

First Occurrence Prime Gaps

By Jeff Young and Aaron Potler

Abstract. An ongoing search for first occurrence prime gaps continues.

An ongoing search for first occurrence prime gaps is being carried out which extends all previous work done on this subject. To date this search has found all such gaps for primes up to 7.263×10^{13} . First occurrence prime gaps had previously been known for primes less than 4.444×10^{12} [2]. Several gaps larger than the previously largest gap of 682 (not a first occurrence) found by Weintraub [4] have been found.

Computer programs were written in FORTRAN and CAL (Cray Assembly Language) on a CRAY-2 supercomputer.

The computation was conducted as follows. A sufficient number of primes were generated to perform a sieve. Odd numbers beginning with 3 were sieved one block at a time, where each block was chosen to contain 40,000,000 numbers based on system resource availability. The even numbers were eliminated during initialization of each block. One number was stored per 64-bit computer word. After sieving each block, the differences between consecutive primes were calculated and stored. This was accomplished by loading 64 elements of the sieved block at a time into a vector register. A 64-bit vector mask was created containing 1's for corresponding nonzero values in the vector register. If the vector mask was zero, the next 64 values of the sieved block were loaded into the vector register. If the mask was nonzero, an instruction to count the number of leading zeros was executed to get the offset from the beginning of the vector register for the next prime. A subtraction of the previous prime was done, thus arriving at the gap. The leftmost 1 of the vector mask was cleared and the method repeated, beginning with checking if the vector mask was zero. This was repeated until the entire block was processed. The last prime in each block was saved in order to calculate the difference between that prime and the first prime in the next block to make sure no gaps were missed. The time to perform the sieve for each block, and to calculate all gaps generated, was about 10.5 seconds for numbers in the range of 7.2×10^{13} . The largest prime in the last block processed to date is 72635119999997, so the table of first occurrence prime gaps is complete to that prime.

The following table lists all the first occurrence prime gaps found. This table agrees with all previously published results [2], [3], [1]. The maximal first occurrence prime gaps are marked with an asterisk (*).

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	following gap the prime	gap	following the prime	gap	following the prime
* 2	3	102	1444309	202	107534587
* 4	7	104	1388483	204	112098817
* 6	23	106	1098847	206	232423823
* 8	89	108	2238823	208	192983851
10	139	110	1468277	* 210	20831323
12	199	* 112	370261	212	215949407
* 14	113	* 114	492113	214	253878403
16	1831	116	5845193	216	202551667
* 18	523	* 118	1349533	218	327966101
* 20	887	120	1895359	* 220	47326693
* 22	1129	122	3117299	* 222	122164747
24	1669	124	6752623	224	409866323
26	2477	126	1671781	226	519653371
28	2971	128	3851459	228	895858039
30	4297	130	5518687	230	607010093
32	5591	* 132	1357201	232	525436489
* 34	1327	134	6958667	* 234	189695659
* 36	9551	136	6371401	236	216668603
38	30593	138	3826019	238	673919143
40	19333	140	7621259	240	391995431
42	16141	142	10343761	242	367876529
* 44	15683	144	11981443	244	693103639
46	81463	146	6034247	246	555142061
48	28229	* 148	2010733	* 248	191912783
50	31907	150	13626257	* 250	387096133
* 52	19609	152	8421251	252	630045137
54	35617	* 154	4652353	254	1202442089
56	82073	156	17983717	256	1872851947
58	44293	158	49269581	258	1316355323
60	43331	160	33803689	260	944192807
62	34061	162	39175217	262	1649328997
64	89689	164	20285099	264	2357881993
66	162143	166	83751121	266	1438779821
68	134513	168	37305713	268	1579306789
70	173359	170	27915737	270	1391048047
* 72	31397	172	38394127	272	1851255191
74	404597	174	52721113	274	1282463269
76	212701	176	38089277	276	649580171
78	188029	178	39389989	278	4260928601
80	542603	* 180	17051707	280	1855047163
82	265621	182	36271601	* 282	436273009
84	461717	184	79167733	284	1667186459
* 86	155921	186	147684137	286	2842739311
88	544279	188	134065829	* 288	1294268491
90	404851	190	142414669	290	1948819133
92	927869	192	123454691	* 292	1453168141
94	1100977	194	166726367	294	5692630189
* 96	360653	196	70396393	296	5260030511
98	604073	198	46006769	298	8650524583
100	396733	200	378043979	300	4758958741

