Addendum to “Large Integral Points on Elliptic Curves”

By Don Zagier

The author has been informed by Serge Lang that the conjecture that all integral solutions of \( y^2 = x^3 + ax + b \) are polynomially bounded in \( a \) and \( b \) (cf. [3, Section 2]), and the observation that a naive probabilistic argument would lead to the expectation that the number \( \rho = (\log x) / \log(\max(|a|^{1/2}, |b|^{1/3})) \) does not ever exceed \( 10 + o(1) \), have been made by him and by Harold Stark; Lang conjectured that all solutions, with the possible exception of a finite number of parametric families, satisfy \( \rho \leq 10 + o(1) \) (the necessity of the caveat about exceptional families followed from a related example of Stark’s). For all this, see [1]. It has been shown by Paul Vojta that Lang’s conjecture in this form follows from his own more general Diophantine conjecture [2]. Recently, Noam Elkies has constructed an infinite family of curves having an integral solution with \( \rho = 12 + o(1) \).


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