CORRIGENDA


We are grateful to Carla Manni for pointing out an error in the proof of Lemma 1. This necessitates corrections in the statement of Lemma 1 and its consequences. In addition to the notations used in the paper, we set

\[ \beta_d^r = \left( \frac{d-r+1}{2} - 2 \left( \frac{r+1}{2} \right) + \left( \frac{r-(d-2r)+1}{2} \right) \right)_+, \]

\[ \gamma_d^r = \left( \frac{d+2}{2} - \left( \frac{2r+2}{2} \right) \right)_+, \]

\[ \sigma_d^r(n) = \begin{cases} 
   \sum_{j=r+1}^{d-r} [r+j+1-nj]_+ & \text{for } d \geq 3r+1, \\
   \sum_{j=r+1}^{d-r} [r+j+1-n(j+d-3r-1)]_+ & \text{for } 2r+1 \leq d \leq 3r, \\
   0 & \text{otherwise.} 
\end{cases} \]

The correct formulation of Lemma 1 is then as follows.

**Lemma 1.** Let \( \Delta_0 \) be a rectilinear grid partition, as described. Then

\[ \dim S_d^r(\Delta_0) = \left( \frac{d+2}{2} \right) + [n_0 \beta_d^r + \sigma_d^r(n_0) - \gamma_d^r]_+. \]

In particular, it is clear that for \( d \geq 4r+1 \) or \( d \leq 2r \) the formulation of this lemma reduces to the formulation of Lemma 1 in the original paper. For \( 2r+1 \leq d \leq 4r \), the mistake in the proof of Lemma 1 occurs in the formulation of equation (24). A correct statement is the following. For \( d \geq 3r+1 \), we have

\[ \text{rank } H' = \min \left( \sum_{j=r+1}^{d-2r} \text{rank } H_j + \sum_{j=d-2r+1}^{d-r} \text{rank } [B_{r+1}^j \cdots B_{j-1}^j B_{j-1}^j H_j], n_0 \beta_d^r \right) \]

\[ = \min \left( \sum_{j=r+1}^{d-r} (r+j+1-[r+j+1-N_0 j]_+), n_0 \beta_d^r \right) \]

\[ = n_0 \beta_d^r - [n_0 \beta_d^r - \gamma_d^r + \sigma_d^r(n_0)]_+. \]
For $2r < d < 3r + 1$, we have
\[
\text{rank } H' = \min \left( \text{rank } H_{r+1} + \sum_{j=r+2}^{d-r} \text{rank} [D_{r+1} \cdots D_{j-1} H_j], n_0 \beta_d^r \right)
= n_0 \beta_d^r - [n_0 \beta_d^r - \gamma_d^r + \sigma_d^r (N_0)]_+.
\]

The above reformulation of Lemma 1 entails changes in the statements of Lemma 2, Theorems 1, 2, 3, 4 and three of the corollaries. The correct statements of these results are given below.

**Lemma 2.** Let $\Delta$ be a rectilinear grid partition and $\epsilon = 0$ or 1 as described. Then
\[
\dim \widehat{S}_d^r (\Delta) = \binom{d + 2}{2} + E_i \beta_d^r - \epsilon [\gamma_d^r - \sigma_d^r (N_0)] + (\epsilon [\gamma_d^r - \sigma_d^r (N_0)] - n_0 \beta_d^r)_+.
\]

**Theorem 1.** Let
\[
D_d^r = \dim \widehat{S}_d^r (\Delta) - \binom{d + 2}{2} - E_i \beta_d^r + V_i \gamma_d^r.
\]
Then
\[
\sum_{i=1}^{V_j} \delta_i \leq D_d^r \leq \sum_{i=1}^{V_j} \beta_i,
\]
where $\delta_i = \sigma_d^r (e_i) + (\gamma_d^r - \sigma_d^r (e_i) - d_i \beta_d^r)_+$, $\beta_i = \sigma_d^r (e_i) + (\gamma_d^r - \sigma_d^r (e_i) - d_i \beta_d^r)_+$ with $d_i$ and $e_i$ denoting the number of edges and the number of edges with different slopes attached to the vertex $A_i$ respectively, and $\beta_i$ and $\delta_i$ denoting the number of edges and the number of edges with different slopes attached to the vertex $A_i$ but not $A_j$, $j < i$, respectively, where $i = 1, \ldots, V_j$. In particular, for $d \geq 3r + 1$ and all $\epsilon_i \geq 2$,
\[
\dim \widehat{S}_d^r (\Delta) = \binom{d + 2}{2} + E_i \beta_d^r - V_i \gamma_d^r.
\]

