REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the American Mathematical Society classification scheme. The 1980 Mathematics Subject Classification (1985 Revision) can be found in the December index volumes of Mathematical Reviews.


This book is destined to become an indispensable reference for any researcher working on the development or analysis of Runge-Kutta (RK) or General Linear (GL) methods for the numerical solution of Initial-Value Problems (IVP's) for Ordinary Differential Equations (ODE's), and it will be used by the research community as a starting point for further work in this area.

As its title and length suggest, it contains a comprehensive in-depth analysis of RK and GL methods, as well as an excellent summary of results from the theory of ODE's, graphs, and combinatorics needed for their development and analysis. Butcher also includes an extensive bibliography of just under one hundred pages. In addition, the book contains a brief discussion of linear multistep, extrapolation and Taylor series methods. These, though, are not analyzed in depth, but rather are included for completeness in an overview of numerical methods for the solution of IVP's.

Many readers, such as myself, who have not closely followed the rapid development of RK and GL methods in research journals during the past decade will welcome this coherent presentation of the fruits of this labor. Further, it is most appropriate that Butcher himself should have written this book since he has contributed more than any other researcher over the past three decades to the development of RK and GL methods.

The reader, though, may be surprised to learn that a well-written book of this length does not contain everything of importance on such a seemingly specialized topic—but this is indeed the case. In the preface, Butcher alludes to a sequel on "practical issues concerning the design of efficient differential equation software". This underscores the observation that the current volume is devoted almost exclusively to theoretical issues. These, though, are covered thoroughly in Butcher's elegant, but terse, mathematical style.

Another indication of the extent of this seemingly narrow topic and its impressive rate of development over the past decade is the recent publication of two other books in this field [1], [2]. Although it addresses other closely related topics, the first monograph focuses on the highly specialized subtopic of the nonlinear stability of Runge-Kutta methods, a research area that has blossomed during the past decade.
Butcher covers much of this material in his book as well, but not in as much depth and often from a different perspective. The authors of the second book, on the other hand, survey numerical methods for solving IVP's for ODE's. Although they place greater emphasis on linear multistep and extrapolation methods than Butcher does, the strength of their book, like his, lies in the analysis of RK methods. But unlike Butcher, they consider practical aspects associated with the numerical solution of IVP's, and even include codes and numerical test results in their book. However, as the title suggests, their first volume deals with nonstiff IVP's only: a second volume on stiff problems is currently being written.

Any researcher working on RK or GL methods or any library that aims at having a comprehensive collection of works on the numerical solution of IVP's should acquire all three of these excellent books. Although they overlap to some degree, there are many aspects discussed in one but not the others. Moreover, as already mentioned above, topics treated in two or more of these books are often presented from different perspectives, giving the reader a deeper understanding of the material.

Hairer, Norsett and Wanner's book is the most suitable of the three for either a text for an advanced course on the numerical solution of IVP's or a practical guide to solving IVP's for scientists and engineers. Its obvious drawback is that it does not treat stiff problems, but this deficiency will be rectified, we hope, with the appearance of their second volume. Dekker and Verwer's book is too specialized for either of these two potential audiences. Butcher's, on the other hand, although not more specialized than Hairer, Norsett and Wanner's, does not treat practical issues to nearly as great an extent and is aimed more at the RK/GL research fraternity who will study it extensively from cover to cover, rather than at the broader scientific computing community looking for insight into solving practical problems. Hairer, Norsett and Wanner's book, on the other hand, is directed more towards this broader audience, although RK/GL specialists will find it a very useful reference. It is more accessible to nonexperts in the field than Butcher's, because of the less mathematically sophisticated style in which it is written. This also contributes to the ease one frequently finds in reading a topic of interest in it without having studied the preceding sections, a characteristic not shared by Butcher's book.

The comparison in the preceding paragraph should not be taken as negative criticism of any one of these three books. Each is excellent in its own way, but written with different audiences in mind.

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REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS


The numerical solution of Volterra equations (VE's) is a rather new subject. A subclass of VE's are ordinary differential equations (ODE's). The numerical treatment of this class is by now well understood. A commonly used approach, when discussing methods for VE's, is simply to transform results from ODE's to VE's. Concepts like convergence and stability are then inherited directly from the ODE theory without verifying their validity when applied to VE's. This book can be viewed as a milestone, in that it establishes the numerical solution of VE's as a subject on its own. The authors succeed in presenting the latest results for VE's in a readable way. A good background in ODE's, however, would be helpful for the reader. Various numerical concepts introduced are then easier to understand. The preface states that a background in calculus is sufficient as a prerequisite. I would recommend to supplement this with a course in basic numerical analysis.

