Addendum to “Large Integral Points on Elliptic Curves”  

By Don Zagier

The author has been informed by Serge Lang that the conjecture that all integral solutions of $y^2 = x^3 + ax + b$ are polynomially bounded in $a$ and $b$ (cf. [3, Section 2]), and the observation that a naive probabilistic argument would lead to the expectation that the number $\rho = (\log x)/ \log(\max( |a|^{1/2}, |b|^{1/3}))$ does not ever exceed $10 + o(1)$, have been made by him and by Harold Stark; Lang conjectured that all solutions, with the possible exception of a finite number of parametric families, satisfy $\rho \leq 10 + o(1)$ (the necessity of the caveat about exceptional families followed from a related example of Stark’s). For all this, see [1]. It has been shown by Paul Vojta that Lang’s conjecture in this form follows from his own more general Diophantine conjecture [2]. Recently, Noam Elkies has constructed an infinite family of curves having an integral solution with $\rho = 12 + o(1)$.

Max-Planck-Institut für Mathematik  
Gottfried-Claren-Strasse 26  
D-5300 Bonn 3, West Germany