**Theorem 2.** Let
\[
F_d^r (n) := (n \beta_d^r + \sigma_d^r (n) - \gamma_d^r)_+.
\]
Then
\[
\dim \widehat{S}_d^r (\Delta_e) = \binom{d + 2}{2} + L \beta_d^r + \sum_{i=1}^{V_i} F_d^r (l_i).
\]

**Corollary 1.** Let $d \geq 3r + 1$. Then
\[
\dim \widehat{S}_d^r (\Delta_{mn}^{(2)}) = mn \left[ 6 \binom{d - r + 1}{2} - 6 \binom{r + 1}{2} - 2 \binom{d + 2}{2} + 2 \binom{2r + 2}{2} \right] + (m + n) \left[ 5 \binom{d - r + 1}{2} - 10 \binom{r + 1}{2} - \binom{d + 2}{2} + \binom{2r + 2}{2} \right] + \binom{2r + 2}{2} + 4 \binom{d - r + 1}{2} - 8 \binom{r + 1}{2}.
\]
Corollary 2. Let $\Delta_m^{(1)}$ be a uniform type-1 triangulation of $\Omega_R$. Then
\[ \dim S_d(\Delta_m^{(1)}) = \left(\frac{d + 2}{2}\right) + mn[3\beta_d^r - \gamma_d^r + \sigma_d(3)] + (2m + 2n + 1)\beta_d^r. \]

Corollary 3. Let $\Delta_m^{(2)}$ be a uniform type-2 triangulation of $\Omega_R$. Then
\[ \dim S_d(\Delta_m^{(2)}) = \left(\frac{d + 2}{2}\right) + mn[6\beta_d^r - 2\gamma_d^r + \sigma_d(2) + \sigma_d(4)] + (2m + 2n + 1)\beta_d^r. \]

In the following, we need the notations:
\[ \beta_d^r = \left[\left(\frac{d - r + 1}{2}\right) - 2\left(\frac{\rho - r + 1}{2}\right) + \left(\frac{\rho - r - (d - \rho) + 1}{2}\right)\right]_, \]
\[ \gamma_d^r = \left[\left(\frac{d + 2}{2}\right) - \left(\frac{\rho + 2}{2}\right)\right]_+ , \]
\[ \sigma_d^{r,\rho}(n) = \begin{cases} \sum_{j=\rho-r+1}^{d-r}(r + j + 1 - nj)_+ & \text{for } d > 2\rho - r, \\ \sum_{j=\rho-r+1}^{d-r}(r + j + 1 - n(j + d - 2\rho + r - 1))_+ & \text{for } \rho < d \leq 2\rho - r, \\ 0 & \text{otherwise.} \end{cases} \]

Theorem 3. Let
\[ D_d^{r,\rho} = \dim S_d^{r,\rho}(\Delta) - \left(\frac{d + 2}{2}\right) + V_i \gamma_d^r - E_i \beta_d^{r,\rho}. \]
Then
\[ \sum_{i=1}^{V_i} \sigma_i^{\rho} \leq D_d^{r,\rho} \leq \sum_{i=1}^{V_i} \sigma_i^{\rho}, \]
where $\sigma_i^{\rho} = \sigma_d^{r,\rho}(e_i) + (\gamma_d^r - \sigma_d^{r,\rho}(e_i) - d_i\beta_d^{r,\rho})_+$, $\sigma_i^{\rho} = \sigma_d^{r,\rho}(\tilde{e}_i) + (\gamma_d^r - \sigma_d^{r,\rho}(\tilde{e}_i) - \tilde{d}_i\beta_d^{r,\rho})_+$, and $e_i$, $d_i$, $\tilde{e}_i$ and $\tilde{d}_i$ have been defined in Theorem 1.

Theorem 4. Let $\rho \geq r$ and
\[ F_d^{r,\rho}(n) := (n\beta_d^{r,\rho} + \sigma_d^{r,\rho}(n) - \gamma_d^r)_+. \]
Then
\[ \dim S_d^{r,\rho}(\Delta_c) = \left(\frac{d + 2}{2}\right) + L \beta_d^{r,\rho} + \sum_{i=1}^{V_i} F_d^{r,\rho}(l_i). \]

Charles K. Chui
Tian Xiao He