In addition to dealing with the construction and convergence of multistep and Runge-Kutta type methods, there is a separate chapter on numerical stability. The authors recognize that the question of stability is still in its infancy. Some results are given, but a good deal more needs to be done in that direction.

In the chapter on Runge-Kutta type methods, the Volterra series approach is only referred to. No details are given. In order to understand the construction of these methods, one has to read the appropriate literature. In my opinion, this is an inconvenience to the reader.

The historical notes are of great interest and value, as they provide a flavor of the evolution of the subject.

This monograph is well suited for any reader who wants to gain insight into the latest results on the numerical solution of VE's.

S.P.N.


The topic of this book is the computation of all solutions of a system of $n$ polynomial equations in $n$ variables, where $n$ is assumed to be small. Such problems arise in many applications, and there is much interest in simple and reliable methods which do not require a deeper analysis. This excludes, for instance, Newton's method, since the determination of suitable initial approximations for all the solutions often requires more information about the system than is readily available.
During recent years there has been a considerable research effort on the application of homotopy methods to this problem. The author is one of the most active contributors to this work and presents here an introduction to his methods for a general audience of potential users.

Briefly, let \((1) \quad f(x) = 0, \quad f \in \mathbb{C}^n \rightarrow \mathbb{C}^n\) be the given system and denote by \(d_1, \ldots, d_n\) the degrees of the \(n\) components of \(f\). The sum of these degrees is the total degree \(d\) of \((1)\). Now an initial system \((2) \quad g(x) = 0\) can be introduced with components \(g_j(x) = p_j^{d_j} x_j^{d_j} - q_j^{d_j}, \quad j = 1, \ldots, n\), where \(p_j\) and \(q_j\) are suitable complex constants. Then the desired homotopy is \((3) \quad h(x,t) = (1 - t)g(x) + tf(x), \quad 0 < t < 1\), which permits the application of a continuation process to follow the \(d\) paths beginning at each of the solutions of \((2)\). Clearly, the implementation of this concept requires much attention to various details, and that takes up a major part of the book.

There are two parts, cohering the method and several applications, respectively. More specifically, after two introductory chapters introducing the basic ideas for one- and two-dimensional equations, Chapter 3 describes the general method, which is followed by a chapter on its implementation. The first part then ends with a chapter on scaling techniques and on some alternative continuation methods. The second part begins with Chapter 7, covering practical considerations of systems reduction, while the final Chapters 8 through 10 are devoted to case studies. More specifically, geometric intersection problems, chemical equilibrium systems, and kinematic problems of mechanisms are considered. There are also six appendices, namely five containing additional mathematical details and another one which presents some 200 pages of FORTRAN source code of all the procedures.

As the title already indicates, the book is intended primarily for engineers and scientists who wish to use these techniques. In line with this, an informal approach was adopted and the mathematical prerequisites were kept at the level of a working knowledge of multivariate calculus, linear algebra, and computer programming. The book certainly succeeds well in its aims and offers a nice introduction to these valuable new methods.

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This is a textbook based on a graduate course given at the University of Antwerp, a supplement to the many elementary books which mainly treat linear techniques. I opened the book with a sense of expectation and curiosity. What is it that can be found here, behind the rather general title?

Quite sensibly (and not unexpectedly) the authors have restricted their presentation to methods based on Padé approximation and “Padé-like” techniques like rational Hermite interpolation. A Padé approximant of order \((m,n)\) to a function
f(x) holomorphic at 0 is a rational function \( r_{m,n}(x) = P_{m,n}(x)/Q_{m,n}(x) \), where \( P_{m,n} \) and \( Q_{m,n} \) are polynomials of degree \( m \) and \( n \), respectively, such that

\[
Q_{m,n}(x)f(x) - P_{m,n}(x) = \sum_{k=1}^{\infty} c_k x^{m+n+k}
\]

