)80(5)100(10)200(25)300(50)600(100)1000$, and for $1 - \alpha$ and $\alpha = 0.005, 0.01, 0.025, 0.05, 0.10, 0.25$.

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21[65-06, 65M50, 65N50, 65R20].—D. J. PADDON & H. HOLSTEIN (Editors), *Multigrid Methods for Integral and Differential Equations*, The Institute of Mathematics and its Applications Conference Series, Clarendon Press, Oxford, 1985, xii + 323 pp., 24 cm. Price $36.40.

These are the proceedings of a Summer School/Workshop held at the University of Bristol, England, in September of 1983. Most of the papers were substantially revised after the meeting and reflect the state of affairs as of the effective closing date of the proceedings (July 1984). The volume opens with four contributions of the guest speakers: A. Brandt, "Introduction—Levels and scales" (10 pp.), outlines the philosophical underpinnings and the scope of the subject; W. Hackbusch, "Multigrid methods of the second kind" (73 pp.), is a substantial review of multigrid methods for integral equations; P. W. Hemker & P. M. de Zeeuw, "Some implementations of multigrid linear system solvers" (32 pp.), provides a guide to the design and implementation of programs for multigrid methods applied to general elliptic problems of the convection-diffusion type; P. Sonneveld, P. Wesseling & P. M. de Zeeuw, "Multigrid and conjugate gradient methods as convergence acceleration techniques" (51 pp.), treats multigrid and conjugate gradient methods as a means of accelerating iterative methods. The remainder of the book is devoted to

contributed papers, some of them, like the paper by J. Ruge & K. Stüben on algebraic multigrid methods (44 pp.), themselves substantial contributions.

W. G.

22[65–06, 65L10].—U. M. ASCHER & R. D. RUSSELL (Editors), *Numerical Boundary Value ODEs*, Progress in Scientific Computing, Vol. 5, Birkhäuser, Boston, 1985, xii + 317 pp., 23 cm. Price $34.95.

These are the proceedings of an international workshop held in Vancouver, Canada, July 10–13, 1984. There are 18 contributions reviewing recent progress in the numerical solution of two-point boundary value problems. They are grouped into five sections entitled: Conditioning, dichotomy and related numerical considerations: Implementation aspects of various methods; Singular perturbation ("stiff") problems; Bifurcation problems and delay differential equations; Special applications.

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