gap	following the prime	gap	following the prime	gap	following the prime
302	6675573497	402	44293346177	502	1258535916601
304	2433630109	404	144999022043	504	747431049203
306	3917587237	406	49306638307	506	1339347750707
308	5490459101	408	134664608389	508	1841086484491
310	4024713661	410	98276144093	510	2209016910131
312	6570018347	412	124221464119	512	1999066711391
314	8948418749	414	49914935177	* 514	304599508537
316	12109172293	416	121972158437	* 516	416608695821
318	4372999721	418	129300694603	518	2296497058133
* 320	2300942549	420	82490815123	520	2336167262449
322	7961074441	422	280974865361	522	1214820695701
324	10958687879	424	264495345259	524	2256065636039
326	5837935373	426	180265084403	526	1620505682371
328	13086861181	428	219950168411	528	1529741785139
330	6291356009	430	250964194171	530	2205492372371
332	5893180121	432	87241770619	* 532	461690510011
334	30827138509	434	127084569923	* 534	614487453523
* 336	3842610773	436	367459059871	536	5371284217763
338	22076314313	438	101328529441	538	2122536905311
340	8605261447	440	141846299801	* 540	738832927927
342	12010745569	442	417470554687	542	2707053887651
344	19724087267	444	36172730063	544	2652427555639
346	11291401837	446	190418076203	546	2164206784721
348	17002876643	448	402872474743	548	3380058341279
350	16808773277	450	63816175447	550	2496646209271
352	30750892801	452	466855187471	552	2210401546601
* 354	4302407359	454	202530831163	554	3621153039299
356	24355072517	* 456	25056082087	556	4338624362173
358	16792321339	458	304040251469	558	5263973982823
360	20068818197	460	131956235563	560	4260199366373
362	35877724601	462	400729567081	562	2081209441279
364	25425617317	* 464	42652618343	564	1480064231153
366	20108776097	466	565855695631	566	4897642179197
368	51430518413	* 468	127976334671	568	6010330572331
370	59942358571	470	681753256133	570	4442109925217
372	20404137779	472	865244709607	572	5441175346967
374	23064761663	* 474	182226896239	574	3108794067079
376	16161669787	476	725978934347	576	8817792098461
378	38116957819	478	367766547571	578	7552870120721
380	23323808741	480	482423533897	580	9383081340541
* 382	10726904659	482	1051602787181	* 582	1346294310749
* 384	20678048297	484	767644374817	584	6993007248239
386	35238645587	* 486	241160624143	586	6364466316577
388	156798792223	488	1275363152099	* 588	1408695493609
390	53241805651	* 490	297501075799	590	20761252261751
392	117215204531	492	910361180689	592	3410069454097
* 394	22367084959	494	804541404419	594	5499789519863
396	50806025873	496	880318998907	596	10487995154603
398	40267027589	498	428315806823	598	5614481773561
400	47203303159	* 500	303371455241	600	4872634110067

gap	following the prime	gap	following the prime	gap	following the prime
* 602	1968188556461	642	14141685364577	690	15712145060693
604	5439564948583	644	41433781612373	692	43603583701331
606	12112937821403	646	51027160468351	694	62088893223739
608	20767330530329	648	9787731507761	696	23333096984797
610	9105981382177	650	5120731250207	698	33785727371453
612	13397310636587	* 652	2614941710599	700	14998144209049
614	17418754709747	654	54916086007427	702	19786638118631
616	8095224517651	656	65862966031241	706	35625755878981
618	4165633395149	658	39883132551139	714	49639993268989
620	9344093035661	660	10653514291843	* 716	13829048559701
622	13059969946711	662	11082394066097	724	59692452738913
624	24923033918059	664	38745678640849	728	57723522921803
626	33605480400197	666	18691113008663	730	24179270588173
628	34140047613391	668	28340177964929	740	57360609786539
630	12644461143649	670	47137733785861	744	42610475373269
632	45678685880759	672	26456514142099	756	70099348325843
634	17659394869309	* 674	7177162611713	758	47581758352253
636	9483480841753	678	37970994487033	* 766	19581334192423
638	17499522060011	682	41459443375351	* 778	42842283925351
640	22099408494481	684	30236507704253		

Cray Research, Inc.
1440 Northland Drive
Mendota Heights, Minnesota 55120

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