in a neighborhood of 0. That is, it is basically interpolation at 0. Indeed, an alternative (but not quite equivalent) definition of a Padé approximant is that the derivatives evaluated at 0 satisfy \( r_{m,n}^{(k)}(0) = f^{(k)}(0) \) for \( k = 0, 1, \ldots, m + n \). A Hermite interpolant interpolates at more than one point. If \( f \) is holomorphic at the distinct interpolation points \( \{x_i\} \), then the requirement is that

\[
r_{m,n}^{(k)}(x_i) = f^{(k)}(x_i) \quad \text{for} \quad k = 0, \ldots, t_i \quad \text{and} \quad i = 1, \ldots, s
\]

for some given constants \( s, t_1, \ldots, t_s \in \mathbb{N} \cup \{0\} \), where \( \sum_{i=1}^{s} (t_i + 1) = m + n + 1 \).

These concepts are intimately connected with continued fractions, which are presented in a separate chapter in the book. The interpolation requirements (1) have their counterpart in correspondence of a continued fraction.

By presenting in turn continued fractions, Padé approximants and rational interpolants in separate, self-contained chapters, the authors manage to repeat the idea of interpolation in varying situations. This has a great pedagogical value and gives the book a unified and compact quality. The economy in the wording adds to this impression.

Within this framework the authors have also managed to present methods for the multivariate case—a considerably more complex task. By carefully following the same lines of presentation, they are able to give a logical and readable introduction to branched continued fractions and multivariate Padé approximation and interpolation. This important and useful contribution makes the book valuable also to users and workers in the field.

For teaching, the most valuable part of this book, perhaps, is the last chapter. It is devoted to applications of the techniques introduced. The headings are: convergence acceleration, nonlinear equations, initial value problems, numerical integration, partial differential equations and integral equations.

With few (rather randomly occurring) exceptions, stability analyses and error analyses are omitted, also in cases where error estimates could be obtained almost for free from arguments given in the text. Instead, the authors have chosen to present numerical examples where the exact answers are known, so that the accuracy obtained by various algorithms can be immediately seen from the output.

Speaking about examples, I would have liked to see some in which a numerical method is needed to solve a practical problem. Maybe also some more examples illustrating the advantages of nonlinear methods over linear methods. But these are minor objections. The book is a clear, well-written introduction to a fascinating and useful field. The extensive list of references and exercises is a valuable addition.

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The successor to Jahnke and Emde’s tables of special functions [2], first published in 1909, was the 1964 handbook edited by Abramowitz and Stegun [1], often called AMS-55. An extensive supplement to the latter, written at an advanced level and emphasizing rational approximations, was published in 1975 by Luke [3]. AMS-55, now 24 years old, seems likely to be superseded by An Atlas of Functions, although the price, not unreasonable in view of glossy paper and multicolored graphs, may limit its popularity. Numerical tables have been replaced by algorithms for a programmable calculator that usually yield better than seven significant figures. Two or three significant figures can often be obtained directly from the many computer-generated graphs because of the admirable device of dotted coordinate lines. There is much more explanatory text and comment than in AMS-55, and the book seems aimed at a wider and mathematically less experienced audience. The whole volume represents an immense labor done with intelligence, skill, and great care.

The return to two-man authorship, as in Jahnke-Emde, has produced more uniform style and coverage, enhanced by rigid adherence to a system of dividing each of the 64 chapters into 13 sections on topics such as Expansions, Numerical Values, and Approximations (but not Inequalities). The section titled Operations of the Calculus gives the authors, who collaborated previously on a well-known book on fractional calculus, an opportunity to include representations by fractional derivatives and integrals. An occasional fourteenth section on Related Topics contains short accounts of subjects such as linear regression, orthogonal coordinate systems, fast Fourier transforms, and Laplace transforms. Like AMS-55, the book is not only a source of numerical values and a collection of graphs but also a formulary that ranks with the best and may prove easier to use than most because of good organization and many helpful comments.

Roughly half the book is devoted to functions on the integers, elementary algebraic functions, classical orthogonal polynomials, and elementary transcendental functions. The Heaviside step function and the Dirac delta function each have a chapter. The second half of the book treats the gamma function and its relatives, the indispensable roster of special confluent and Gaussian hypergeometric functions, the Struve function, elliptic integrals and functions, and the Hurwitz zeta function. One must still turn to AMS-55 for Coulomb wave functions, Mathieu functions, and spheroidal wave functions. The book ends with an appendix of utility algorithms, a second appendix with tables of physical units and constants, a list of references (not including Whittaker and Watson’s Modern Analysis nor Watson’s Bessel Functions), a subject index, and a symbol index.

Despite admiration and gratitude for a superb piece of work, one may still regret strongly a few of the authors’ decisions. They treat special functions as functions of a real variable instead of a complex variable; in contrast with AMS-55, the reader will not learn here, for example, that the gamma function is analytic except for poles at the nonpositive integers nor that its reciprocal is entire. In each chapter there is a section titled Complex Argument, but it usually contains only a separation into real
and imaginary parts. In an age when complex function theory is an undergraduate course, when analytic continuation in the complex energy or angular-momentum plane is commonplace in physics, when engineers use analytic transfer functions to describe electrical and mechanical systems, it seems a step backward to survey the properties of special functions with no mention of analyticity, poles, or branch points.

Secondly, the authors have relegated to the background the one notational device, $pF_q$, that has done more than any other to bring order and unity into the welter of redundant definitions and notations that have accumulated during two centuries. One must thank them for a long list (Table 18.14.2) identifying hypergeometric series having various parameters with functions discussed elsewhere in the book. However, they eschew the $pF_q$ notation for such series because they consider it more general (see Section 60:13 and p. 155) to specify the coefficient of $x^n$ rather than $x^n/n!$. That is, they prefer to insert 1 as a parameter in the denominator when necessary rather than remove the $n!$ when necessary by a parameter 1 in the numerator. This is really a question of taste, not generality. Their choice has the effect that closely related functions (for example, arcsin and arctan, special cases of $2F_1$) are listed with different numbers of parameters, and a linear transformation of $2F_1$ can change the number of parameters. Worse yet, it has the effect that the $pF_q$ notation is not used in the other chapters to show the reader that some order and simplicity underlie the chaotic throng of special hypergeometric functions. The shifted factorial or Pochhammer symbol, $(a)_n$, fares somewhat better with a chapter to itself, but in the rest of the book it gives way to semifactorials and even to the notations $n!!$, and $n!!!$.

What one reader deplores, another may applaud; no book can satisfy everyone in all respects. This one should please nearly everyone in most respects and should have a deservedly bright future in the Citation Index.

B. C. C.


Applied spectral analysis has undergone significant changes in recent years. While traditional spectral analysis has evolved around Fourier methods, the modern approach emphasizes parametric modeling and makes extensive use of matrix analysis. The advent of powerful hardware for numerical processing, together with the ever increasing demand for speed, accuracy and complexity, are largely responsible for this shift of emphasis.
Lawrence Marple's new book well reflects this trend. While its title states that it is a book about digital spectral analysis, its contents place an emphasis on the so-called "modern" spectral estimation. This book was written by an engineer for engineers and often sacrifices mathematical rigor for readability. The book is very clearly written, has a lucid, informal style, and it continuously stresses intuition and engineering judgment. Each of the major chapters starts with a summary of the algorithms discussed in that chapter, in a form suitable for immediate use. This is very helpful for readers who are already familiar with the theory, and who need the book mainly for reference. The book contains a diskette with 35 FORTRAN programs that are carefully prepared, well documented, and presented in a form ready to compile and run.

Here is a chapter by chapter description of the book's contents.

Chapter 1 gives a very nice historical introduction to spectral analysis, followed by some illustrations of the problems and difficulties involved in this field.

Chapter 2 provides a brief review of linear system theory, to the extent that it is used later in the book. The exposition is rather informal, in the general spirit of the book. Section 2.7 contains an interesting discussion of the relationship between continuous and discrete transforms.

Chapter 3 similarly reviews basic matrix theory. Among standard results, it includes some facts about Toeplitz matrices not frequently encountered in linear algebra texts.

Chapter 4, which reviews random process theory, is undoubtedly the weakest in the book. Here, informality often extends to inaccuracy. The author proceeds to describe properties of estimators without ever mentioning what estimation theory is all about, skipping basic definitions and facts. For a book whose main concern is estimation, this is a serious omission. The discussion of ergodicity in Section 4.4 is flawed. In fact, the reader would be better off skipping this section completely. A good presentation of ergodicity is given in the text by Grenander and Rosenblatt [1].

Chapter 5 is the only one in the book devoted to classical spectral estimation. It gives a rather concise description of the main issues involved in nonparametric spectral estimation—windowing, averaging, trade-off between variance and resolution, etc. There is an error in Eq. (5.26)—the right-hand side is incorrect in general, unless \( L = N - 1 \).

Chapter 6 serves as an introduction to parametric models for stationary time series, in particular AR, MA and ARMA models. This chapter is, perhaps, too concise. The reader may want to consult the book by Box and Jenkins [2] for additional material on parametric models of time series.

Chapters 7, 8, and 9, give an extensive exposition of autoregressive (AR) estimation. AR modeling is an extremely popular technique, and there is a very rich literature on the subject. The author's choice of material here is well balanced, and both the theoretical and practical aspects of AR estimation are well explained. I have two minor comments about the material in these chapters. In Section 8.8, the author does not alert the reader to the difficulties associated with noise subtraction. Additive noise does not just increase the zero correlation term. It affects the
probability distribution of all the estimated correlations, hence a simple subtraction of the noise variance from the zero correlation seldom yields acceptable results. In Section 9.3, the author quotes conditions for convergence of the LMS that are known to be incorrect for AR processes. In fact, conditions for convergence of the LMS algorithm for general AR processes are still unknown.

Chapter 10 is devoted to autoregressive moving average (ARMA) estimation. ARMA estimation is considerably more difficult than AR estimation. Simple algorithms, based on linear approximations, are relatively inaccurate. High performance algorithms (such as maximum likelihood) are iterative in nature, prone to convergence problems, and require intensive computations. Therefore, they are seldom used in real-time applications. The author concentrates on relatively simple algorithms, the performance of which is known to be inadequate in many cases. For ARMA estimation, the book by Box and Jenkins [2] is probably still the best reference.

Chapter 11 describes Prony's method and some of its variants.

Chapters 12 and 13 deviate from the parametric approach, concentrating on methods based on the estimated covariance matrix. These methods take advantage of the special eigenstructure of the covariance matrix, and most of them are restricted to sinusoidal signals in white noise. Chapter 12 presents the simplest of these methods—the so-called minimum variance algorithm. Chapter 13 presents more advanced methods, such as Pisarenko harmonic decomposition and the multiple signal classification (MUSIC) method of Schmidt.

Chapter 14 summarizes the estimation methods for single time series, and attempts at giving them a unified description (in Table 14.1).

Chapters 15 and 16 are devoted to two specialized topics: estimation of multiple time series, and two-dimensional spectral estimation. Multiple time series have not received a decent treatment in textbooks since Hannan's classic text [3]. Unfortunately, Hannan's book has never gained popularity in the engineering community. Chapter 15 in Marple’s book thus serves to fill a certain gap in this area. Similarly, two-dimensional spectral estimation has received only little treatment. Chapter 16 in Marple’s book is fair, but it considerably overlaps with Chapter 6 in the book by Dudgeon and Mersereau [4].

In summary, Marple has succeeded in organizing the vast material available in the technical literature, and presenting it in a very readable form. The book should prove a useful reference. As a text for, say, an engineering graduate course, it should be supplemented by additional reading, to better cover the theoretical aspects. As a book for self-study, it is very good, provided the reader has some background in random process and estimation theory.

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A welcome tradition seems to be evolving in England to organize a conference once a decade to review progress made in numerical analysis during the past ten years. The volume under review contains the proceedings of the third such conference, held at the University of Birmingham, April 14–18, 1986. (For the two preceding conferences, see [1], [2].) An attempt has again been made to survey the entire field of numerical analysis. This resulted in 23 contributions, written by acknowledged experts, covering such areas as numerical linear algebra (eigenvalues, statistical applications, sparse matrices), approximation theory (multivariate approximation, splines, best approximation and regression analysis, complex elementary functions), optimization (linear and quadratic programming, nonlinear constraints), nonlinear equations (tensor methods, bifurcation problems, secant updating techniques), the influence of machine architectures on numerical analysis (vector and parallel processors), ordinary differential equations (stability theory, stiff problems, order stars), integral equations (Fredholm and Volterra equations, boundary integral equations), and partial differential equations (multigrid, Galerkin, and finite element methods, free and moving boundary problems, nonlinear conservation laws).

W. G.


W. G.